SYDNEY GRAMMAR SCHOOL

NAME _____ MATHS MASTER _____

2023 Annual Examination

Form III Mathematics

Tuesday 31st October 2023

Session C

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Total Marks: 104

Questions 1 – 8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.

3A: BLR	3B: NJL	3C: PKS	3D: PC
3E: WJW/BR	3F: LRP/GMC	3G: OMD	3H: NWS
3I: LYL	3J: YH		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 202 pupils

Writer: BLR

QUESTION ONE (13 marks) Start a new page. **Marks**

(a) Find the value of $\frac{a}{3} + 4b$ when $a = 27$ and $b = -2$. 1

(b) State the y-intercept of $y = -3x + 10$. 1

(c) Evaluate $\tan 84^\circ$. Give your answer correct to two decimal places. 1

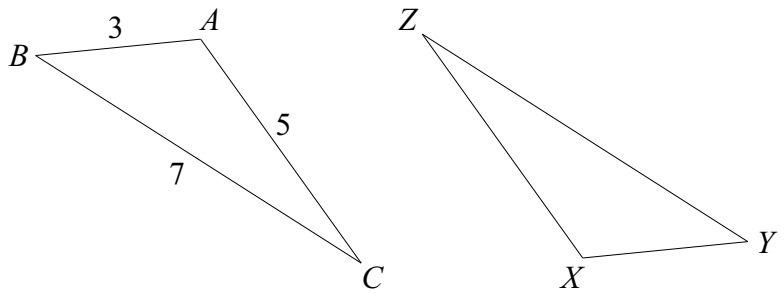
(d) Simplify:

(i) $\left(x^7\right)^2$ 1

(ii) $\frac{6y^5}{3y^4}$ 1

(iii) $\frac{9}{z} + \frac{8}{z}$ 1

(e)



In the diagram above, $\triangle ABC \equiv \triangle XYZ$. You do **not** need to prove this. 1

Given that $AB = 3$, $BC = 7$, and $AC = 5$, write down the length of YZ.

(f) Write down the radius of the circle with equation $x^2 + y^2 = 16$. 1

(g) Solve $3(x + 2) < 12$. 1

(h) Express each of the following with positive indices:

(i) x^{-4} 1

(ii) $\sqrt{y}$ 1

(i) Expand and simplify:

(i) $(m - 8)(m + 7)$ 1

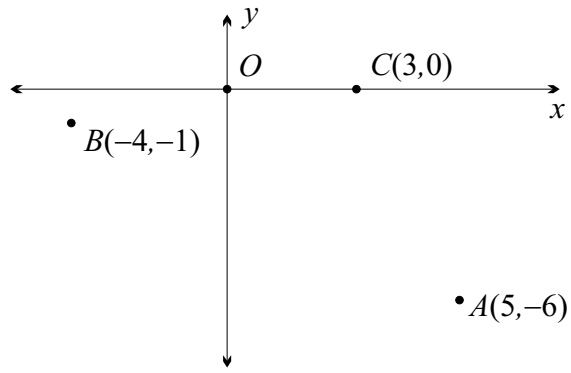
(ii) $(n + 6)^2$ 1

QUESTION TWO (13 marks) Start a new page. **Marks**

(a) Calculate the surface area of a cube with side lengths of 5 m. [1]

(b) Last week, Ash worked for 38 hours at his normal hourly rate and worked an additional 3 hours at time-and-a-half. Given that his normal hourly rate of pay is \$19.20, calculate Ash's total earnings for the week. [2]

(c)



The diagram above shows the origin O and the points A, B, and C.

