SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2020 Annual Examination

Form III Mathematics

Tuesday 3rd November 2020

8:39am

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Total Marks: 104

Questions 1–8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.

Collection

- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Write your name and master on this page and on each page of writing paper.
- Place everything inside this question booklet.

3A: LRP	3B: YH	3C: LL	3D: FMW
3E: LYL	3F: RCF	3G: NJL	3H: SDP
3I: RR	3J: AEA		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 191 pupils

Writer: YH

	Marks
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
TOTAL	

QUESTION ONE (13 marks)

(a) Simplify $2x - 5x$.

(b) Find 12% of 160.

(c) Simplify $p^2 \times p^3$.

(d) Factorise $2x - 4$.

(e) Solve $3x - 2 = 13$.

(f) If $a = -2$, evaluate $3a^2$.

(g) Solve $(x - 1)(x + 2) = 0$.

(h) Simplify $\sqrt{40}$.

(i) Use your calculator to evaluate $\sin 75°$. Give your answer correct to two decimal places.

(j) Expand and simplify $(x + 3)(x + 5)$.

(k) Express 315 000 in scientific notation. Give your answer to 3 significant figures.

(l) Convert $3\,\text{cm}^2$ to square millimetres.

(m) Find the simple interest earned on \$200 invested at 5% p.a. over 3 years.

QUESTION TWO (13 marks)

(a) Express 12 out of 75 as a percentage.

(b) Write down the equation of the line which has gradient 5 and y-intercept $(0, -1)$. Give your answer in gradient-intercept form.

(c) Find the distance between the points $(1, 14)$ and $(6, 2)$.

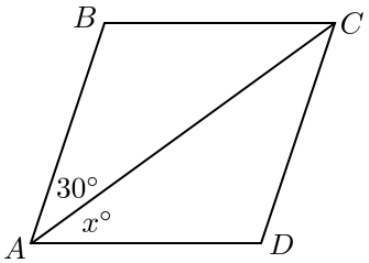
(d) Find the midpoint M of the interval joining the points $(5, 6)$ and $(3, -2)$.

(e) Factorise the following:

(i) $x^2 - 9$

(ii) $x^2 + 5x + 6$

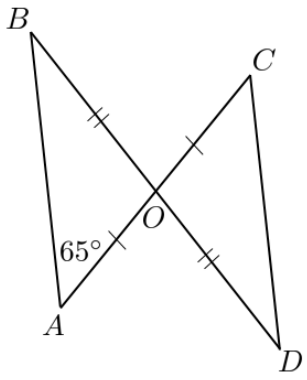
(f)



The diagram above shows a rhombus $ABCD$. Let $\angle CAD = x°$. If $\angle BAC = 30°$ state the value of x, giving a clear geometric reason.

(g) Express $\dfrac{2}{\sqrt{10}}$ with a rational denominator. Give your answer in simplest form.

(h)



In the diagram above $OB = OD$, $OA = OC$ and $\angle BAO = 65°$.

(i) Which congruency test would be used to prove that $\triangle OAB \equiv \triangle OCD$?

(ii) State the value of $\angle OCD$, giving a clear geometric reason.

QUESTION THREE (13 marks)

(a) Factorise fully $2xy - 2y + 3x - 3$.

(b)

stem	leaf					
1	1	3	3	4		
2	2	2	2	8		
3	4	4	6	8	8	9
4	1	2				

1 | 3 represents 13

The stem-and-leaf plot above shows the 16 results of a class test.

(i) State the mode.

(ii) Find the median.

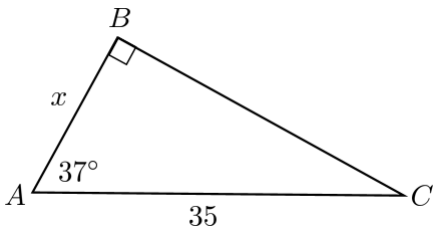
(c) Make q the subject of $p = 2q + 3$.

(d) Simplify:

(i) $\left(\frac{1}{3}\right)^{-2}$

(ii) $(64x^6y^2)^{\frac{1}{2}}$

(e)



Find the value of x in the diagram above, correct to two decimal places.

(f) Simplify $\dfrac{4x}{2x^2 - 4xy}$.

QUESTION FOUR (13 marks)

(a) State the equation of the vertical line which passes through the point $(-1, 2)$.

