SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2022 Annual Examination

Form III Mathematics

Thursday 27th October 2022

Session A

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Total Marks: 104

Questions 1 – 8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Place everything inside this question booklet.

3A: OMD	3B: PJE	3C: JAZ/LYL	3D: YH
3E: CWG/RR	3F: AEA/WJM	3G: FMW	3H: NJL
3I: LRP/GMC	3J: BR		

Checklist

- 7 sheets of writing paper per boy
- Candidature: 198 pupils

Writer: NJL

QUESTION ONE (13 marks) Start a new page.

(a) Simplify $2\sqrt{2} + 3\sqrt{2}$.

(b) If $a = -4$ and $b = 2$, evaluate $\dfrac{a^2}{b}$.

(c) Solve $2x - 3 = x + 9$.

(d) State the gradient of the line $y = 4x - 1$.

(e) Use your calculator to find $\cos 32^\circ$. Give your answer correct to three decimal places.

(f) Factorise $6a - a^2$.

(g) Simplify $\dfrac{2x}{y} + \dfrac{x}{y}$.

(h) Given $\tan\theta = 1{\cdot}804$, find θ. Give your answer correct to the nearest degree.

(i) Express $\dfrac{2}{\sqrt{7}}$ with a rational denominator.

(j) Express b^{-4} with a positive index.

(k) Write $2{\cdot}15 \times 10^4$ as a basic numeral.

(l) Simplify $16^{\frac{1}{4}}$.

(m) Make x the subject of the formula $y = 6x - 5$.

QUESTION TWO (13 marks) Start a new page.

(a) Solve $(x-2)(x+7)=0$.

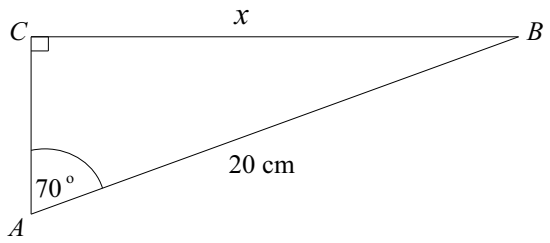
(b) Factorise:

(i) $x^2+7x+12$

(ii) x^2-36

(c) Find the length of the interval AB given $A(3,7)$ and $B(6,11)$.

(d)

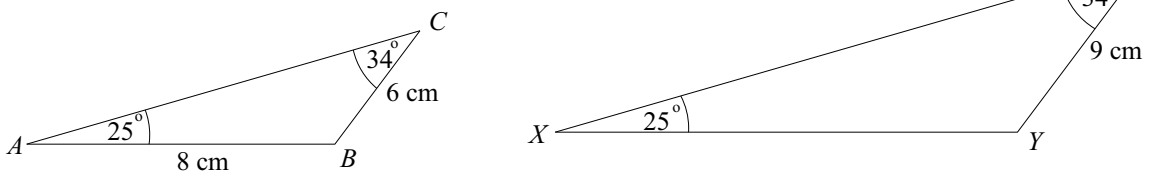


In $\triangle ABC$ above, find the value of x. Give your answer correct to one decimal place.

(e) (i) Solve $9-2x \geq 6$.

(ii) Show your solution on a number line.

(f)



In the diagram above $\triangle ABC|||\triangle XYZ$.

(i) State a similarity test which can be used to justify that $\triangle ABC|||\triangle XYZ$.

(ii) Hence calculate the length XY.

QUESTION THREE (13 marks) Start a new page.

The first 4 parts of this question are multiple-choice. Record your response A, B, C or D clearly on your answer page. You may give working, but only the response will be marked. There is only one correct answer for each.

(a) What are the coordinates of the midpoint of AB, given $A(-3, 1)$ and $B(5, -7)$?

(A) $(-1, -2)$

(B) $(-4, 4)$

(C) $(1, -3)$

(D) $(4, -4)$

(b) Which of the areas below is equivalent to $4\,\text{cm}^2$?

(A) $40\,\text{mm}^2$

(B) $400\,\text{mm}^2$

(C) $0{\cdot}4\,\text{mm}^2$

(D) $0{\cdot}04\,\text{mm}^2$

(c) Which expression is equivalent to $(x + 2)(x + 4) - (x - 2)(x - 5)$?

(A) $18 - x$

(B) $13x + 18$

(C) $-2 - x$

(D) $13x - 2$

(d) If $bx + 5d = ax + 2c$ then x is equal to which of the expressions below?

(A) $\dfrac{5d - 2c}{a - b}$

(B) $\dfrac{a + 2c - 5d}{b}$

(C) $\sqrt{\dfrac{2c - 5d}{b - a}}$

(D) $\dfrac{5d - (a + 2c)}{b}$

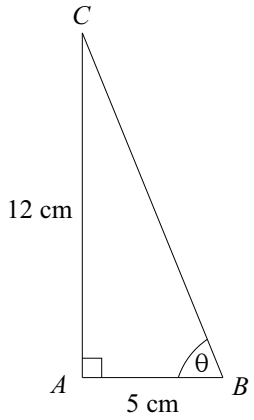
The question continues on the next page

QUESTION THREE (Continued)

(e) Write down the equation of the vertical line that passes through $(2, 3)$.

