NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL

2018 Annual Examination

FORM III

MATHEMATICS

Wednesday 31st October 2018

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: LRP	3B: LL	3C: OMD	3D: RPL
3E: PKH	3F: BR	3G: LYL	3H: RCF
3I: BDD	3J: WJM		

Checklist

- Writing paper — approx. 16 sheets
- Candidature — 192 boys

Examiner

BR

QUESTION ONE (13 marks) Start a new page.

(a) Simplify $\sqrt{8}$.

(b) If $a = 4$ and $b = -2$, evaluate ab^2.

(c) Simplify $15t^2 \div 5t^2$.

(d) Expand $5(r - 2)$.

(e) Write down the gradient of the line $y = 5x - 7$.

(f) Use your calculator to evaluate $\sin 35°$. Give your answer correct to 3 decimal places.

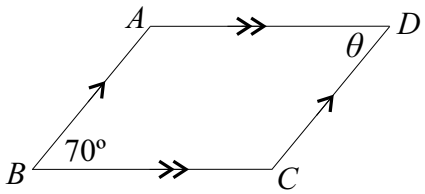
(g) Expand and simplify $(y + 3)(y - 7)$.

(h) Find the midpoint M of the interval AB for the points $A(2, 8)$ and $B(6, 6)$.

(i) Write down the equation of the line which has a gradient of 7 and a y-intercept of $(0, 2)$.

(j) Solve $(x - 9)(x + 10) = 0$.

(k)



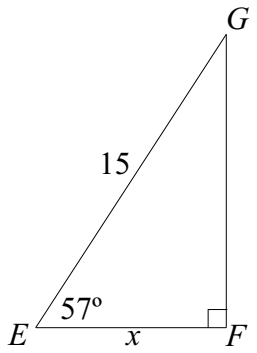
Find the value of θ in the diagram above giving a clear geometrical reason(s).

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Simplify $3\sqrt{2} - \sqrt{2}$.

(b)



Find the value of x. Give your answer correct to 2 decimal places.

(c) Factorise:

(i) $x^2 + 7x + 6$

(ii) $a^2 - 25$

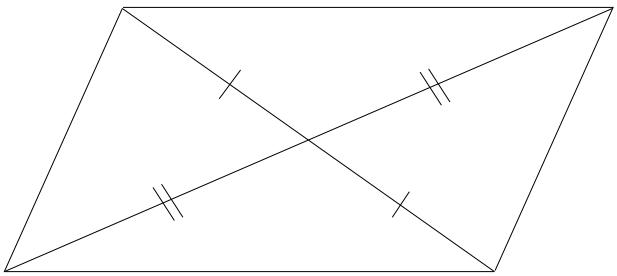
(d) Find the gradient of the line PQ for the points $P(5, 1)$ and $Q(6, -2)$.

(e) Sketch the line $y = 2x + 1$ on a number plane, clearly showing x- and y-intercepts.

(f) Find the length of the interval AB for the points $A(4, -5)$ and $B(2, 7)$. Give your answer as a simplified surd.

(g) Find the area of a circle with radius 3·5 cm. Give your answer in exact form.

(h)



Using the information on the diagram above, state which test for special quadrilaterals could be used to prove that it is a parallelogram.

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

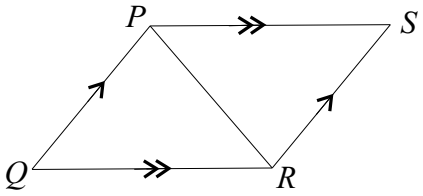
(a) Write the following numbers in scientific notation:

(i) 302 000

(ii) 0·000 082 6

(b) Simplify $\left(64a^3b^6\right)^{\frac{1}{3}}$.

(c)



The diagram above shows the parallelogram $PQRS$. Prove $\triangle PQR \equiv \triangle RSP$.

(d) Make x the subject of the formula $y = 3x + 5$.

(e) Solve $x^2 + 7x + 10 = 0$.

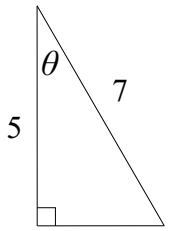
(f) Write $\dfrac{7}{\sqrt{2}}$ with a rational denominator.

(g) In a test out of 5, the marks of 12 pupils were recorded below.