(b) Express each of the following as a single fraction in its simplest form:

(i) $\dfrac{1}{x} + \dfrac{3}{2x}$

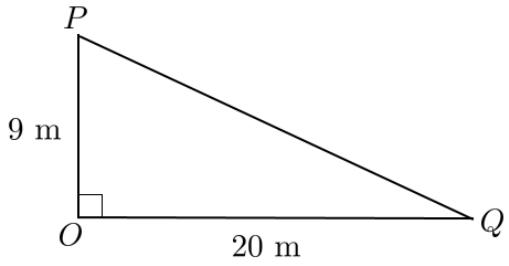
(ii) $\dfrac{2}{x-1} - \dfrac{1}{(x-1)^2}$

(c) By factorising, solve the following:

(i) $x^2 - 6x + 8 = 0$

(ii) $3x^2 + 11x + 6 = 0$

(d)



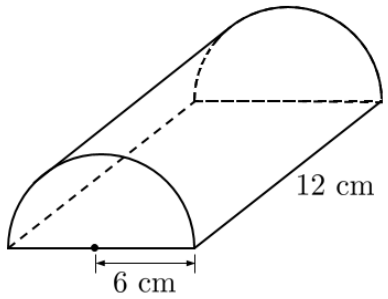
In the diagram above, a vertical tower OP has a height of 9 m. A point Q along the horizontal ground is 20 m away from the base of the tower.

Find the angle of depression from the top of the tower to the point Q. Give your answer correct to the nearest degree.

(e) The average of 25 scores is 30. A new score x is added to increase the average to 32. Find the value of x. Show your working.

QUESTION FIVE (13 marks)

(a)



The diagram above shows a solid piece of wood in the shape of a half-cylinder with radius 6 cm and height 12 cm.

(i) Find the volume of the solid. Give your answer as an exact value in terms of π.

(ii) Find the surface area of the solid. Give your answer correct to two decimal places.

(b) Find the equation of the line which is parallel to $y = -2x - 1$ and passes through the point $(-5, 3)$. Give your answer in gradient-intercept form.

(c) The price of a yacht increased by 20% then decreased by 40%. Determine the net percentage decrease from the original price.

(d) Complete the square for $x^2 + 10x$.

(e) Anna invested \$5000 in an account paying interest of 2·5% p.a., compounded annually for 20 years.

(i) What is the total value of Anna's investment after 20 years? Give your answer to the nearest dollar.

(ii) How much interest did Anna earn over the 20 years? Give your answer to the nearest dollar.

(iii) Paul wishes to invest \$$P$ into an account which pays **simple** interest at 2·5% p.a. for 20 years. If he wishes to earn the same amount of interest as Anna, find P to the nearest dollar.

QUESTION SIX (13 marks)

(a) A line passes through the points $(6, -2)$ and $(4, -5)$. Find the equation of this line, giving your answer in gradient-intercept form.

(b) The line $5x - 4y + 20 = 0$ cuts the x- and y-axes at points A and B respectively.

(i) Sketch the graph of the line $5x - 4y + 20 = 0$, clearly labelling both intercepts.

(ii) Find the area of triangle OAB, where O is the origin.

(c) Solve the following pairs of simultaneous equations:

(i) $3x - 2y = 7$
$y = x - 2$

(ii) $4x + 3y = 5$
$7x - 4y = -19$

(d) Simplify:

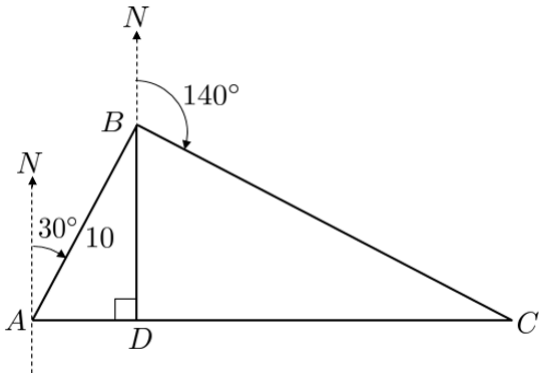
(i) $\dfrac{x^2 + 5x}{x^3} \times \dfrac{x^2}{x^2 + 3x - 10}$

(ii) $\dfrac{4x^2 - y^2}{4} \div \dfrac{2x + y}{2}$

QUESTION SEVEN (13 marks)

(a) Solve $\dfrac{4}{x-1} - \dfrac{5}{x+2} = \dfrac{3}{x}$.

(b)

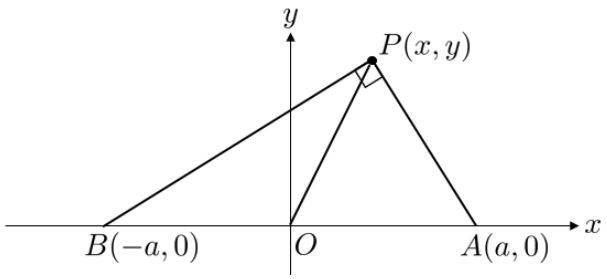


In the diagram above, a car is initially at point A. From this point the car travels 10 km on a bearing of 030°T to reach point B. The car then travels on a bearing of 140°T to point C which is due east of A. The point D lies on AC such that BD is perpendicular to AC.

