NAME MASTER

SYDNEY GRAMMAR SCHOOL

2019 Annual Examination

FORM III

MATHEMATICS

Wednesday 30th October 2019

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: SO	3B: NJL	3C: LYL/RR	3D: REJ/PKH
3E: WJM/RCF	3F: DWH/GMC	3G: LRP	3H: RDP
3I: DNW	3J: FMW		

Checklist

- Writing paper — approx. 16 sheets
- Candidature — 196 boys

Examiner

NJL

QUESTION ONE (13 marks) Start a new page.

(a) Find 3% of 200 .

(b) Factorise $4x + 2$.

(c) Express 200 millimetres in metres.

(d) A rectangle is x cm wide and $2x$ cm long. Write a simplified expression for:

(i) the perimeter,

(ii) the area.

(e) Find the mode of the following data: 9, 7, 6, 8, 1, 3, 7 .

(f) Simplify:

(i) $(2x)^3$

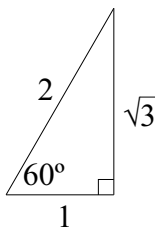
(ii) $\sqrt{50}$

(g) Find the simple interest earned on \$100 invested at 10% p.a. over 2 years.

(h) Expand and simplify $(x + 2)(x + 7)$.

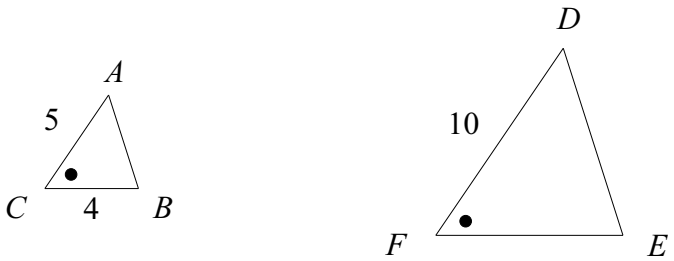
(i) Write $5{\cdot}08 \times 10^4$ as a basic numeral.

(j)



Using the diagram above, or otherwise, state the exact value of $\tan 60°$.

(k)



In the diagram above, $\triangle ABC$ is similar to $\triangle DEF$, $\angle ACB = \angle DFE$, $AC = 5$, $BC = 4$ and $DF = 10$. Find the length of side EF.

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

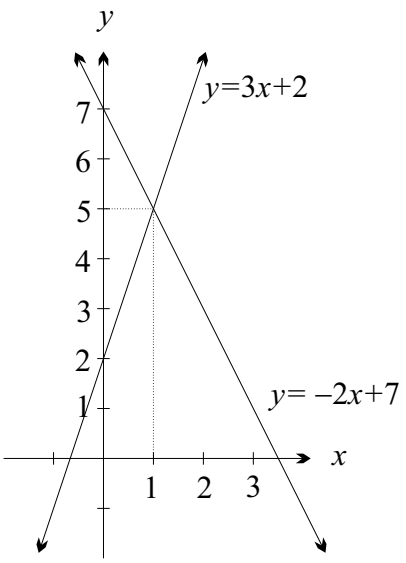
(a) Solve $(x+2)(x-1)=0$.

(b) Write down the value of $\sin 26°$. Give your answer correct to two decimal places.

(c) Write down the gradient of the line $y=6x-4$.

(d) Write down the y-intercept of the line $x-3y=12$.

(e)



Using the graph above, solve $y=3x+2$ and $y=-2x+7$ simultaneously.

(f) Express $\dfrac{2}{\sqrt{6}}$ as a simplified fraction with a rational denominator.

(g) Find the distance between the points $A(2,-1)$ and $B(5,3)$.

(h) Express $20\,000\,\text{cm}^3$ in m^3.

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

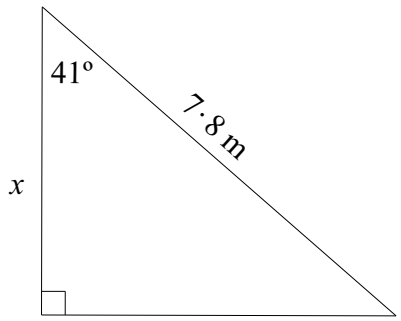
(a) Factorise:

(i) $x^2 + x - 6$

(ii) $2ax + 3a + 6x + 9$

(b) Find the gradient of the line between the points $A(1, 4)$ and $B(3, 10)$.