(f) Find the x-intercept and y-intercept of the line $y = -x + 3$.

(g)



In the triangle above, find the angle θ. Give your answer correct to the nearest degree.

(h) Solve the following equations simultaneously:

$$3x - y = 6$$
$$y = x + 2$$

(i) Write $\dfrac{a}{b+1} - \dfrac{a}{(b+1)^2}$ as a single fraction in simplest form.

QUESTION FOUR (13 marks) Start a new page.

(a) Show that the line $2x + y = 10$ is perpendicular to the line $y = \frac{1}{2}x + 2$.

(b) Solve the following equations simultaneously by the method of elimination:

$$3x + 5y = 8$$
$$x - 2y = -1$$

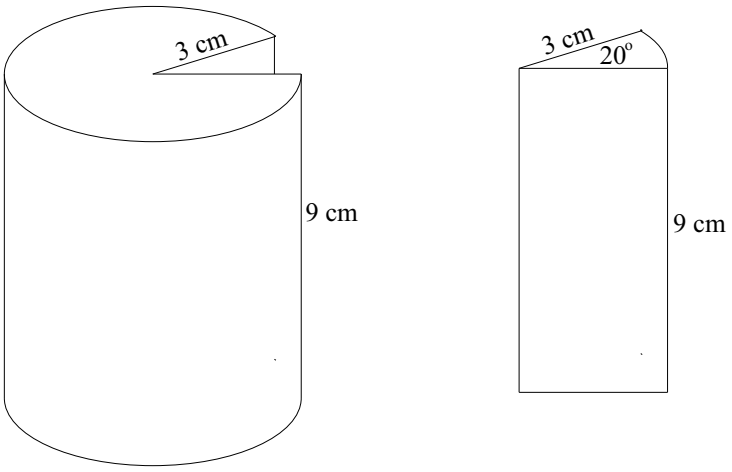
(c) Factorise and simplify $\dfrac{x^2 - 9x + 14}{x^2 - 2x - 35}$.

(d) Josh invests \$50 000 into an account which earns 6% interest p.a, compounded annually.

(i) What is the value of the account at the end of eight years? Give your answer correct to the nearest cent.

(ii) How many years will it take for him to accumulate a total interest of \$200 000? Give your answer rounded up to the nearest year.

(e)



remaining shape slice

The diagram above shows a cylinder of height 9 cm and radius 3 cm with a slice cut out of it, as shown. The slice that was cut out has a 20° angle at the centre of the original circular face.

Calculate the exact volume of the remaining shape once the slice has been removed.

QUESTION FIVE (13 marks) Start a new page.

(a) A straight line passes through the points $(1, 5)$ and $(5, -7)$.

(i) Find the gradient of the line.

(ii) Find the equation of the line.

(b) A hot chocolate costs \$2 more than a bubble tea. Three hot chocolates and five bubble teas cost \$70.

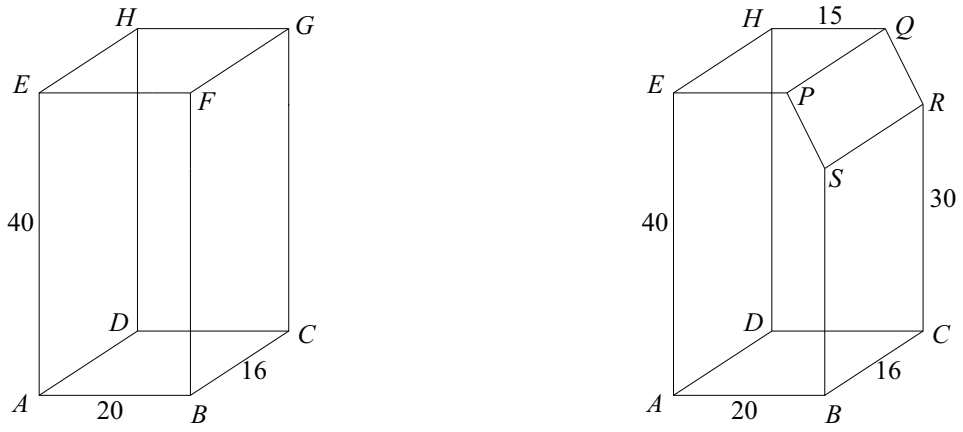
(i) Let the price of one hot chocolate be x dollars and the price of one bubble tea be y dollars. Write two equations to represent the information given.

(ii) Solve your equations to find the cost of each drink.