(i) Find the length of AD.

(ii) Find the exact length of BD. Hence find the length of AC, giving your answer correct to two decimal places.

(c)

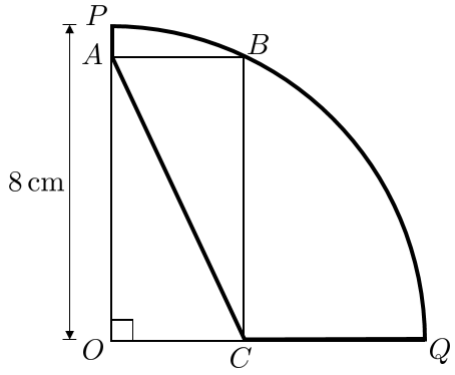


On the number plane above, points A and B have coordinates $(a, 0)$ and $(-a, 0)$ respectively, where a is a positive constant. Point $P(x, y)$ lies in the first quadrant such that PA is perpendicular to PB.

(i) Show that $x^2 + y^2 = a^2$.

(ii) If $\angle BAP = 75°$, find the value of $\angle BOP$ where O is the origin, giving clear geometric reasons.

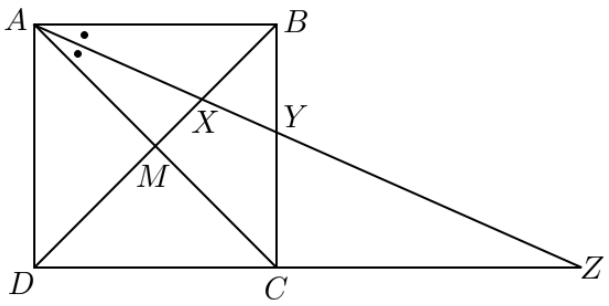
(d)



In the diagram above, arc PQ is one quarter of a circle with centre O and radius 8 cm. The perimeter of rectangle $OABC$ is 22 cm, find the perimeter of the figure $APBQC$. Give your answer correct to two decimal places.

QUESTION EIGHT (13 marks)

(a)



In the diagram above, the diagonals AC and BD of the square $ABCD$ intersect at M. The bisector of $\angle BAC$ cuts BM at X and BC at Y. The intervals DC and AY are produced to meet at Z.

(i) Prove that $\triangle AMX \,|||\, \triangle ZCY$.

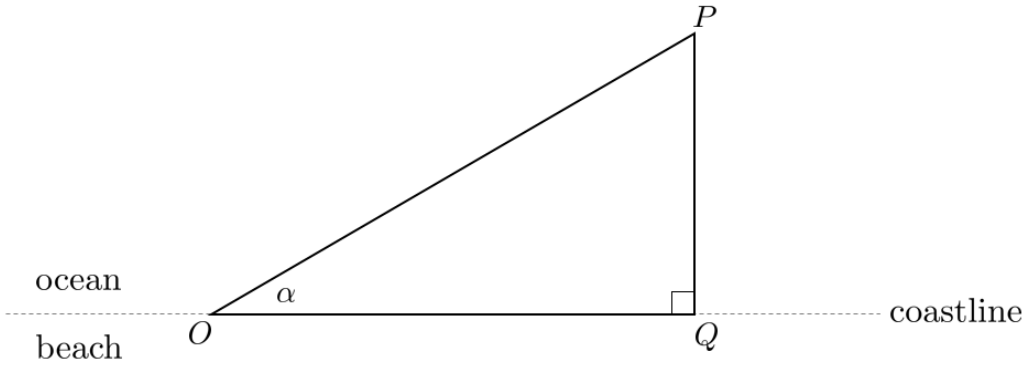
(ii) Find the value of $\dfrac{MX}{CY}$, giving clear geometric reasons.

(b) A wire of length 12 cm is cut into two sections. One section is bent into a square, and the other into a rectangle which is three times as long as it is wide. The sum of the areas of the square and the rectangle is $10\,\text{cm}^2$. Let x be the width of the rectangle.

(i) Show that $x^2 - \dfrac{12}{7}x - \dfrac{1}{7} = 0$.

(ii) By completing the square, find the exact value of x.