(c)



Calculate the length of the side labelled x in the diagram above. Give your answer correct to two significant figures.

(d) \$100 000 is invested in an account paying 5% p.a. compounded annually.

(i) What is the value of the investment after one year?

(ii) What is the value of the investment after ten years? Give your answer correct to the nearest dollar.

(iii) To the nearest whole number, what is the percentage increase of the original investment at the end of ten years?

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) Evaluate $27^{-\frac{1}{3}}$.

(b) Simplify $\dfrac{x^2+xy}{x+y}$.

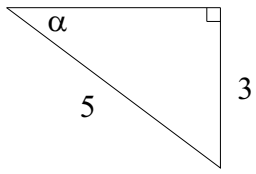
(c) Express as a single fraction:

(i) $\dfrac{2}{3x}+\dfrac{1}{x}$

(ii) $\dfrac{1}{(x-4)^2}-\dfrac{2}{x-4}$

(d) Suppose $M(6,8)$ is the midpoint between $P(a,7)$ and $Q(8,9)$. Find the value of a.

(e)



Find the value of α in the triangle above. Give your answer correct to the nearest degree.

(f) (i) Copy and complete the table below for the equation $y=-x^2+4$.

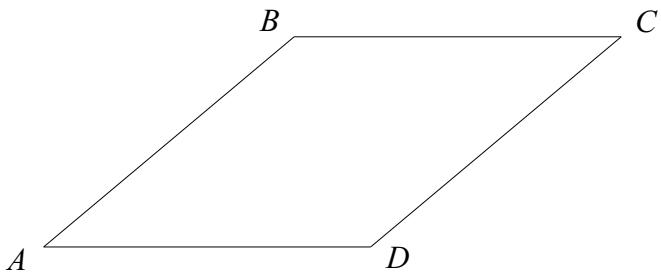
x	-3	-2	-1	0	1	2	3
y							

(ii) Use your completed table to plot the graph of $y=-x^2+4$. Marks are awarded for suitably scaled and labelled axes.

Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a)



In the diagram above, $ABCD$ is a rhombus. Use congruent triangles to prove that the opposite angles of a rhombus are equal. You must provide clear geometric reasoning.

(b) Line l is perpendicular to the line $y = -2x - 2$ and passes through the point $(4, -2)$.

(i) Write down the gradient of line l.

(ii) Hence find the equation of line l. Give your answer in the form $y = mx + c$.

(c) Solve the following quadratic equations:

(i) $x^2 + 9x + 14 = 0$

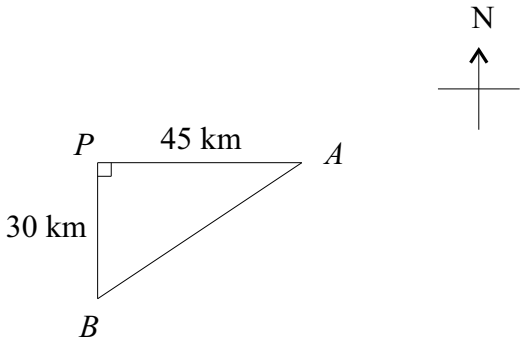
(ii) $6x^2 - 17x + 12 = 0$

(iii) $\dfrac{x+5}{x} = \dfrac{8}{x-1}$

Examination continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a)



The diagram above shows the positions of boats A and B relative to port P. A is 45 km due East of P and B is 30 km due South of P. Find the true bearing of B from A. Give your answer correct to the nearest degree.

(b) Solve the following pairs of simultaneous equations:

(i) $2x - 3y = -8$
$y = x + 3$

(ii) $-7x + 3y = 22$
$3x - 6y = -11$

(c) Sketch the graphs of the following equations, showing all x- and y-intercepts:

(i) $x^2 + y^2 = 16$

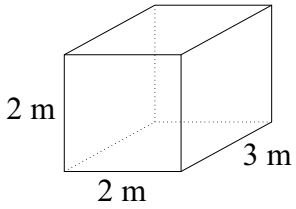
(ii) $3x - 4y = 12$

(d) Simplify $\dfrac{x^2 - 1}{3x^2 - 6x} \div \dfrac{5x + 4 + x^2}{8 - 2x - x^2}$.