0, 1, 1, 1, 2, 3, 3, 4, 5, 5, 5, 5.

Write down the median score.

(h)

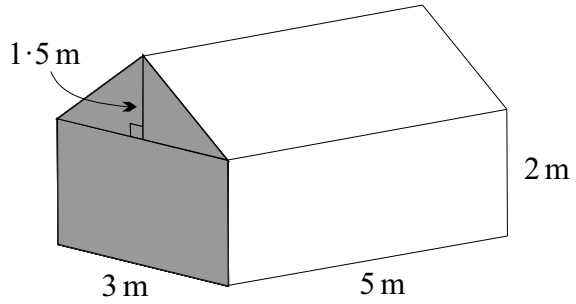


Find the value of θ. Give your answer correct to the nearest degree.

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a)



The shed shown in the diagram above consists of a rectangular prism and a triangular prism.

(i) What is the area of the front face of the shed, as shaded in the diagram?

(ii) Hence find the volume of the shed.

(b) Find the exact value of $\sin 60°$.

(c) Solve the following equations simultaneously:

$$x + y = 1$$
$$y = 3x - 7$$

(d) Expand and simplify $\left(\sqrt{2}+2\right)\left(3-\sqrt{2}\right)$.

(e) Solve $\dfrac{3a+2}{4} = \dfrac{2a-4}{3}$.

(f) Factorise and simplify $\dfrac{x^2-x-2}{x^2-5x+6}$.

(g) Express $\dfrac{2}{x-1} - \dfrac{3}{x+2}$ as a single simplified fraction.

Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Factorise $6x^2 + 7x - 5$.

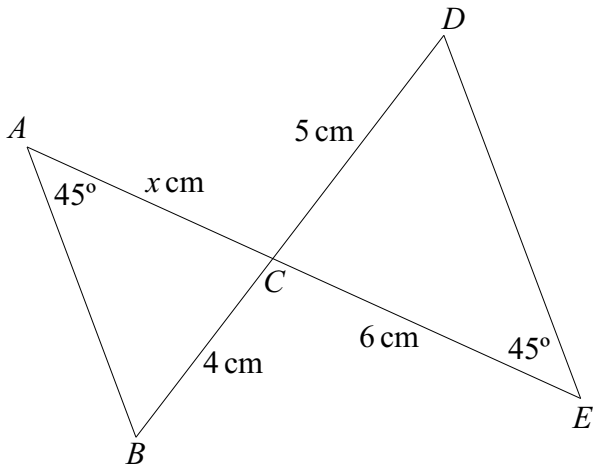
(b) The table below shows the number of dresses sold in Fashion Warehouse last week, organised by dress size.

Dress Size	Frequency
8	5
10	9
12	7
14	4

(i) Use the table above to find the mean dress size.

(ii) What is the modal dress size and why is the mode a better measure to use in this instance for future statistical analysis?

(c)



In the diagram above, C lies on the lines AE and BD.

(i) Prove that $\triangle ABC \,|||\, \triangle EDC$, giving clear geometrical reasons.

(ii) Find the value of x. You must give a geometrical reason.

(d) Write down the equation of the line which is parallel to $x = 3$ and passes through the point $(-1, 4)$.

(e) Solve $19x - x^2 = 84$.

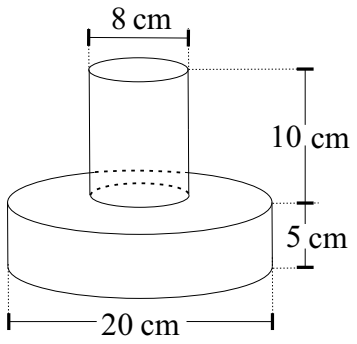
(f) Raphael invested \$5000 in rare stamps and in successive years their valued showed a 23% gain, a 17% gain and a 6% loss. What is the value of Raphael's investment at the end of the 3 year period? Give your answer to the nearest dollar.

Examination continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a) Find the equation of the line which is perpendicular to $x + 3y + 4 = 0$ and passes through the point $(1, 4)$. Write your answer in gradient-intercept form.

(b)



What is the surface area of the solid shown above? Give your answer in exact form.

(c) Two cars travelling at different speeds leave from the same starting point S.

Car X drives in the direction due west.

Car Y drives in the direction with a bearing of 195°T.