(c)



Peter and Quentin participate in a race at the beach. The race starts at point O on the coastline and finishes at point P in the ocean, as shown in the diagram above. Peter swims from O to P at constant speed u m/s. Quentin runs at constant speed v m/s and also swims at constant speed u m/s, where $v > u$. Quentin chooses to run along the beach from O to Q, where $\angle OQP = 90^\circ$ and then he swims from Q to P. Let $\angle POQ = \alpha$.

(i) Given that Peter beats Quentin in the race, show that $\tan\alpha > \dfrac{v^2 - u^2}{2uv}$.

(ii) Rory, who has the same running and swimming speeds as Quentin, also participates in this race. He chooses to run from O to a point R, where R is a point on the line OQ, somewhere in between O and Q. He then swims from R to P. Let $RQ = x$ metres and show that if Peter beats Quentin, then Rory will beat Quentin in this race for all possible values of x.

——— END OF PAPER ———

B L A N K P A G E

Question One - 13 Marks

(a) [1]

$2x - 5x = -3x$ (✓)

(b) [1]

$0{\cdot}12 \times 160 = 19{\cdot}2$ (✓)

(c) [1]

$p^2 \times p^3 = p^5$ (✓)

(d) [1]

$2x - 4 = 2(x - 2)$ (✓)

(e) [1]

$3x - 2 = 13$

$3x = 15$

$x = 5$ (✓)

(f) [1]

$3a^2$

$= 3(-2)^2$

$= 3(4)$

$= 12$ (✓)

(g) [1]

$x = 1,\ x = -2$ (✓)

(h) [1]

$\sqrt{40} = 2\sqrt{10}$ (✓)

(i) [1]

$\sin 75^\circ = 0.97$ (2dp) (✓)

(j) [1]

$(x + 3)(x + 5) = x^2 + 8x + 15$ (✓)

(k) [1]

$315\,000 = 3{\cdot}15 \times 10^5$ (3 significant figures) (✓)

(l) [1]

$3\,\text{cm}^2 = 300\,\text{mm}^2$ (✓)

(m) [1]

$I = Prn$

$I = 200 \times 0{\cdot}05 \times 3$

$I = \$30$ (✓)

Question Two - 13 Marks

(a) [1]

$\frac{12}{75} \times 100 = 16\%$ (✓)

(b) [1]

$y = 5x - 1$ (✓)

(c) [2]

$D = \sqrt{(6-1)^2 + (2-14)^2}$ (✓)

$D = \sqrt{169}$

$D = 13\,\text{u}$ (or $5, 12, 13$ triads) (✓)

(d) [1]

$M = \left(\frac{5+3}{2}, \frac{6-2}{2}\right)$

$M = (4, 2)$ (✓)

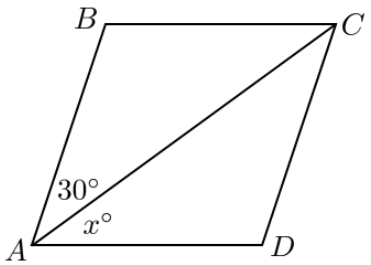
(e) i. [1]

$x^2 - 9 = (x+3)(x-3)$ (✓)

ii. [1]

$x^2 + 5x + 6 = (x+2)(x+3)$ (✓)

(f) [2]



$x = 30$ (diagonals of a rhombus bisects the angle at the vertex) (✓✓)

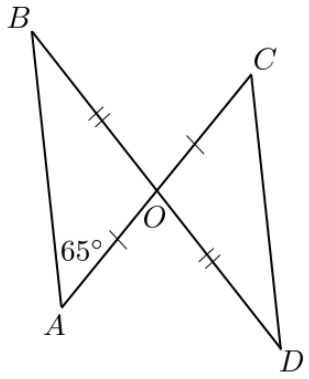
(g) [1]

$\frac{2}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{2\sqrt{10}}{10}$

$= \frac{\sqrt{10}}{5}$ (✓)

(h)



i. [1]

SAS (✓)

ii. [2]

$\angle OCD = 65^\circ$ (matching angles of congruent triangles) (✓✓)

Question Three - 13 Marks

(a) [2]

$2xy - 2y + 3x - 3$

$= 2y(x-1) + 3(x-1)$ (✓)

$= (x-1)(2y+3)$ (✓)

(b) i. [1]

22 (✓)

ii. [1]

$\dfrac{28+34}{2} = 31$ (✓)

(c) [2]

$p = 2q + 3$

$2q = p - 3$ (✓)

$q = \dfrac{p-3}{2}$ (✓)

(d) i. [1]

$\left(\dfrac{1}{3}\right)^{-2}$

$= (3)^2$

$= 9$ (✓)

ii. [2]