Examination continues overleaf ...

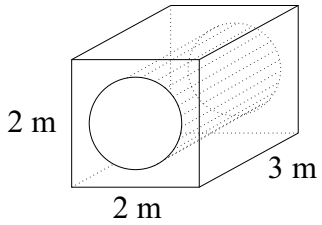
QUESTION SEVEN (13 marks) Start a new page.

(a) (i)



A solid concrete square prism, as shown above, is 2 m wide, 2 m high and 3 m long. Calculate the volume of the prism.

(ii)



A cylindrical hole is drilled through the prism so that it is open at both ends, as shown in the diagram above. The remaining volume after the hole is removed is $8\,\text{m}^3$.

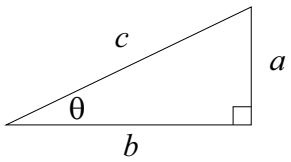
(α) Show that the radius of the hole must be $\dfrac{2}{\sqrt{3\pi}}$ m.

(β) Hence calculate the exact total surface area of the prism.

(b) Steven the seagull has found an open lunchbox full of grapes and strawberries. He eats the grapes and strawberries at different but constant rates. It takes Steven two and a half minutes for him to eat five grapes and two strawberries, and four minutes and eighteen seconds to eat six grapes and five strawberries. Use simultaneous equations to determine how long would it take Steven to eat twenty strawberries.

(c) Find the equation of the circle with centre $(0, 0)$ passing through the point $\left(2\sqrt{3}, \sqrt{6}\right)$.

(d)



(i) Use the triangle above to find an expression for $\sin\theta$.

(ii) Show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Consider a regular hexagon of side length x. Find a simplified expression for the distance between any two parallel sides.

(b) Newton, Euler and Jock solve a problem together in x hours. Newton would have taken four hours longer to solve it on his own. Euler would have taken three hours longer to solve it on his own and Jock would have taken three times as long to solve it on his own.

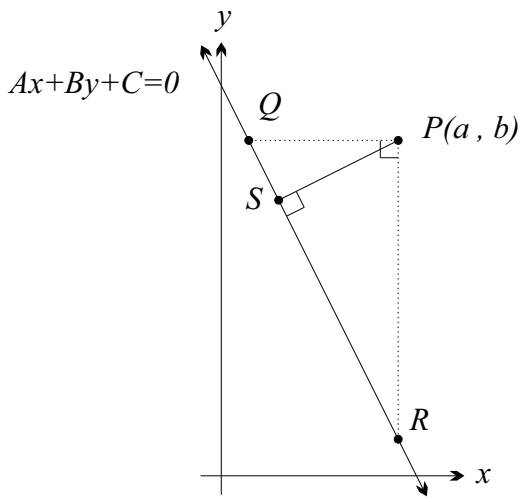
(i) Show that $x^2 + \frac{7}{4}x - 6 = 0$.

(ii) How long did it take them to solve the problem together? Give your answer in exact form.

(c) Every year on the first of January, the Mayor of Gausstown counts the population of his town. Although the population increases every year, he notices that it increases by 4% less each year, as compared to the year before. The population increased by four thousand during the 2015 calendar year, and was three hundred and sixty thousand on the first of January 2017.

What was the population of Gausstown on the first of January 2013?

(d)



Point $P(a,\ b)$ lies to the right of line $Ax+By+C=0$ as shown in the diagram above. Points Q, R, and S lie on the line $Ax + By + C = 0$, with R vertically below P, Q horizontally to the left of P and $\angle PSR = 90^\circ$.

(i) Find expressions for the coordinates of R and Q in terms of A, B, C, a and b.

(ii) Show that $\triangle RPQ \ ||| \ \triangle RSP$.

(iii) Hence show that $PS^2 = \dfrac{(Aa + Bb + C)^2}{A^2 + B^2}$.

END OF EXAMINATION

SGS Form III Annual Examination Solutions.