(c) Factorise $4x^2 + 16x + 7$.

(d) Use the method of completing the square to solve $x^2 - 4x - 4 = 0$.

(e)



The diagram on the left above shows a rectangular prism of clay. The prism on the right is what remains after a corner has been sliced off. The face $PQRS$ is a rectangle and $AB = 20\,\text{cm}$, $BC = 16\,\text{cm}$, $EA = 40\,\text{cm}$, $EP = HQ = 15\,\text{cm}$, and $SB = RC = 30\,\text{cm}$.

Calculate the exact surface area of the prism that remains.

QUESTION SIX (13 marks) Start a new page.

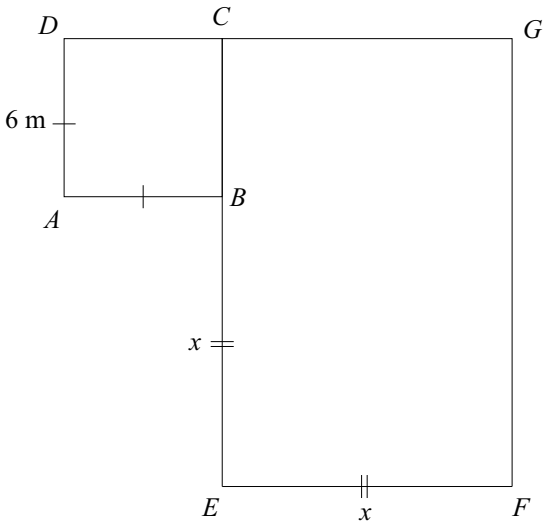
(a) The straight line l is parallel to $y = 5x - 8$ and passes through the point $(-1, -14)$. Find the equation of line l.

(b) Solve:

(i) $\dfrac{x}{20} = \dfrac{3}{x-4}$

(ii) $\dfrac{x}{x-2} - \dfrac{x+1}{x+4} = 1$

(c)



In the diagram above, $ABCD$ is a square of side length 6 metres and $EFGC$ is a rectangle. The points D, C and G form a straight line, and $EF = BE = x$ metres. The composite area of the square and rectangle is $223\,\text{m}^2$.

(i) Create a quadratic equation relating the area of the composite shape to x.

(ii) Solve your equation to find the value of x.

(d) Alex's average homework effort, measured in minutes per week, depreciates by 15% a week over a ten week term. How long did Alex work on homework in the first week of term if, in the tenth week of term, he completed two hours. Give your answer correct to the nearest minute.

QUESTION SEVEN (13 marks) Start a new page.

(a) The points $A\left(2+\sqrt{2},-1\right)$, $B\left(8+\sqrt{2},3\right)$, $C\left(6+\sqrt{2},6\right)$ and $D\left(\sqrt{2},2\right)$ are the vertices of the quadrilateral $ABCD$.

Prove that $ABCD$ is a rectangle.

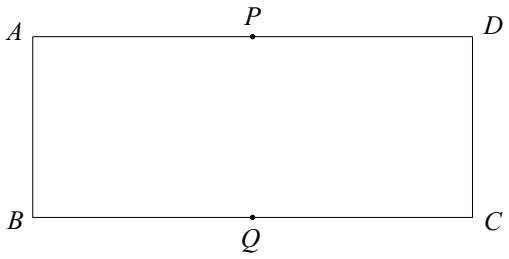
(b) A passenger on a train travelling due north and in a straight line at 160 km/h sees a light from a house on the horizon. At 9 pm the light is due east of the passenger's position. At 9:45 pm the light is on a bearing of 150°T from the passenger's position.

(i) Calculate how far the passenger travelled between 9 pm and 9:45 pm.

(ii) How far away is the light from the passenger at 9:45 pm? Give your answer as an exact value.

(c) Simplify $\dfrac{3x^2-12}{30x+15} \div \dfrac{2x^2-3x-2}{4x^2+4x+1}$.

(d)



Consider the rectangle $ABCD$ above.

The points P and Q are the midpoints of AD and BC respectively.

Prove, with full geometric reasoning, that PQ bisects BD.

(e) Name a special quadrilateral which is not necessarily a trapezium.