$(64x^6y^2)^{\frac{1}{2}}$

$= 8x^3y$ (✓✓)

(e) [2]

$\cos 37^\circ = \dfrac{x}{35}$ (✓)

$x = 35\cos 37^\circ$

$x = 27.95$ (2dp) (penalise rounding) (✓)

(f) [2]

$$\frac{4x}{2x^2 - 4xy}$$

$$= \frac{4x}{2x(x - 2y)}$$ (✓)

$$= \frac{2}{x - 2y}$$ (✓)

Question Four - 13 Marks

(a) [1]

$x = -1$ (✓)

(b) i. [2]

$$\frac{1}{x} + \frac{3}{2x}$$

$$= \frac{2}{2x} + \frac{3}{2x}$$ (✓)

$$= \frac{5}{2x}$$ (✓)

ii. [2]

$$\frac{2}{x-1} - \frac{1}{(x-1)^2}$$

$$= \frac{2(x-1)}{(x-1)^2} - \frac{1}{(x-1)^2}$$ (✓)

$$= \frac{2x-3}{(x-1)^2}$$ (✓)

(c) i. [2]

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$ (✓)

$\therefore x = 2,\ x = 4$ (✓)

ii. [2]

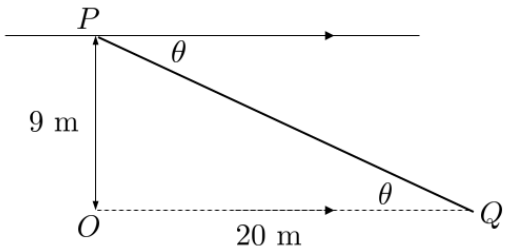
$3x^2 + 11x + 6 = 0$

$(3x+2)(x+3) = 0$ (✓)

$\therefore x = -\dfrac{2}{3},\ x = -3$ (✓)

Question Four continued

(d) [2]



Let $\angle PQO = \theta$ and so $\tan\theta = \dfrac{9}{20}$ (✓)

$\theta = \tan^{-1}\left(\dfrac{9}{20}\right)$

$\theta = 24^\circ$ (nearest degree)

$\therefore$ the angle of depression is 24°(alternate angles on parallel lines) (✓)

(e) [2]

Total of original scores is $25 \times 30 = 750$ (✓)

Let x be the new score.

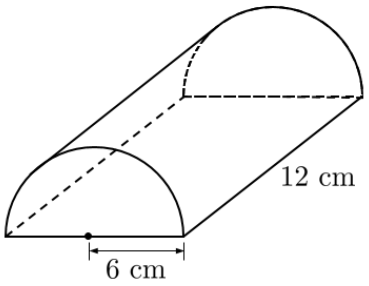
$\dfrac{750 + x}{26} = 32$

$750 + x = 832$

$\therefore x = 82$ (✓)

Question Five - 13 Marks

(a)



i. [1]

$V = \dfrac{1}{2} \times \pi(6)^2 \times 12$

$V = 216\pi\,\text{cm}^3$ (penalise units) (✓)

ii. [2]

$A = \pi(6)^2 + 12^2 + \pi(6)(12)$

$A = 108\pi + 144$

$A = 483{\cdot}29\,\text{cm}^2$ (2dp) (penalise units) (✓✓)

(b) [2]

$y = -2x - 1$

$\therefore m = -2$ (✓)

$y - 3 = -2(x + 5)$

$y = -2x - 7$ (✓)

(c) [2]

Let x be the original price.

$x\,(1{\cdot}2)(0{\cdot}6) = 0{\cdot}72x$ (✓)

$\therefore$ Net decrease of 28%. (✓)

(d) [2]

$x^2 + 10x$

$= x^2 + 10x + 25 - 25$ (✓)

$= (x + 5)^2 - 25$ (✓)

(e) i. [2]

$A = 5000(1.025)^{20}$

$A = \$8193$ (nearest dollar)

(one mark for any appearance of 1.025, one mark for final answer) (✓✓)

ii. [1]

$I = 5000(1.025)^{20} - 5000$

$I = \$3193$ (nearest dollar) (✓)

iii. [1]

$P(0.025)(20) = I$

$P = \dfrac{I}{0.5}$

$P = \$6386$ (nearest dollar) (✓)

Question Six - 13 Marks

(a) [2]