1) a) $\frac{3}{100} \times 200$

$= 6.$ [✓]

b) $2(2x+1)$ [✓]

c) $200\ mm = 0.2\ m.$ [✓]

d) i) $x+x+2x+2x = 6x\ cm$ [✓]

ii) $2x \times x = 2x^2\ cm^2$ [✓]

e) most often occurring $= 7$. [✓]

f) i) $2x \times 2x \times 2x = 2^3 \times x^3 = 8x^3$ ✓

ii) $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ [✓]

g) $100 \times \frac{10}{100} \times 2 = \$20.$ [✓]

h) $x^2 + 7x + 2x + 14$

$= x^2 + 9x + 14$ [✓]

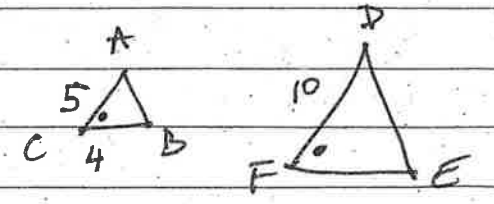
i) 5.08×10^4

$= 50,800$ [✓]

j) $\tan 60^\circ = \frac{\sqrt{3}}{1}$

$\therefore \tan 60^\circ = \sqrt{3}$ [✓]

k)



$EF = 4 \times 2 = 8$ units. [✓]

2) a) $x = -2$ [✓]

or

$x = 1$ [✓]

b) $\sin 26^\circ$

$= 0.4383371468$ [✓]

$= 0.44$ (to 2 d.p.) Don't penalise rounding here.

c) $m = 6$ [✓]

d) $x = 0$, $-3y = 12$

$y = -4$ [✓]

e) $x = 1$ [✓]

and

$y = 5$ [✓] Don't penalise $(1, 5)$

f) $\frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$\frac{2\sqrt{6}}{6}$ [✓]

$= \frac{\sqrt{6}}{3}$ [✓]

g) $d = \sqrt{(5-2)^2 + (3+1)^2}$ [✓]

$= \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5$ [✓]

h) $20{,}000 \times \left(\frac{1}{100}\right)^3$ [✓]

$= 0.02 \text{ m}^3$ [✓] Don't penalise units.

3) a) i) $x^2 + x - 6$

$= (x-2)(x+3)$ [✓✓] one for numbers, one for signs.

ii) $2ax + 3a + 6x + 9$

$= a(2x+3) + 3(2x+3)$ [✓]

$= (2x+3)(a+3)$ [✓]

b) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10-4}{3-1}$ [✓]

$= \frac{6}{2}$

$= 3$ [✓]

c) $\cos 41° = \frac{x}{7.8}$ [✓]

$x = 7.8 \cos 41°$

$x = 5.886734726$ m [✓]

$x = 5.9$ m (to 2 sig fig) [✓]

Don't penalise units
Do penalise sig. fig.

d) i) $100{,}000 \times 1.05 = \$105{,}000$ [✓]

ii) $100{,}000 \times 1.05^{10} = \$162{,}889.46$ [✓]

iii) $\frac{\text{increase}}{\text{original}} \times 100$

$= \frac{62889.4627\ldots}{100{,}000} \times 100$ [✓]

$= 62.889\%$ [✓]

$= 63\%$

4) a) $27^{-1/3}$

$= \frac{1}{\sqrt[3]{27}}$

$= 1/3$ [✓]

b) $\frac{x^2+xy}{x+y}$

$= \frac{x(x+y)}{x+y}$ [✓]

$= x$ [✓]

c) i) $\frac{2}{3x} + \frac{1}{x}$

$= \frac{2}{3x} + \frac{3}{3x}$

$= \frac{5}{3x}$ [✓]

ii) $\frac{1}{(x-4)^2} - \frac{2(x-4)}{(x-4)^2}$ [✓]

$= \frac{1-2x+8}{(x-4)^2}$

$= \frac{9-2x}{(x-4)^2}$ [✓]

d) $\frac{a+8}{2} = 6$ [✓]

$a + 8 = 12$

$a = 4$ [✓]

e) $\sin\alpha = 3/5$ [✓]

$\alpha = \sin^{-1}(3/5)$

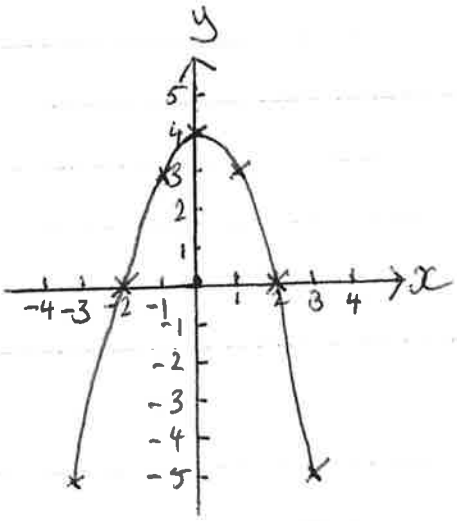
$\alpha = 36.87^\circ$ [✓]

$\alpha = 37^\circ$ to nearest degree.

f) i)

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	3	0	-5

[✓]



[✓] Shape.