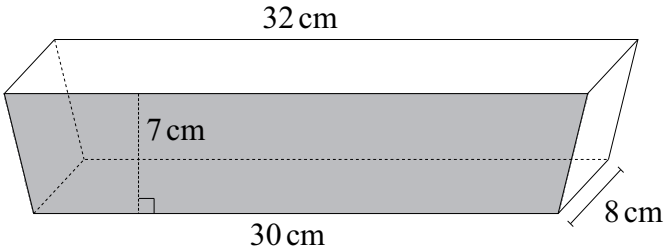
(i) Find the midpoint of AC. [2]

(ii) Calculate the gradient of BC. [1]

(iii) Determine the length of AB. Give your answer correct to the nearest integer. [2]

(iv) Write down the equation of the line passing through O and C. [1]

(d)

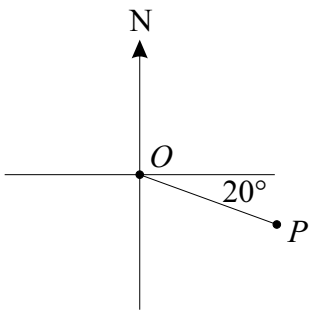


The diagram above shows a flower planter in the shape of a trapezoidal prism.

(i) Calculate the area of the shaded trapezoidal cross-section. [2]

(ii) Hence, calculate the volume of the flower planter. [1]

(e)

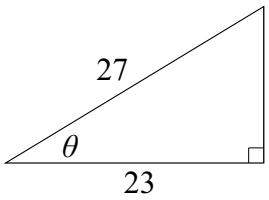


Write down the true bearing of P from O. [1]

QUESTION THREE (13 marks) Start a new page. **Marks**

(a) Solve $x(x-6)=0$. 2

(b)



Find the size of the angle θ in the triangle above. Give your answer correct to the nearest degree. 2

(c)

Blood Type	*O*	*A*	*B*	*AB*
Frequency	19	21	8	4

The frequency table above shows the blood types of 52 donors at a blood donation clinic.

(i) Write down the mode. 1

(ii) Rhonda wants to present the data in a stem-and-leaf plot. Is this an appropriate choice? Briefly justify your answer. 1

(d) Factorise each expression.

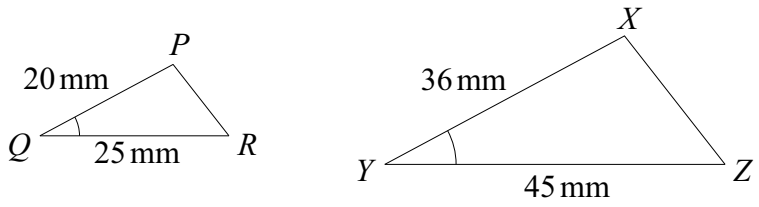
(i) $8a^2-16a$ 1

(ii) b^2-64 1

(iii) $n^2-3n-28$ 1

(iv) $x(x-4)+y(x-4)$ 1

(e)



State the similarity test that could be used to prove that $\triangle PQR$ is similar to $\triangle XYZ$. You do **not** need to prove this result. 1

(f) Solve the following equations simultaneously using the elimination method: 2

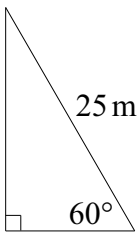
$a+b=21$

$a-b=5$

QUESTION FOUR (13 marks) Start a new page. **Marks**

(a) Natalie is a carpenter who makes and sells furniture. She sells coffee tables for \$396. How much does it cost Natalie to make a coffee table, given that she makes a profit of 10% on each table? 2

(b)



The diagram above shows a 25 m long guy-wire attached to the top of a radio mast. The guy-wire makes an angle of 60° with the ground. Calculate the exact height of the radio mast. 2

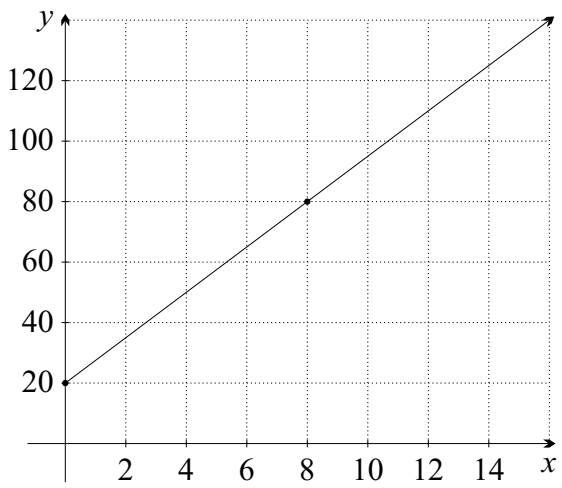
(c) Solve the following equations simultaneously using the substitution method: 2

$$x + 6y = 7$$
$$3x + 2y = 13$$

(d) Factorise $5x^2 + 17x + 6$. 2

(e) Solve $2x^2 - 8x = 10$. 2

(f)