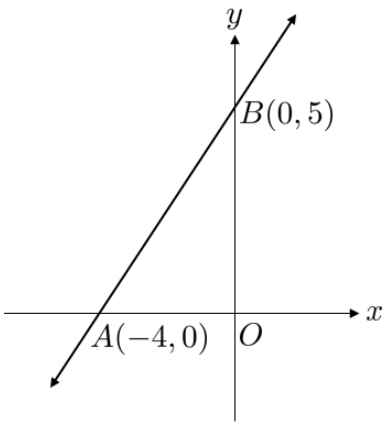
$$m = \frac{-2+5}{6-4} = \frac{3}{2} \quad (\checkmark)$$

$$y + 2 = \frac{3}{2}(x-6)$$

$$y = \frac{3}{2}x - 11 \quad (\checkmark)$$

(b)

i. [2]



(one mark for each intercept) (✓✓)

ii. [1]

$$A = \frac{1}{2}(4)(5)$$

$$A = 10\,u^2 \quad (\checkmark)$$

(c) i. [2]

$$3x - 2y = 7 \cdots ①$$

$$y = x - 2 \cdots ②$$

sub ② into ① :

$$3x - 2(x-2) = 7 \quad (\checkmark)$$

$$3x - 2x + 4 = 7$$

$$\therefore x = 3$$

$$\therefore y = 1 \quad (\checkmark)$$

ii. [2]

$4x + 3y = 5 \cdots ①$

$7x - 4y = -19 \cdots ②$

$① \times 4:$

$16x + 12y = 20 \cdots ③$

$② \times 3:$

$21x - 12y = -57 \cdots ④$ (✓)

$③ + ④:$

$37x = -37$

$\therefore x = -1$

$\therefore y = 3$ (✓)

(d) i. [2]

$$\frac{x^2 + 5x}{x^3} \times \frac{x^2}{x^2 + 3x - 10}$$

$$= \frac{x(x+5)}{x^3} \times \frac{x^2}{(x+5)(x-2)} \quad (✓)$$

$$= \frac{1}{x-2} \quad (✓)$$

ii. [2]

$$\frac{4x^2 - y^2}{4} \div \frac{2x + y}{2}$$

$$= \frac{(2x+y)(2x-y)}{4} \times \frac{2}{2x+y} \quad (✓)$$

$$= \frac{2x - y}{2} \quad (✓)$$

Question Seven - 13 Marks

(a) [3]

$$\frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{x}$$

multiply throughout by $x(x-1)(x-2)$

$4x(x+2) - 5x(x-1) = 3(x-1)(x+2)$ (✓)

$4x^2 + 8x - 5x^2 + 5x = 3x^2 + 3x - 6$

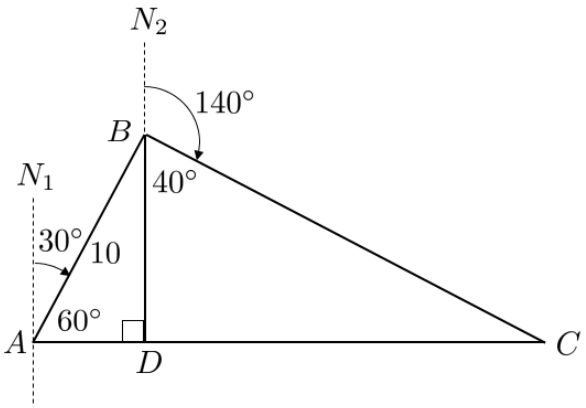
$4x^2 - 10x - 6 = 0$ (✓)

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$x = -\frac{1}{2},\ x = 3$ (✓)

(b)



i. [1]

$\cos 60^\circ = \frac{AD}{10}$

$AD = 10\cos 60^\circ$

$AD = 5\,\text{km}$ (✓)

ii. [2]

$BD = \sqrt{10^2 - 5^2}$

$BD = 5\sqrt{3}\,\text{km}$ (✓)

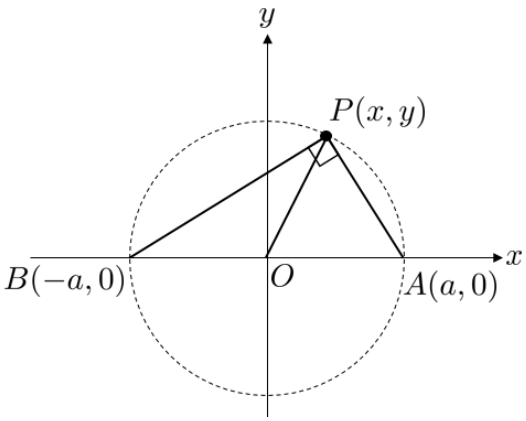
$\tan 40^\circ = \frac{DC}{BD}$

$DC = 5\sqrt{3}\tan 40^\circ$

$\therefore AC = 5 + 5\sqrt{3}\tan 40^\circ = 12.27\,\text{km}$ (2dp) (✓)