[✓] Points, labels, accuracy.

(only deduct 1 mark on entire paper for unlabelled axes.)

5.)a) Construct BD.

In ΔABD and ΔCDB,

$AB = CD$ (opposite sides of a rhombus are equal.)
$AD = CB$ (opposite sides of a rhombus are equal.) } [✓]

BD is common.

$\therefore \Delta BAD \equiv \Delta BCD$ (SSS Congruency test). [✓]

$\therefore \angle BAD = \angle BCD$ (Matching angles of congruent triangles.) [✓]

Similarly, $\angle ABC = \angle ADC$

b)i) $m = \frac{1}{2}$ [✓]

ii) $y - y_1 = m(x - x_1)$ $(4, -2)$

$y - -2 = \frac{1}{2}(x - 4)$ [✓]

$y + 2 = \frac{1}{2}(x - 4)$

$y = \frac{1}{2}x - 2 - 2$

$y = \frac{1}{2}x - 4$ [✓]

c) i) $x^2 + 9x + 14 = 0$

$(x + 2)(x + 7) = 0$ [✓]

$x = -2$
or
$x = -7$ } [✓]

ii) $6x^2 - 17x + 12 = 0$

$(2x - 3)(3x - 4) = 0$ [✓]

$2x = 3$
$x = 3/2$

or
$3x - 4 = 0$
$3x = 4$
$x = 4/3$ } [✓]

iii) $\frac{x+5}{x} = \frac{8}{x-1}$

$(x+5)(x-1) = 8x$ [✓]

$x^2 - x + 5x - 5 = 8x$

$x^2 + 4x - 8x - 5 = 0$

$x^2 - 4x - 5 = 0$

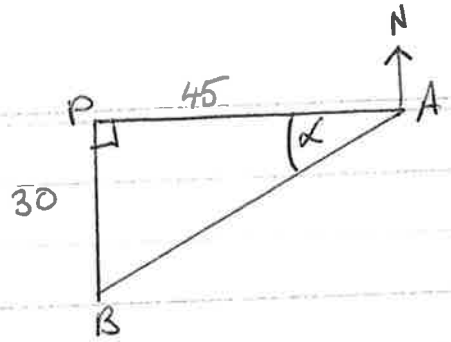
$(x+1)(x-5) = 0$ [✓]

$x = -1$

or

$x = 5$ [✓]

6.) a)



$\tan\alpha = \frac{30}{45}$ [✓]

$\alpha = 33.69°$

∴ Bearing of B from A:

$270 - 33.69° = 236.31°$ [✓]

$= 236°T$ to nearest degree.

b) i) $2x - 3y = -8$

$y = x + 3$

$2x - 3[x+3] = -8$ [✓]

$2x - 3x - 9 = -8$

$-x - 9 = -8$

$-x = 1$

$x = -1$

sub $x = -1$ into $y = x + 3$ } [✓]

$y = -1 + 3$

$y = 2$

ii) $-7x + 3y = 22$ ①

$3x - 6y = -11$ ②

① × 2

$-14x + 6y = 44$

$3x - 6y = -11$

$-11x = 33$

$x = -3$

Valid Strategy [✓]

both answers [✓]