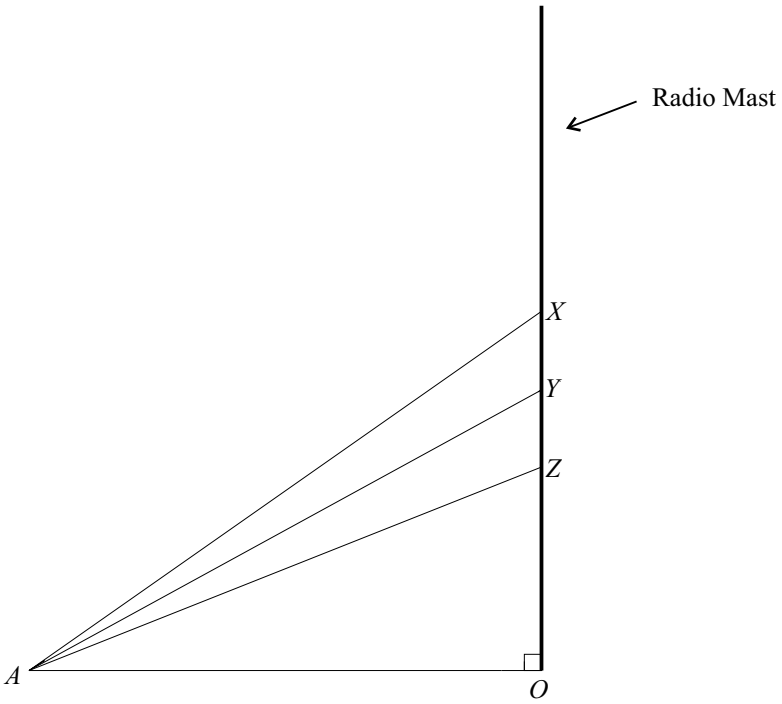
QUESTION EIGHT (13 marks) Start a new page.

(a) Find an expression for b given $\dfrac{2a+1}{b} + \dfrac{1}{c} = \dfrac{1}{3}$. Express your answer as a single fraction.

(b) The numbers a and b can be expressed as $a = 2^p$ and $b = 2^q$, where p and q are integers. It is also known that $ab = 32$ and $2a^2b = 32$.

Find the values of p and q.

(c)



An engineer erects a vertical radio mast on horizontal ground at O. Three wires from point A are fixed to the mast at points X, Y and Z. The wires form straight lines and $XY = YZ$.

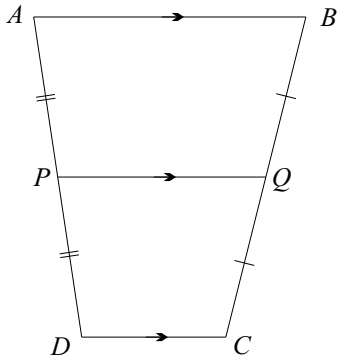
(i) Prove that $OY = \dfrac{1}{2}(OX + OZ)$.

(ii) Hence prove $\tan(\angle XAO) + \tan(\angle ZAO) = 2\tan(\angle YAO)$.

The question continues on the next page

QUESTION EIGHT (Continued)

(d)



Consider the quadrilateral $ABCD$ above, in which $AB \parallel PQ \parallel DC$, $AP = PD$ and $BQ = QC$. Intervals AB, PQ and DC are related by $AB = 2x - 7$, $PQ = x + 1$ and $DC = x - 3$, for some value x.

Prove that $x = 12$.

(e) A professor is due to be picked up at university by her husband immediately after her last lecture of the day is scheduled to finish. She unexpectedly finishes 3 hours early and immediately begins to walk home at a constant speed of 6 km/h.

Her husband, who knows nothing of her early finish, leaves home in his car at exactly the right time in order to meet with her at the university at the originally anticipated completion time of her last lecture. On the way to the university he sees his wife and picks her up 9 km from home. They immediately drive home together and reach home 2 hours earlier than was scheduled.

Calculate the average speed of the car for its entire journey.

END OF PAPER

B L A N K P A G E

Q1

a) $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$ ✓

b) $\frac{(-4)^2}{2} = \frac{16}{2} = 8$ ✓

c) $2x - 3 = x + 9$

$x - 3 = 9$

$x = 12$ ✓

d) $m = 4$ ✓

e) $\cos(32°) = 0.848048096 2$

$= 0.848$ ✓

f) $6a - a^2 = a(6 - a)$ ✓

g) $\frac{2x}{y} + \frac{x}{y} = \frac{3x}{y}$ ✓

h) $\tan\theta = 1.804$

$\theta = \tan^{-1}(1.804)$

$\theta = 60.9993\ldots$ ✓

$\theta = 61°$ (Don't penalise rounding)

i) $\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{2\sqrt{7}}{7}$ ✓

j) $6^{-4} = 1/6^4$ ✓

k) 2.15×10^4

$= 21500$ ✓

l) $16^{1/4} = \sqrt[4]{16} = 2$ ✓

m) $y = 6x - 5$

$y + 5 = 6x$

$x = \frac{y+5}{6}$ ✓

Q2

a) $(x-2)(x+7)=0$

$x=2$ or $x=-7$ ✓✓ (1 mark for correct numbers but incorrect signs.)