After car X has travelled 15 km, it is due north of car Y.

(i) Draw a diagram representing the information given above.

(ii) How far apart are the two cars? Give your answer correct to the nearest kilometre.

(d) Solve $\dfrac{7}{2x+3} + 1 = x$.

(e) Solve the following equations simultaneously:

$$2x - 3y = 4$$
$$4x + 5y = 30$$

Examination continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

(a) Jonty has \$50 000 to invest. He will invest the money for a 4 year period.

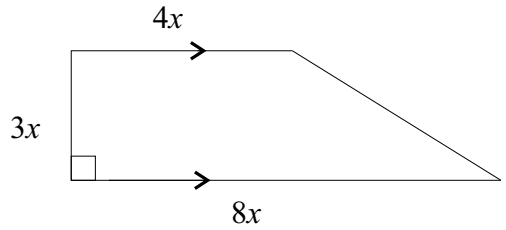
(i) Calculate the simple interest earned if he places the money in an St George Bank account which pays 12% per annum.

(ii) Calculate the final amount if he places the money in an ANZ account which pays 10% per annum compounded annually.

(iii) Which bank account will earn Jonty more interest over the 4 year period? By how much?

(b) Vishal and Devon are testing out their new compasses in the playground. Vishal observes that Devon is on a bearing of 163°T. What is the true bearing of Vishal from Devon?

(c)



Find the area of the trapezium shown above if it has a perimeter of 60 centimetres.

(d) Make u the subject of the formula $v^2 = u^2 + 2as$.

(e) Hugo has twice as much money as Jamie. If Hugo gives Jamie \$10.25, Jamie will have three times as much money as Hugo. By forming a pair of simultaneous equations, calculate how much money Hugo has.

(f) Simplify $\dfrac{4x^2 - x - 5}{x^2 - 4x - 5} \div \dfrac{4x^2 - 5x}{3x^2 - 75}$.

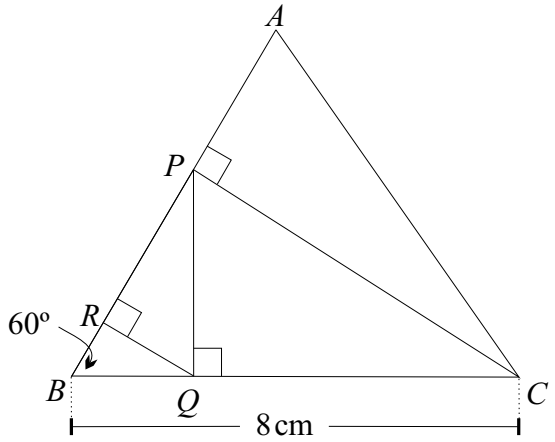
Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $\dfrac{5}{x+2} - \dfrac{8}{x-7} + \dfrac{3x+2}{x^2-5x-14}$.

(b) Solve $x^2 - 3x - 8 = 0$ by completing the square.

(c)



In the diagram above, triangle ABC has lines CP, PQ and QR drawn perpendicular to AB, BC and AB respectively, $\angle ABC = 60^\circ$ and $BC = 8\,\text{cm}$.

Find the exact value of QR.

(d) A circle with centre O is fully contained within an isosceles triangle ABC such that the circle touches the sides AB, BC and AC only once at P, Q and R respectively. The radii OP, OQ and OR are perpendicular to the sides of the triangle at P, Q and R.

Given $AB = AC = 85\,\text{cm}$ and $BC = 80\,\text{cm}$, find the length of the radius of the circle.

(e) $x^2 - 19x + 94$ is a perfect square where x is an integer. Find the largest possible value of x.