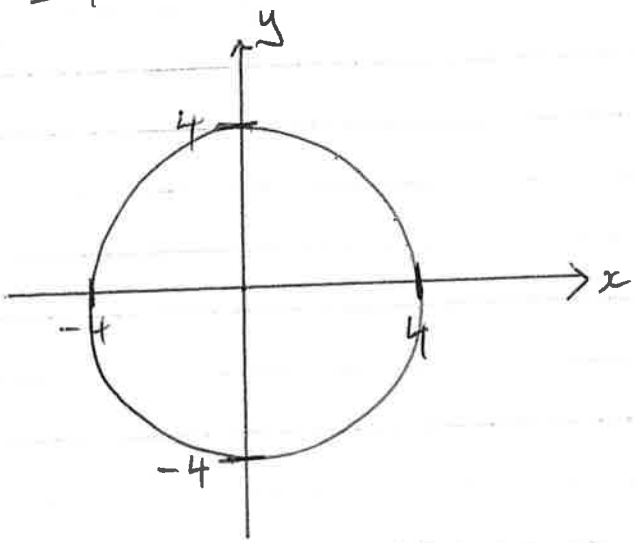
② $3[-3] - 6y = -11$

$-9 - 6y = -11$

$-6y = -2$

$y = 1/3$

c) i) $x^2 + y^2 = 4^2$



radius [✓]

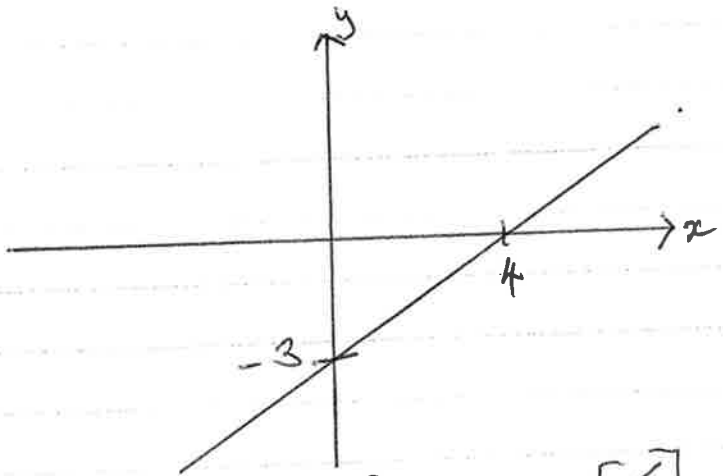
labelled, shape [✓]

ii) let $x = 0$, $-4y = 12$

$y = -3$ $(0, -3)$

let $y = 0$, $3x = 12$

$x = 4$ $(4, 0)$



Intercepts [✓]

labelled, shape, clear [✓]

d) $\dfrac{(x-1)(x+1)}{3x(x-2)} \times \dfrac{-x^2-2x+8}{x^2+5x+4}$ [✓]

$\dfrac{(x-1)(x+1)}{3x(x-2)} \times -\dfrac{(x-2)(x+4)}{(x+1)(x+4)}$ [✓]

$\dfrac{-(x-1)}{3x}$

$= \dfrac{1-x}{3x}$ [✓]

7.) i) $V = 2 \times 2 \times 3$

$= 12 \text{ m}^3$ [✓] Deduct for missing m^3

ii) α) $V_{\text{cylinder}} = 4$

$\pi r^2 h = 4$ [✓]

$r^2 = \frac{4}{\pi h}$

$r^2 = \frac{4}{3\pi}$ [✓] or equiv. method.

$r = \frac{2}{\sqrt{3\pi}}$ m.

β) S.A of hole $= \pi d \times 3$

$= \pi \times \frac{4}{\sqrt{3\pi}} \times 3$

$= \frac{12\pi}{\sqrt{3\pi}}$ [✓]

S.A of front and back face

$= \left(4 - \pi \times \left(\frac{2}{\sqrt{3\pi}}\right)^2\right) \times 2$

$= 2\left(4 - \frac{4\pi}{3\pi}\right)$

$= 2(4 - 4/3)$

$= 16/3$ [✓]

S.A of remaining sides

$= 3 \times 2 \times 4$

$= 24 \text{ m}^2$

Total S.A $= 24 + 16/3 + \frac{12\pi}{\sqrt{3\pi}}$

$= \frac{88}{3} + \frac{12\pi}{\sqrt{3\pi}}$

$= \frac{88}{3} + 4\sqrt{3\pi}$ m^2 [✓] No deduction for units.

b) let g be time to eat 1 grape. (in seconds)
let x be time to eat 1 strawberry. (in seconds)

(t in seconds) } [✓] (both)

① $5g + 2x = 150$.
② $6g + 5x = 258$

⑤ $= ① \times 6 - ② \times 2$.

$12g + 10x = 516$
$25g + 10x = 750$
$13g = 234$
$g = 18$.

← [✓] either.