The graph above shows the temperature y, in degrees Celsius, inside an oven x minutes after it is turned on. The initial temperature inside the oven is 20°C. After 8 minutes, the temperature inside the oven has increased to 80°C.

(i) Calculate the gradient of the line. 1

(ii) Write down the equation of the line. 1

(iii) Use your equation from part (ii) to predict the temperature inside the oven 20 minutes after it is turned on, assuming that the oven continues to heat at the same rate. 1

QUESTION FIVE (13 marks) Start a new page. **Marks**

(a) Simplify $\dfrac{x^2+9x+18}{x+6} \div \dfrac{x^2-9}{5x-15}$. [3]

(b) Each year, Australian cherry growers produce 2.015×10^7 kg of cherries. If the average weight of a single cherry is 5×10^{-3} kg, approximately how many cherries are produced in Australia each year? Give your answer in scientific notation correct to three significant figures. [2]

(c) The table below shows the income tax rates for Australian residents.

Taxable income	Tax on this income
$0 - $18,200	Nil
$18,201 - $45,000	19 cents for each $1 over $18,200
$45,001 - $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 - $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,000 and over	$51,667 plus 45 cents for each $1 over $180,000

(i) Teddy is an Australian resident. During the most recent financial year, his total income was $46,096. Calculate the total amount of income tax Teddy owes. Do not include the Medicare levy. [2]

(ii) During the year, Teddy paid a total of $7980 in PAYG instalments. Is Teddy entitled to a refund, or does he owe more in tax? Calculate the amount to be refunded or paid. [1]

(d) Find the equation of the line that is perpendicular to $y = -\dfrac{1}{7}x + 5$ and passes through the point $(-2, 9)$. Give your answer in gradient-intercept form. [2]

(e) Solve $x^2 - 14x + 5 = 0$ by completing the square. [3]

QUESTION SIX (13 marks) Start a new page. **Marks**

(a) Simplify $\dfrac{1}{x^2-2x-3}-\dfrac{1}{4x-12}$. [3]

(b) Data was collected from a sample of Form III pupils about the number of hours they spend playing video games each night. The data is summarised in the frequency distribution table below, where x is the number of hours and f is the frequency.

x	f
0	9
1	12
2	6
3	N
4	1

(i) Calculate the range. [1]

(ii) Given that the mean is $1\frac{1}{4}$ hours spent playing video games each night, find the value of N. [3]

(c) Simplify $\sqrt{\dfrac{9x}{8}}-\sqrt{\dfrac{8x}{9}}$. Express your answer with a rational denominator. [3]

(d) A right-angled triangle has an acute angle of θ. In this triangle, $\cos\theta=\dfrac{2}{5}$.

(i) Draw a diagram to represent a possible triangle and use Pythagoras' theorem to find the exact length of the third side. [2]

(ii) Hence, find the exact value of $\tan\theta$. [1]

QUESTION SEVEN (13 marks) Start a new page. **Marks**

(a) A park ranger observed the number of wallabies that visited a water hole and recorded the total number each day for six days. Her data set has a mean of 10 and a median of 12. Find a possible data set that gives the smallest possible value of the range. Briefly explain your answer. [2]

(b) Consider the equation $\dfrac{x-3}{9} = \dfrac{n\,(x+10)}{5}$.