b) i) $x^2+7x+12$

$=(x+3)(x+4)$ ✓

ii) x^2-36

$=(x+6)(x-6)$ ✓

c) $A(3,7)$ $B(6,11)$

$AB=\sqrt{(6-3)^2+(11-7)^2}$ ✓

$=\sqrt{9+16}$

$=\sqrt{25}$

$=5$ ✓

d) $\sin 70=\frac{x}{20}$ ✓

$x=20\sin 70$

$x=18.7938$

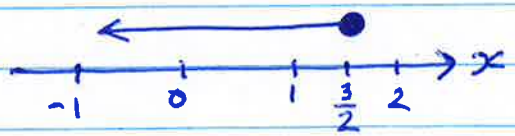
$x=18.8$ cm ✓ (Deduct for missing units.)

e) i) $9-2x\geq 6$

$-2x\geq -3$

$x\leq 3/2$ ✓

ii)



✓

f) i) AAA ✓ (accept AA)

ii) $6\times r=9$

$r=9/6$ ✓ (or similar)

$r=3/2$

$8\times\frac{3}{2}=12$

$XY=12$ cm ✓

(Q3)

a) $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{-3+5}{2}, \frac{1-7}{2} \right)$

$= (1, -3)$

C ✓

b) $1\ cm = 10\ mm$

$1\ cm^2 = 100\ mm^2$

$4\ cm^2 = 400\ mm^2$

B ✓

c) $(x+2)(x+4) - (x-2)(x-5)$

$= x^2 + 4x + 2x + 8 - (x^2 - 5x - 2x + 10)$

$= x^2 + 6x + 8 - x^2 + 7x - 10$

$= 13x - 2$

D ✓

d) $bx + 5d = ax + 2c$

$bx - ax + 5d = 2c$

$bx - ax = 2c - 5d$

$x(b-a) = 2c - 5d$

$x = \frac{2c - 5d}{b-a}$

$x = \frac{5d - 2c}{a - b}$

A ✓

e) $x = 2$ ✓

f) $y = -x + 3$

let $x = 0$, $y = 3$ ✓ y-intercept

let $y = 0$ $0 = -x + 3$

$x = 3$ ✓ x-intercept

g) $\tan\theta = \frac{12}{5}$ ✓

$\theta = \tan^{-1}(12/5)$

$\theta = 67.3801\ldots$

$\theta = 67^\circ$ ✓ (to nearest degree). (deduct for rounding.)

h) $3x - y = 6$ ①

$y = x + 2$ ②

① + ② $3x = 8 + x$ ✓ (or equivalent)

$2x = 8$

$x = 4$

② $y = 4 + 2$ } ✓ (Both answers req. for 2nd mark.)

$y = 6$

i) $\frac{a}{b+1} - \frac{a}{(b+1)^2}$

$= \frac{a(b+1)}{(b+1)^2} - \frac{a}{(b+1)^2}$

$= \frac{a(b+1) - a}{(b+1)^2}$ ✓

$= \frac{ab + a - a}{(b+1)^2}$

$= \frac{ab}{(b+1)^2}$ ✓

Q4.

a) $2x + y = 10$

$y = -2x + 10$

$m_1 = -2$ ✓ $m_2 = 1/2$ (for $m = -2$)

$-2 \times \frac{1}{2} = -1$ ✓

b) $3x + 5y = 8$ (1)

$x - 2y = -1$ (2)

(2)×3 $3x - 6y = -3$ (3) ✓

(1)-(3) $11y = 11$ ✓

$y = 1$

(2) $x - 2 = -1$

$x = 1$

$x = 1, y = 1$ ✓ (Both)

c) $\frac{x^2 - 9x + 14}{x^2 - 2x - 35}$

$= \frac{(x-2)(x-7)}{(x-7)(x+5)}$ ✓

$= \frac{x-2}{x+5}$ ✓

d) i) $50\,000 \times 1.06^8$

$= \$79692.40$ ✓

ii) Interest of $200,000 means account balance of $250,000.

$50,000 \times 1.06^n = 250,000$ ✓

$1.06^n = 5$

guess, check $n = 28$ years ✓

e) $V = \frac{340}{360} \times \pi \times 3^2 \times 9$

$\frac{340}{360}$ seen ✓

$= \frac{17}{18} \times \pi \times 9 \times 9$

$\pi r^2 h$ used ✓

$= \frac{17}{2} \times \pi \times 9$

Correct answer ✓

$= \frac{153\pi}{2}$ cm^3

Q5.)

a) i) $m = \frac{5 - -7}{1 - 5}$

$= \frac{12}{-4}$

$= -3$ ✓

ii) $y - y_1 = m(x - x_1)$

$y - 5 = -3(x - 1)$ ✓ (or similar)

$y = -3x + 3 + 5$

$y = -3x + 8$ ✓ (or equivalent)

b) i) $x - y = 2$

$3x + 5y = 70$

} both ✓

ii) $x = y + 2$

$3[y + 2] + 5y = 70$

$3y + 6 + 5y = 70$

$8y = 64$

$y = 8$

$x = 10$

} either ✓

The price of a hot chocolate is \$10 and the price of a bubble tea is \$8. } ✓

(Both, worded, required.)