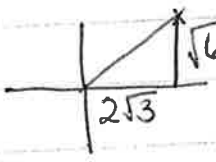
Takes Steven 18 seconds to eat 1 grape.

① $5 \times 18 + 2x = 150$.
$90 + 2x = 150$
$2x = 60$
$x = 30$.

Takes 30 seconds for 1 strawberry.

$20 \times 30 = 600$.
Takes Steven 600 seconds (10 minutes) to eat 20 strawberries. [✓]

c)



$(2\sqrt{3})^2 + (\sqrt{6})^2 = r^2$ [✓]
$r^2 = 4 \times 3 + 6$
$r^2 = 18$

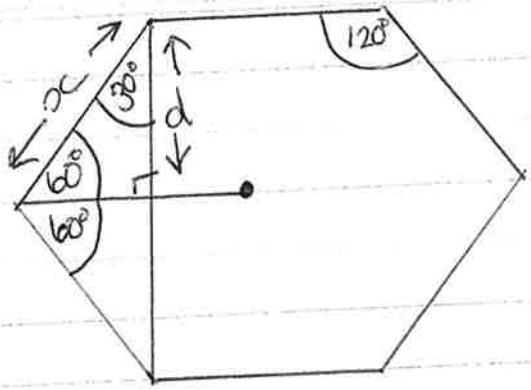
$x^2 + y^2 = 18$ [✓]

d) i) $\sin\theta = \frac{a}{c}$ [✓]

ii) $c\sin\theta = a$, $c\cos\theta = b$
$(c\sin\theta)^2 + (c\cos\theta)^2 = c^2$ [✓]

$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$

8a)



[✓]

$\sin 60° = \frac{d}{x}$

$\frac{\sqrt{3}}{2} = \frac{d}{x}$

$d = \frac{x\sqrt{3}}{2}$.

$2d = x\sqrt{3}$

∴ Distance between parallel sides $= x\sqrt{3}$ u [✓]

b) i) $\frac{1}{x+4} + \frac{1}{x+3} + \frac{1}{3x} = \frac{1}{x}$ [✓]

$3x(x+3) + 3x(x+4) + (x+4)(x+3) = 3(x+4)(x+3)$

$3x^2 + 9x + 3x^2 + 12x - 2(x+4)(x+3) = 0$

$6x^2 + 21x - 2(x^2 + 7x + 12) = 0$

$6x^2 + 21x - 2x^2 - 14x - 24 = 0$ [✓]

$4x^2 + 7x - 24 = 0$

$x^2 + \frac{7}{4}x - 6 = 0$

ii) $\left(x + \frac{7}{8}\right)^2 - \frac{49}{64} - 6 = 0$ [✓]

$\left(x + \frac{7}{8}\right)^2 = \frac{433}{64}$

$x + \frac{7}{8} = \pm\frac{\sqrt{433}}{8}$

$x = -\frac{7}{8} \pm \frac{\sqrt{433}}{8}$

$x > 0$, $x = \frac{-7 + \sqrt{433}}{8}$ hours. [✓]

c)

Increases in	Yearly Increase	Population (end of yr)
2012.	$\frac{4006}{0.96^2} = 4340.27$	[✓]
2013	$\frac{4000}{0.96} = 4166.6$	
2014	4000	
2015	$4000 \times 0.96 = 3840$	
2016.		360,000
2017		

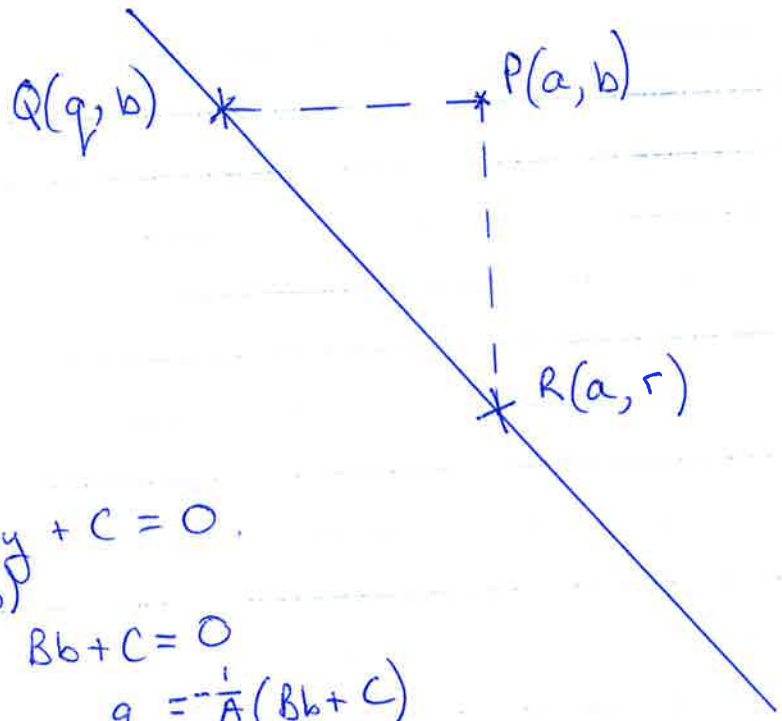
$$360,000 - 3840 - 4000 - \frac{4000}{0.96} - \frac{4000}{0.96^2}$$