END OF EXAMINATION

Form III Annual 2018 Solutions

Question 1

a) $2\sqrt{2}$ ✓

b) $a = 4, b = -2 \therefore ab^2 = (4)(-2)^2$
$= 16$ ✓

c) 3 ✓

d) $5r - 10$ ✓

e) $m = 5$ ✓

f) $\sin 35^\circ = 0 \cdot 5735\,7643\,6\ldots$ ✓
$\doteq 0 \cdot 574$ ✓ (rounding)

g) $y^2 - 4y - 21$ ✓

h) $M\left(\frac{2+6}{2}, \frac{8+6}{2}\right) = M(4, 7)$ ✓

i) $y = 7x + 2$ ✓

j) $x = 9, -10$ ✓

k) $\theta = 70^\circ$ ✓ [opposite angles of a parallelogram] ✓

[or equivalent]

Question 2

a) $2\sqrt{2}$ ✓

b) $\cos 57^\circ = \frac{x}{15}$ ✓

$x = 15 \cos 57^\circ$

$\doteq 8.17$ ✓

c) i) $(x+6)(x+1)$ ✓

ii) $(a+5)(a-5)$ ✓

d) $P(5, 1)$ $Q(6, -2)$

$m = \frac{1+2}{5-6}$ ✓

$= -3$ ✓

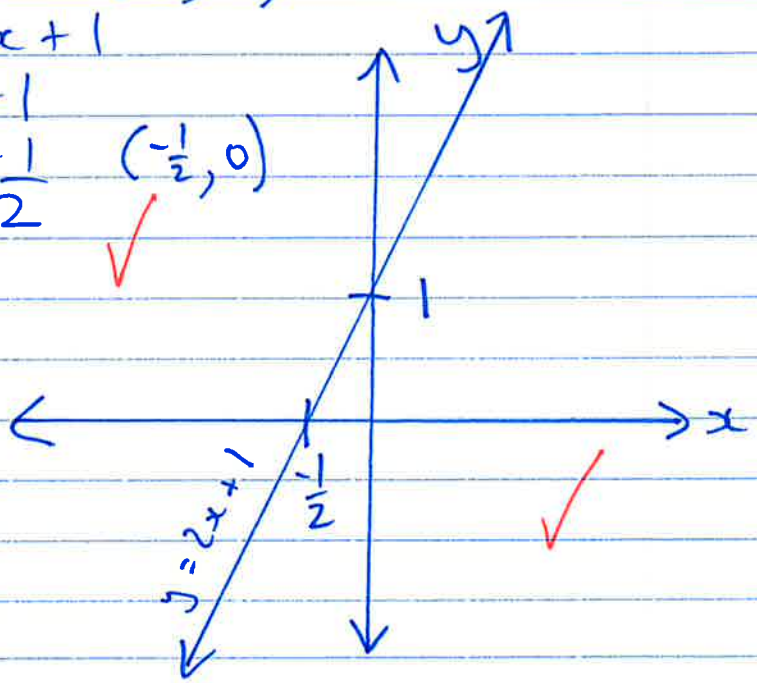
e) $y = 2x + 1$

$x = 0, \quad y = 1 \quad (0, 1)$

$y = 0, \quad 0 = 2x + 1$

$2x = -1$

$x = -\frac{1}{2} \quad (-\frac{1}{2}, 0)$ ✓



✓

f) $A(4,-5)$ $B(2,7)$

$AB^2 = (4-2)^2 + (-5-7)^2$ ✓

$\therefore AB^2 = 148$

$\therefore AB = 2\sqrt{37}$ units ✓ (must be simplified)

g) $A = \pi(3.5)^2$

$= 12.25\pi \text{ cm}^2$ ✓

h) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

✓

Question 3

a) i) 3.02×10^5 ✓

ii) 8.26×10^{-5} ✓

b) $4ab^2$ ✓

c) (can be done by SSS, SAS, AAS)

In $\triangle$'s PQR, RSP

$\angle RPQ = \angle PRS$ [alternate angles, PQ || SR] ✓

$\angle QRP = \angle SPR$ [alternate angles, PS || QR]