c) $(2x + 1)(2x + 7)$

Correct integers ✓

Correct signs ✓

d) $x^2 - 4x - 4 = 0$

$(x-2)^2 - 4 - 4 = 0$

$(x-2)^2 - 8 = 0$

$(x-2)^2 = 8$ ✓

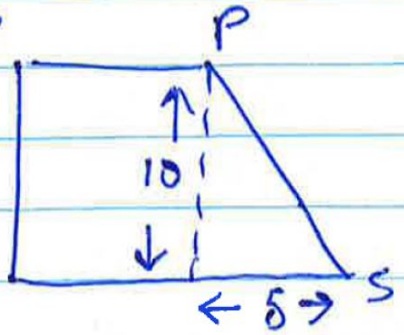
$x - 2 = \pm\sqrt{8}$

$x = 2 \pm 2\sqrt{2}$

$x = 2 + 2\sqrt{2}$ or $x = 2 - 2\sqrt{2}$ ✓

(award $x = 2 \pm \sqrt{8}$ full marks)

e)



$PS^2 = 10^2 + 5^2$

$PS = \sqrt{125}$ ✓

$PS = 5\sqrt{5}$ cm

$S.A = 5\sqrt{5} \times 16 + 30 \times 16 + 40 \times 16 + 20 \times 16 + 15 \times 16 + 2\left(80 \times 20 + \left(\frac{15+20}{2}\right)10\right)$

✓ (for any correct general strategy)

- $80\sqrt{5} + 1680 + 2(600 + 175)$

$= 80\sqrt{5} + 1680 + 1550$

$= 80\sqrt{5} + 3230 \text{ cm}^2$ ✓

6)

a) $y = 5x - 8$
$m = 5$ ✓ $(-1, -14)$

$y + 14 = 5(x + 1)$ ✓
$y = 5x + 5 - 14$
$y = 5x - 9$ ✓

b) i) $\frac{x}{20} = \frac{3}{x-4}$

$x(x-4) = 60$
$x^2 - 4x - 60 = 0$ ✓
$(x + 6)(x - 10) = 0$
$x = -6$ or $x = 10$ ✓ (Both)

ii) $\frac{x}{x-2} - \frac{x+1}{x+4} = 1$

$\frac{x(x+4)}{(x-2)(x+4)} - \frac{(x+1)(x-2)}{(x-2)(x+4)} = 1$

$x(x+4) - (x+1)(x-2) = (x-2)(x+4)$ ✓ (or similar)
$x^2 + 4x - (x^2 - 2x + x - 2) = x^2 + 4x - 2x - 8$
$x^2 + 4x - (x^2 - x - 2) = x^2 + 2x - 8$
$x^2 + 4x - x^2 + x + 2 = x^2 + 2x - 8$
$5x + 2 = x^2 + 2x - 8$
$0 = x^2 - 3x - 10$
$x^2 - 3x - 10 = 0$ ✓
$(x + 2)(x - 5) = 0$
$x = -2$ or $x = 5$ ✓ (Both)

c) i) $223 = 36 + (x+6)(x)$ ✓ (or equivalent)

ii) $223 = 36 + x^2 + 6x$

$x^2 + 6x - 187 = 0$

$(x - 11)(x + 17) = 0$ ✓

$x = 11$ or $x = -17$

$x > 0$

$\therefore x = 11$ m ✓

d) Let x be mins 1st wk of term.

$x \times 0.85^9 = 120$ ✓

$x = \frac{120}{0.85^9}$

$= 518.10$ mins.

$= 518$ mins ✓

$(= 8$ hrs 38 mins$)$

Q7

a) $A(2+\sqrt{2}, -1)$ $B(8+\sqrt{2}, 3)$ $C(6+\sqrt{2}, 6)$ $D(\sqrt{2}, 2)$

$$m_{AB} = \frac{3+1}{(8+\sqrt{2})-(2+\sqrt{2})} = \frac{4}{6} = \frac{2}{3}$$

$$m_{CD} = \frac{6-2}{6+\sqrt{2}-\sqrt{2}} = \frac{4}{6} = \frac{2}{3}$$

$$m_{BC} = \frac{6-3}{6+\sqrt{2}-(8+\sqrt{2})} = \frac{3}{-2} = -\frac{3}{2}$$

$$m_{DA} = \frac{2--1}{\sqrt{2}-(2+\sqrt{2})} = \frac{3}{-2} = -\frac{3}{2}$$ ✓

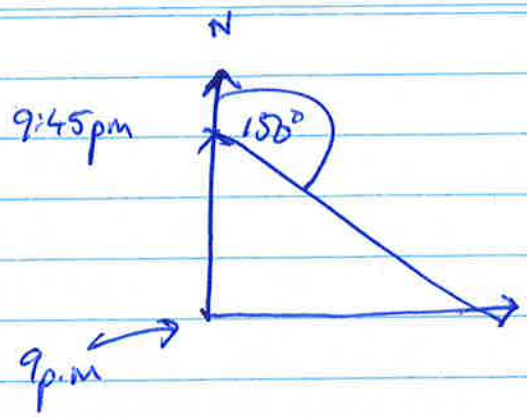
$m_{AB} = m_{CD}$, $m_{BC} = m_{AD}$, $\therefore$ AB // CD and BC // AD ✓

Also $m_{AB} \times m_{BC} = \frac{2}{3} \times -\frac{3}{2} = -1$

$\therefore$ AB $\perp$ BC.