[✓]

$= 348,653.06$ people.

$= 343,653$ people.

d)



i) $Ax + By + C = 0$.

$Q(q, b)$

$Aq + Bb + C = 0$

$q = -\frac{1}{A}(Bb + C)$

$Q\left(-\frac{1}{A}(Bb+C), b\right)$

$Ax + By + C = 0$

$R(a, r)$

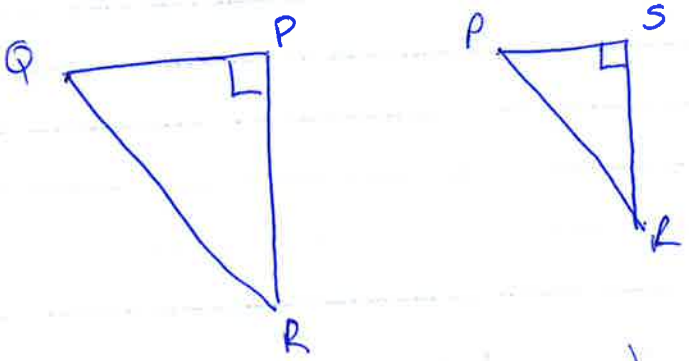
$Aa + Br + C = 0$

$r = -\frac{1}{B}(Aa + C)$

$R\left(a, -\frac{1}{B}(Aa+C)\right)$

Both [✓]

ii)



$\angle SRP = \angle PRQ$ (Common)

$\angle QPR = \angle PSR = 90°$ (Given)

$\therefore \triangle RPQ \equiv \triangle RSP$ (AAA Similarity test) [✓]

iii) $\frac{PS^2}{PR^2} = \frac{PQ^2}{QR^2}$. [✓]

$$\frac{PS^2}{\left(b+\frac{1}{B}(Aa+C)\right)^2} = \frac{\left(a+\frac{1}{A}(Bb+C)\right)^2}{\left(-\frac{1}{A}(Bb+C)-a\right)^2+\left(b+\frac{1}{B}(Aa+C)\right)^2}$$ [✓]

$$PS^2 = \frac{\left(a+\frac{1}{A}(Bb+C)\right)^2\left(b+\frac{1}{B}(Aa+C)\right)^2}{\left(a+\frac{1}{A}(Bb+C)\right)^2+\left(b+\frac{1}{B}(Aa+C)\right)^2}$$

$$PS^2 = \frac{\frac{1}{A^2}(Aa+Bb+C)^2 \times \frac{1}{B^2}(Bb+Aa+C)^2}{\frac{1}{A^2}(Aa+Bb+C)^2+\frac{1}{B^2}(Bb+Aa+C)^2}.$$

$$PS^2 = \frac{\frac{1}{A^2B^2}(Aa+Bb+C)^2(Aa+Bb+C)^2}{\left(\frac{1}{A^2}+\frac{1}{B^2}\right)(Aa+Bb+C)^2}$$

$$PS^2 = \frac{\frac{1}{A^2B^2}(Aa+Bb+C)^2}{\frac{1}{A^2}+\frac{1}{B^2}}$$

$$PS^2 = \frac{\frac{1}{A^2B^2}(Aa+Bb+C)^2}{\frac{A^2+B^2}{A^2B^2}}$$ [✓]

$$PS^2 = \frac{(Aa+Bb+C)^2}{A^2+B^2}$$ as required.