(i) Find the value of x when $n = 0$. [1]

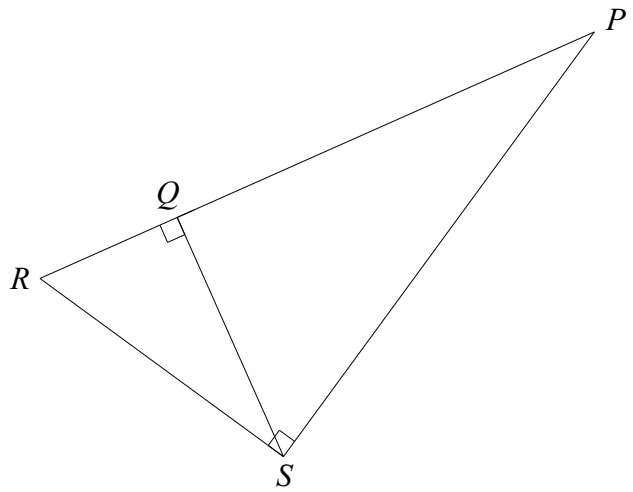
(ii) Find the value of n for which the equation has no solution for x. [2]

(c) Link is standing on top of Hebra Peak, a mountain that is 4260 m above sea level. The angle of depression from the top of Hebra Peak to Talonto Peak is $4°$. On Link's map, these two peaks are represented by points that are 6 cm apart.

(i) Given that the scale on Link's map is $1 : 180\,000$, calculate the horizontal distance between the peaks. Give your answer in metres. [1]

(ii) How many metres above sea level is Talonto Peak? Give your answer correct to the nearest metre. [3]

(d)



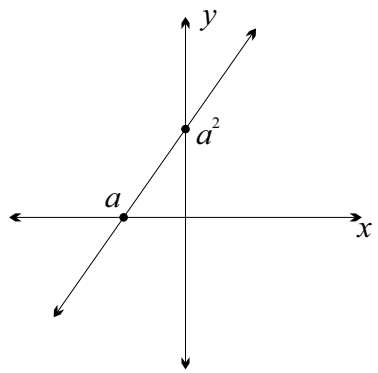
In the diagram above, $\angle RQS$ and $\angle RSP$ are right-angles.

(i) Prove that $\triangle PRS \;|||\; \triangle SRQ$. [2]

(ii) If $PQ = 16$ and $QR = 2$, find the length of RS. Give a geometric reason for your answer. [2]

QUESTION EIGHT (13 marks) Start a new page. **Marks**

(a)



A line with a positive gradient has x-intercept at a and y-intercept at a^2, as shown in the diagram above.

(i) Write down an expression for the gradient of the line in terms of a. 1

(ii) Given that the line passes through the point $(4, 3)$, find the value of a. 3

(b) Gabe won a programming competition and was awarded P dollars in prize money. He uses half of his prize money to purchase a new motorcycle, which depreciates at a compound rate of $R\%$ each year. He invests the other half of his prize money in a term deposit account which earns interest at the same rate of $R\%$ p.a. compounded annually. At the end of two years, the total value of Gabe's term deposit is worth \$700 more than his motorcycle, and the combined value of the two is \$17 507.

(i) Show that the situation can be represented using the pair of equations below, where P dollars is the amount of prize money Gabe won and $r = R\%$. 2

$$2Pr = 700$$
$$P\left(1 + r^2\right) = 17\,507$$

Make clear use of the compound interest and depreciation formulas in your work.

(ii) By solving the equations from part (i) simultaneously, find the value of r, given that $r \leq 1$. 2

(iii) Hence, find the original cost of the motorcycle. 1

(c) (i) Sophie Germain was a French mathematician in the 17th century. She found that expressions of the form $a^4 + 4b^4$ can be factorised as: 1

$$a^4 + 4b^4 = \left(a^2 + 2ab + 2b^2\right)\left(a^2 - 2ab + 2b^2\right)$$

Show that the factorisation is correct by expanding and simplifying the expression on the right-hand side of the factorisation above.