Question Seven continued

(c)



i. [2]

$\dfrac{y-0}{x-a} \times \dfrac{y-0}{x+a} = -1$ (✓)

$\dfrac{y}{x-a} \times \dfrac{y}{x+a} = -1$

$y^2 = -(x+a)(x-a)$

$y^2 = -x^2 + a^2$

$x^2 + y^2 = a^2$ (✓)

alternatively:

$AP^2 + BP^2 = AB^2$

$(x-a)^2 + y^2 + (x+a)^2 + y^2 = (2a)^2$ (✓)

$x^2 - 2ax + a^2 + y^2 + x^2 + 2ax + a^2 + y^2 = 4a^2$

$2x^2 + 2y^2 + 2a^2 = 4a^2$

$2x^2 + 2y^2 = 2a^2$

$x^2 + y^2 = a^2$ (✓)

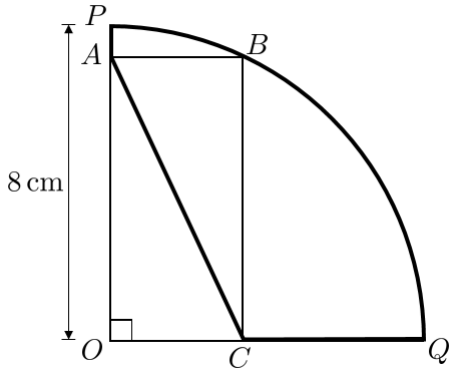
ii. [2]

$OP = OA$ (radii)

$\angle OPA = 75^\circ$ (equal angles opposite equal sides of isosceles $\triangle OAP$) (✓)

$\angle BOP = 75 \times 2 = 150^\circ$ (exterior angle of $\triangle OAP$) (✓)

(d) [3]



Perimeter $= PQ + AC + PA + CQ$

$PQ = \frac{1}{4} \times 2\pi(8)$

$PQ = 4\pi$ (✓)

$AC = OB = 8$ (diagonals of a rectangle are equal) (✓)

$PA + CQ = PA + AO + OC + CQ - (AO + OC)$

but $AO + OC$ is half of the perimeter of the rectangle, so $AO + OC = 11$

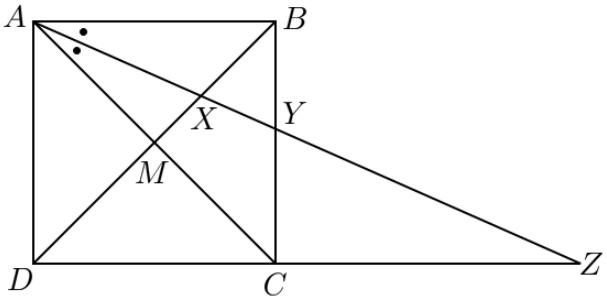
$\therefore PA + CQ = 16 - 11 = 5$

$\therefore$ Perimeter $= 4\pi + 8 + 5$

$\therefore$ Perimeter $= 25{\cdot}57$ cm (2dp) (✓)

Question Eight - 13 Marks

(a)



i. [2]

In $\triangle AMX$ and $\triangle ZCY$:

$\angle AMX = 90^\circ$ (diagonals of a square bisect at 90°)

$\angle DCY = 90^\circ$ (angle in a square)

$\angle ZCY = 90^\circ$ (straight angle)

$\therefore \angle AMX = \angle ZCY$ (✓)

$AB \parallel ZD$ (opposite sides of square are parallel)

$\angle YZC = \angle YAB$ (alternate angles, $AB \parallel ZD$)

$\therefore \angle YZC = \angle XAM$ (AZ bisects $\angle BAC$)

$\therefore \triangle AMX \,|||\, \triangle ZCY$ (equiangular) (✓)

ii. [2]

$\dfrac{AM}{AC} = \dfrac{1}{2}$ (diagonals of a square bisect) (✓)

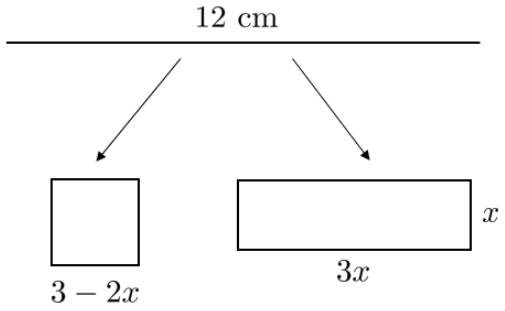
But $AC = ZC$ (equal sides opposite equal angles of isosceles $\triangle ACZ$)