$\therefore$ ABCD is a parallelogram with a right-angle.
$\therefore$ ABCD is a rectangle ✓

b)



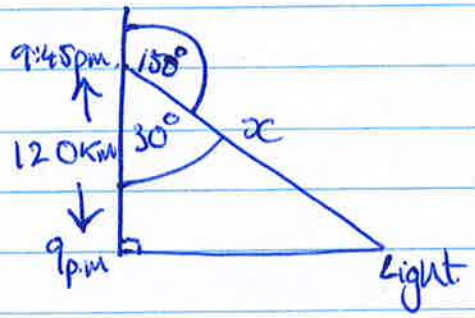
i) $s = \frac{d}{t}$

$d = s \times t$

$= 160 \times \frac{3}{4}$

$= 120$ km. ✓

ii)



$\cos 30^\circ = \frac{120}{x}$ ✓

$x = \frac{120}{\left(\frac{\sqrt{3}}{2}\right)}$

$x = \frac{240}{\sqrt{3}}$

$= \frac{240\sqrt{3}}{3}$

$= 80\sqrt{3}$ km. ✓

(accept $\frac{240 \text{ km}}{\sqrt{3}}$)

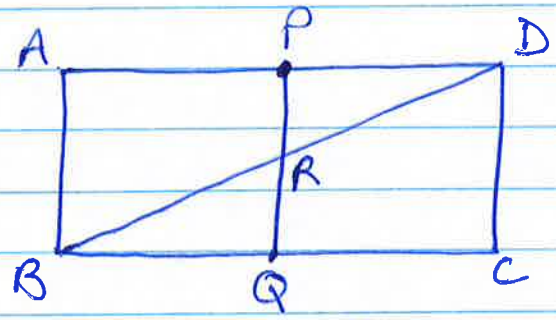
c) $\frac{3x^2-12}{30x+15} \times \frac{4x^2+4x+1}{2x^2-3x-2}$

$= \frac{3(x^2-4)}{15(2x+1)} \times \frac{(2x+1)(2x+1)}{(2x+1)(x-2)}$ ✓

$= \frac{3(x-2)(x+2)}{15} \times \frac{(2x+1)(2x+1)}{(2x+1)(x-2)}$ ✓

$= \frac{x+2}{5}$ ✓

d)



Let PQ meet BD at R.
ABCD is a rectangle, hence AD // BC and AB // DC
∠PDR = ∠QBR (Alternate angles PD // BQ)
∠PRD = ∠BRQ (Vertically opposite) } ✓
BQ = DP ($\frac{1}{2}$AD = $\frac{1}{2}$BC since AD = BC, rectangle).
∴ ΔBQR ≡ ΔDPR (AAS) ✓
∴ BR = DR (Matching sides of congruent triangles.) ✓

e) Kite ✓

Q8) a)

$$\frac{2a+1}{b} + \frac{1}{c} = \frac{1}{3}$$

$\times 3bc \qquad \times 3bc$

$$3c(2a+1) + 3b = bc$$

$$3c(2a+1) = bc - 3b$$

$$3c(2a+1) = b(c-3)$$

$$b = \frac{3c(2a+1)}{c-3} \quad \checkmark$$

or $\qquad b = \frac{6ac+3c}{c-3}$

b) $a = 2^p$ $b = 2^q$ $ab = 32$ $2a^2b = 32$

$2^p \times 2^q = 2^5$

$p + q = 5$ ①

$2^1 \times 2^{2p} \times 2^q = 2^5$

$1 + 2p + q = 5$

$2p + q = 4$ ②

} any valid method ✓

② - ① $p = -1$

① $-1 + q = 5$

$q = 6$

} ✓ (Both)

c) i)

Let $XY = YZ = \rho$

$$\left.\begin{aligned} OX &= OY + \rho \quad (1) \\ OZ &= OY - \rho \quad (2) \end{aligned}\right\} \checkmark$$

$(1) + (2)$: $OX + OZ = 2OY$ ✓ (method clear).