(ii) By using the result from part (i), simplify $\dfrac{(x+1)^4 + 4x^4}{x^2 + (2x+1)^2} - \dfrac{(x-1)^4 + 4x^4}{x^2 + (2x-1)^2}$. 3

END OF PAPER

B L A N K P A G E

Form III Annual 2023

Question 1

a) 1 ✓

b) 10 ✓

c) 9.51 ✓ must be to 2 d.p.

d) i) x^{14} ✓

ii) $2y$ ✓

iii) $\frac{17}{z}$ ✓

e) $YZ = 7$ ✓ geometric reason not required

f) 4 ✓

g) $x + 2 < 4$

$x < 2$ ✓

h) i) $\frac{1}{x^4}$ ✓

ii) $y^{\frac{1}{2}}$ ✓

i) i) $m^2 - m - 56$ ✓

ii) $n^2 + 12n + 36$ ✓

Question 2

a) $A = 6 \times 5^2$

$= 150\ m^2$ ✓ ok if no units

b) ✓ $38 \times 19.20 + 1.5 \times 3 \times 19.20$

$= \$816$ ✓ correct total

c) i) ✓ ✓ $(4, -3)$ must have brackets

ii) $\frac{1}{7}$ ✓

iii) $AB = \sqrt{9^2 + 5^2}$ ✓

$\doteq 10$ ✓ nearest integer

iv) $y = 0$ ✓

d) i) $A = \frac{7}{2}(30 + 32)$ ✓ correct working

$= 217\ cm^2$ ✓ answer with units

ii) $V = 217 \times 8$

$= 1736\ cm^3$ ✓ ok if no/incorrect units

e) $110°\ T$ ✓ ok if T omitted

Question 3

a) $x = 0$, $x = 6$ ✓ ✓

b) $\cos\theta = \frac{23}{27}$ ✓

$\theta \doteq 32°$ ✓ ignore rounding error

c) i) A ✓

ii) no; need quantitative data for stem-and-leaf ✓

d) i) $8a(a - 2)$ ✓

ii) $(b + 8)(b - 8)$ ✓

iii) $(n - 7)(n + 4)$ ✓

iv) $(x - 4)(x + y)$ ✓

e) SAS ✓

f) ①: $a + b = 21$ ②: $a - b = 5$ elimination method

① + ②: $2a = 26$ ① − ②: $2b = 16$ ✓

$a = 13$ sub into ① $b = 8$

$b = 8$ $a = 13$ ✓

Question 4

a) 110% is \$396 ✓ any correct start

10% is \$36

100% is \$360 ∴ costs her \$360 ✓

b) $\sin 60° = \frac{h}{25}$ ✓

$h = 25\sin 60°$

$h = \frac{25\sqrt{3}}{2}\ m$ ✓ ok if no units. must be exact

c) ①: $x = 7 - 6y$ ②: $3x + 2y = 13$ substitution method

sub ① into ②: $3(7 - 6y) + 2y = 13$ ✓

$-16y = -8$

$y = \frac{1}{2}$, sub into ①

$x = 4$ ✓

✓ correct start P: 30 S: 17

d) $(5x + 2)(x + 3)$ ✓ factorised. ok if no working

e) $2(x - 5)(x + 1) = 0$ ✓

$x = 5, -1$ ✓

f) i) $\frac{15}{2}$ or 7.5 ✓

ii) $y = 7.5x + 20$ ✓ m from (i) and y-int

iii) $y = 7.5 \times 20 + 20$ using equation from (ii)

$= 170°C$ ✓ ok if no units

Question 5

a) $\frac{(x+6)(x+3)}{x+6} \times \frac{5(x-3)}{(x+3)(x-3)}$ ✓ ✓

$= 5$ ✓

b) $\frac{2.015 \times 10^7}{5 \times 10^{-3}}$ ✓ or evaluating quotient w/calc gives: $= 4030000000$

$= 0.403 \times 10^{10}$ $= 4.03 \times 10^9$

$= 4.03 \times 10^9$ cherries ✓

c) i) $5092 + 0.325(46096 - 45000)$ ✓ evidence of correct tax bracket

$= \$5448.20$ ✓

ii) entitled to a refund of $2531.80 ✓

d) gradient of new line is 7 ✓

equation:

$y - 9 = 7(x+2)$

$y - 9 = 7x + 14$

$y = 7x + 23$ ✓

e) $x^2 - 14x + 7^2 = 7^2 - 5$

$(x-7)^2 = 44$ ✓

$x - 7 = \pm\sqrt{44}$ ✓

$x = 7 \pm 2\sqrt{11}$ ✓ with simplified surd

b) i) 4 ✓

ii) $\frac{0\times9 + 1\times12 + 2\times6 + 3N + 4\times1}{9+12+6+N+1} = \frac{5}{4}$ ✓

$\frac{28 + 3N}{28 + N} = \frac{5}{4}$

$112 + 12N = 140 + 5N$ ✓

$7N = 28$

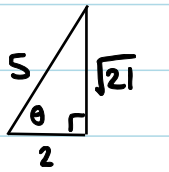
$N = 4$ ✓

c) $\frac{\sqrt{9x}}{\sqrt{8}} - \frac{\sqrt{8x}}{\sqrt{9}}$ 1 mark for demonstrating each skill, in any order.

$= \frac{9\sqrt{x} - 8\sqrt{x}}{3\times2\sqrt{2}}$ ✓ common denominator

$= \frac{\sqrt{x}}{6\sqrt{2}}$ ✓ some simplifying

$= \frac{\sqrt{2x}}{12}$ ✓ rational denominator

d) i) 

✓ diagram ✓ side length

ii) $\tan\theta = \frac{\sqrt{21}}{2}$ ✓

Question 6

a) $\frac{1}{(x-3)(x+1)} - \frac{1}{4(x-3)}$ ✓ factorising both denominators

$= \frac{4 - (x+1)}{4(x-3)(x+1)}$ ✓ forming a common denominator

$= \frac{3-x}{4(x-3)(x+1)}$

$= \frac{-(x-3)}{4(x-3)(x+1)}$

$= \frac{-1}{4(x+1)}$ ✓

Question 7

a) For mean to be 10, must be 60 wallabies in total. For median to be 12, middle two values must add to 24. Smallest range when middle two both 12. Make upper two 12 as well.

—, —, 12, 12, 12, 12

As 60 wallabies in total, lower two must add to 12. Need these two as close to each other as possible, so must both be 6. Smallest range is 6, with final data set: 6, 6, 12, 12, 12, 12.

✓ some correct progress

✓ data set, with justification

b) i) $\frac{x-3}{9} = 0$

$x = 3$

ii) $5(x-3) = 9_n(x+10)$

$5x - 15 = 9_n x + 90_n$

$(5 - 9_n)x = 90_n + 15$

$x = \frac{90_n + 15}{5 - 9_n}$ ✓ making x the subject

no solution when $5 - 9_n = 0$

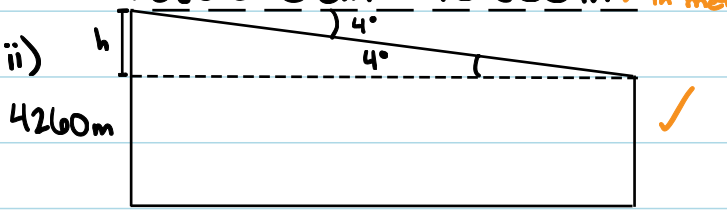
$n = \frac{5}{9}$ ✓

c) i) $1 : 180000$

$6 : 1080000$

$1080000\text{ cm} = 10800\text{ m}$ ✓ in metres

ii)



✓

10800 m

let h be the difference in height

$\tan 4° = \frac{h}{10800}$ ✓

$h = 10800 \tan 4$

$\doteq 755\text{ m}$

Talanto Peak's elevation $= 4260 - 755$

$= 3505\text{ m}$ ✓

d) i) in ΔPRS and ΔSRQ,

$\angle PSR = \angle SQR = 90°$ (given) ✓

$\angle PRS = \angle SRQ$ (common)