$\therefore \dfrac{AM}{ZC} = \dfrac{1}{2}$

$\therefore \dfrac{MX}{CY} = \dfrac{AM}{ZC} = \dfrac{1}{2}$ (ratio of matching sides of similar triangles) (✓)

Question Eight continued

(b)



i. [2]

the length of the square is $\frac{12-8x}{4} = 3-2x$

$3x^2 + (3-2x)^2 = 10$ (✓)

$3x^2 + 9 - 12x + 4x^2 = 10$

$7x^2 - 12x - 1 = 0$

$x^2 - \frac{12}{7}x - \frac{1}{7} = 0$ (✓)

ii. [2]

$$x^2 - \frac{12}{7}x - \frac{1}{7} = 0$$

$$x^2 - \frac{12}{7}x = \frac{1}{7}$$

$$x^2 - \frac{12}{7}x + \left(\frac{6}{7}\right)^2 = \frac{1}{7} + \left(\frac{6}{7}\right)^2$$

$$x^2 - \frac{12}{7}x + \frac{36}{49} = \frac{1}{7} + \frac{36}{49}$$

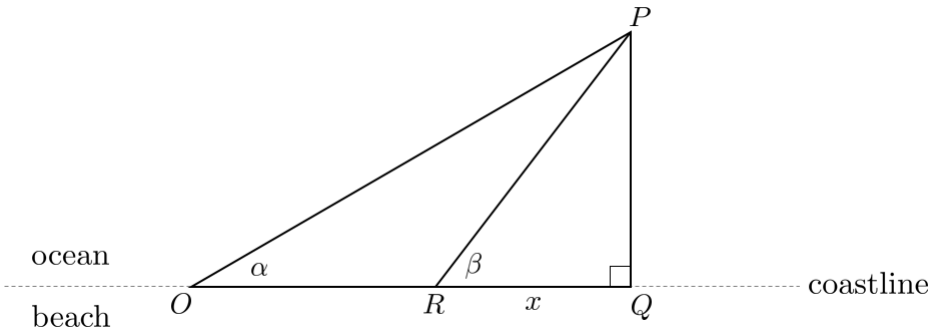
$$\left(x - \frac{6}{7}\right)^2 = \frac{43}{49} \quad (\checkmark)$$

$$x - \frac{6}{7} = \frac{\sqrt{43}}{7} \text{ or } x - \frac{6}{7} = -\frac{\sqrt{43}}{7}$$

$$x = \frac{6+\sqrt{43}}{7} \text{ or } x = \frac{6-\sqrt{43}}{7}$$

$x = \frac{6+\sqrt{43}}{7}$ cm only as $x > 0$ (✓)

(c)



i. Let t_1 be the time it takes for Quentin to run from O to Q. [3]

Let t_2 be the time it takes for Quentin to swim from Q to P.

Peter beats Quentin if the distance swum by Peter in the time Quentin takes to complete the race exceeds OP, that is:

$u(t_1 + t_2) > OP$ (✓)

$u(t_1 + t_2) > \sqrt{(vt_1)^2 + (ut_2)^2}$

$u^2(t_1 + t_2)^2 > v^2t_1^2 + u^2t_2^2$

$u^2(t_1^2 + 2t_1t_2 + t_2^2) > v^2t_1^2 + u^2t_2^2$

$u^2t_1^2 + 2u^2t_1t_2 + u^2t_2^2 > v^2t_1^2 + u^2t_2^2$

$u^2t_1^2 + 2u^2t_1t_2 > v^2t_1^2$ (✓)

$2u^2t_1t_2 > v^2t_1^2 - u^2t_1^2$

$\dfrac{2u^2t_1t_2}{2uvt_1^2} > \dfrac{v^2t_1^2 - u^2t_1^2}{2uvt_1^2}$

$\dfrac{ut_2}{vt_1} > \dfrac{v^2 - u^2}{2uv}$

$\therefore \tan\alpha > \dfrac{v^2 - u^2}{2uv}$ (✓)

ii. Let $\angle PRQ = \beta$. Rory and Quentin will be tied from O to R, so Rory will beat Quentin if Rory's swim from R to P is faster than Quentin's trip from R to Q then Q to R. But this is the same problem as part (i), [2]

so Rory will win if $\tan\beta > \dfrac{v^2 - u^2}{2uv}$. (✓)

But $\tan\beta = \dfrac{PQ}{x} > \dfrac{PQ}{OQ}$ (as $x < OQ$), and as $\dfrac{PQ}{OQ} = \tan\alpha$,

we have $\tan\beta > \tan\alpha > \dfrac{v^2 - u^2}{2uv}$. Therefore Rory will beat Quentin. (✓)