$\therefore OY = \frac{1}{2}(OX + OZ)$

ii) $\tan(\angle XAO) = \frac{OX}{OA}$

$\tan(\angle ZAO) = \frac{OZ}{OA}$

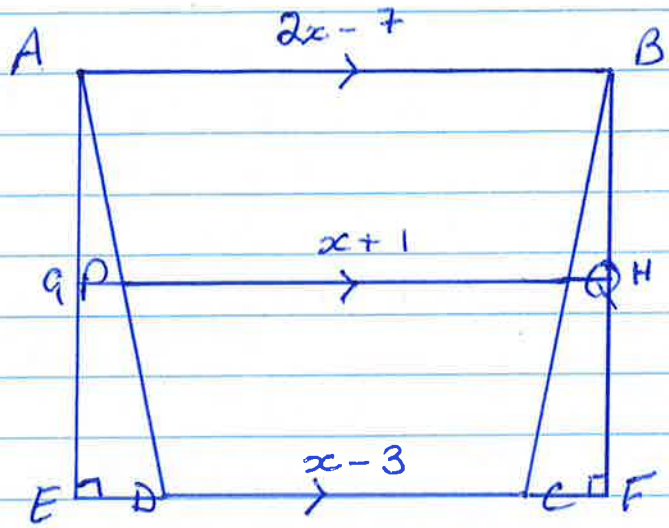
$\tan(\angle XAO) + \tan(\angle ZAO) = \frac{OX}{OA} + \frac{OZ}{OA}$ ✓

$= \frac{OX + OZ}{OA}$

$= \frac{2OY}{OA}$ ✓

$= 2\tan(\angle YAO)$

d)



Construct ΔAED and ΔBFC, with $\angle AED = \angle BFC = 90^\circ$
Extend PQ to meet AE and BF at G and H respectively.
$\angle PAG = \angle DAE$ (Common)
$\angle AGP = \angle AED = 90^\circ$ (Corresponding angles $GH \parallel EF$).
$\angle APG = \angle ADE$ (Corresponding angles $GH \parallel EF$).
$\therefore \Delta PAG \equiv \Delta DAE$ (AAA) ✓ (Full reasoning required).

Similarly $\Delta HBQ \equiv \Delta FBC$ (AAA)
Let $GP = a$, $HQ = b$.
$AG = GE$ (Given)
$ED = 2a$ and $CF = 2b$ (matching sides of similar triangles.) ✓

$\therefore$ $2x - 7 = a + b + x + 1$ ①
$2x - 7 = 2a + 2b + x - 3$ ②
① × 2 $4x - 14 = 2a + 2b + 2x + 2$ ③
③ − ② $2x - 7 = x + 5$
$x = 12$ ✓

e)

Let x be distance walked by professor, km.
s speed car, km/h.

If car had been at university 3 hrs earlier, they would have arrived home 3 hrs earlier. She walked x km and was driven $x+9$ km but only arrives home 2 hours early.

Total journey time = Total walking time + driving time

$$= \frac{x}{6} + \frac{9}{s}$$

Total journey time − expected journey time = 3 − 2

$$= 1$$

$$\therefore \left(\frac{x}{6} + \frac{9}{s}\right) - \frac{x+9}{s} = 1$$

$$\frac{sx+54}{6s} - \frac{6(x+9)}{6s} = 1$$

$$\frac{sx+54-6(x+8)}{6s} = 1$$

$$\frac{sx+54-6x-54}{6s} = 1$$

$$\frac{x(s-6)}{6s} = 1 \quad (1) \checkmark$$

For simplicity, assume, (for moment), car was due to arrive at university at 6 p.m. She therefore starts walking at 3 p.m.
Let place where car meets professor be point A.

time car arrives at A = 6 p.m − time would have taken to drive from A to university.

$= 6\text{ p.m} - \frac{x}{5}$

time professor gets to point A $= 3\text{ p.m} + \frac{x}{6}$.

Since they are both at point A at same time:

$6\text{ pm} - \frac{x}{5} = 3\text{ pm} + \frac{x}{6}$

$6\text{pm} - 3\text{pm} = \frac{x}{5} + \frac{x}{6}$

Difference in expected and actual learning time → $3 = \frac{6x + 5x}{6s}$ (2) ✓

(1) ÷ (2)

$1 \div 3 = \frac{x(s-6)}{6s} \div \frac{x(6+s)}{6s}$

$\frac{1}{3} = \frac{x(s-6)}{6s} \times \frac{6s}{x(s+6)}$

$$s + 6 = 3s - 18$$
$$24 = 2s$$
$$s = 12.$$

$\therefore$ The average speed of the car journey was 12 km/h. ✓

Alternative Solution

Professor travels x km at 6 km/h; this takes $\frac{x}{6}$ hrs. She is picked up 1 hr early (half the time saved by the car). ✓

ie $$\frac{x}{6} = 3 - 1$$ ✓

$$x = 12$$

Hence the car would travel 12 km/h. ✓