$\therefore \Delta PRS \,|||\, \Delta SRQ$ (AA) ✓

ii) $\frac{RQ}{RS} = \frac{RS}{RP}$ (matching sides of $\Delta PRS \,|||\, \Delta SRQ$) ✓

$\frac{2}{RS} = \frac{RS}{18}$

$RS^2 = 36$

$RS = 6$ ✓ (ok if missing this) $(RS \neq -6$ as length is positive$)$

Question 8

a) i) $m = \frac{a^2 - 0}{0 - a}$

$= -a$ ✓

ii) line: $y = -ax + a^2$

sub $(4, 3)$: $3 = -4a + a^2$ ✓

$a^2 - 4a + 4 = 7$

$(a-2)^2 = 7$

$a = 2 \pm \sqrt{7}$ ✓

$a = 2 - \sqrt{7}$ $(a \neq 2 + \sqrt{7}$ as $a < 0)$ ✓ with justification

b) i) value of motorcycle $= \frac{P}{2}(1-r)^2$

value of term deposit $= \frac{P}{2}(1+r)^2$ ✓ both

$\frac{P}{2}(1+r)^2 - \frac{P}{2}(1-r)^2 = 700$

$\frac{P}{2}\left[(1 + 2r + r^2) - (1 - 2r + r^2)\right] = 700$

$\frac{P}{2} \times 4r = 700$

$2Pr = 700$ ①

$\frac{P}{2}(1+r)^2 + \frac{P}{2}(1-r)^2 = 17507$

$\frac{P}{2}\left[1 + 2r + r^2 + 1 - 2r + r^2\right] = 17507$

$\frac{P}{2}(2 + 2r^2) = 17507$

$P(1 + r^2) = 17507$ ② ✓ both ① and ②

ii) sub ① into ②

$\frac{350}{r}(1 + r^2) = 17507$

$350(1 + r^2) = 17507r$ ✓

$50(1 + r^2) = 2501r$

$50r^2 - 2501r + 50 = 0$

$(50r - 1)(r - 50) = 0$

$r = 50, 0.02$

$r \neq 50$, as $r \leq 1$

$\therefore r = 0.02$, or 2% ✓ ok if $r \neq 50$ omitted

iii) sub $r = 0.02$ into ①

$P = \frac{700}{2 \times 0.02}$

$= \$17500$

$\therefore$ motorcycle cost \$8750 ✓

c) i) $(a^2+2ab+2b^2)(a^2-2ab+2b^2)$

$= a^4 - 2a^3b + 2a^2b^2 + 2a^3b - 4a^2b^2 + 4ab^3 + 2a^2b^2 - 4ab^3 + 4b^2$ ✓ expanding and simplifying

$= a^4 + 4b^4$

ii) $\dfrac{(x+1)^4+4x^4}{x^2+(2x+1)^2} - \dfrac{(x-1)^4+4x^4}{x^2+(2x-1)^2}$

✓ applying result from (i) to one fraction

$= \dfrac{[(x+1)^2+2x^2+2x(x+1)][(x+1)^2+2x^2-2x(x+1)]}{x^2+4x^2+4x+1} - \dfrac{[(x-1)^2+2x^2+2x(x-1)][(x-1)^2+2x^2-2x(x-1)]}{x^2+4x^2-4x+1}$

$= \dfrac{(x^2+2x+1+2x^2+2x^2+2x)(x^2+2x+1+2x^2-2x^2-2x)}{5x^2+4x+1} - \dfrac{(x^2-2x+1+2x^2+2x^2-2x)(x^2-2x+1+2x^2-2x^2+2x)}{5x^2-4x+1}$

✓ expanding at least 3 perfect squares

$= \dfrac{(5x^2+4x+1)(x^2+1)}{5x^2+4x+1} - \dfrac{(5x^2-4x+1)(x^2+1)}{5x^2-4x+1}$

$= x^2+1-(x^2+1)$

$= 0$ ✓