PR is common [PR = RP] ✓

$\therefore \triangle PQR \equiv \triangle RSP$ [AAS] ✓

d) $y = 3x + 5$

$3x + 5 = y$

$3x = y - 5$

$x = \frac{y-5}{3}$ ✓

e) $x^2 + 7x + 10 = 0$

$(x+5)(x+2) = 0$ ✓

$\therefore x = -5, -2$ ✓

f) $\frac{7}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$ ✓

g) 3 ✓

h) $\cos\theta = \frac{5}{7}$ ✓

$\theta = \cos^{-1}\left(\frac{5}{7}\right)$

$\doteq 44^\circ$ ✓

Question 4

a) i) $A = (3)(2) + \frac{1}{2}(3)(1.5)$

$= 8.25 \text{ m}^2$ ✓

ii) $V = (8.25)(5)$

$= 41.25 \text{ m}^3$ ✓

b) $\frac{\sqrt{3}}{2}$ ✓

c) $x + y = 1 \quad \ldots (1)$

$y = 3x - 7 \quad \ldots (2)$

sub (2) into (1) $\Rightarrow$ $x + 3x - 7 = 1$ ✓

$4x - 7 = 1$

$4x = 8$

$x = 2$

$y = -1$ ✓

$\therefore \quad x = 2, \; y = -1$

d) $(\sqrt{2} + 2)(3 - \sqrt{2}) = 3\sqrt{2} - 2 + 6 - 2\sqrt{2}$ ✓

$= 4 + \sqrt{2}$ ✓

e) $\frac{3a+2}{4} = \frac{2a-4}{3}$

$3(3a+2) = 4(2a-4)$ ✓

$9a + 6 = 8a - 16$

$a + 6 = -16$ ✓

$\therefore a = -22$ ✓

f) $\frac{x^2-x-2}{x^2-5x+6} = \frac{(x+1)(x-2)}{(x-2)(x-3)}$ ✓

$= \frac{x+1}{x-3}$ ✓

g) $\frac{2}{x-1} - \frac{3}{x+2} = \frac{2(x+2)-3(x-1)}{(x-1)(x+2)}$ ✓

$= \frac{2x+4-3x+3}{(x-1)(x+2)}$

$= \frac{7-x}{(x-1)(x+2)}$ ✓

Question 5

a) $6x^2 + 7x - 5$

P	-30
S	10, -3

$= 6x^2 + 10x - 3x - 5$

$= 2x(3x+5) - 1(3x+5)$

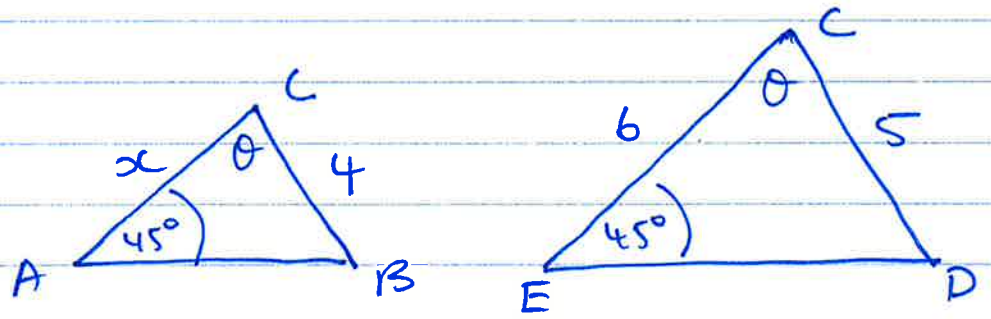
$= (3x+5)(2x-1)$ ✓

b) i) $\text{mean} = \dfrac{8(5) + 10(9) + 12(7) + 14(4)}{5+9+7+4}$

$= 10 \cdot 8$ ✓

ii) The mode is 10. ✓ It is not a fraction/decimal and it is the most popular size. This is helpful for future ordering. ✓

c)



i) In $\triangle$'s ABC, EDC

$\angle CAB = \angle CED = 45°$ [given]

$\angle ACB = \angle ECD$ [vertically opposite angles] ✓

$\therefore \triangle ABC \,|||\, \triangle EDC$ [AA] ✓

ii) $\frac{x}{6} = \frac{4}{5}$ [matching sides of similar triangles] ✓ (must have reason)

$\therefore x = \frac{24}{5}$ cm ✓

d) $x = -1$ ✓

e) $19x - x^2 = 84$

$x^2 - 19x + 84 = 0$

$(x-12)(x-7) = 0$ ✓

$\therefore x = 12, 7$ ✓

f) $A = 5000(1{\cdot}23)(1{\cdot}17)(0{\cdot}94)$ ✓

$\doteq \$6764$ ✓

Question 6

a) $x + 3y + 4 = 0$

$3y = -x - 4$

$y = -\frac{1}{3}x - \frac{4}{3}$

$m = -\frac{1}{3}$

$m_\perp = 3$ ✓ $(1, 4)$

$y - 4 = 3(x - 1)$

$y - 4 = 3x - 3$

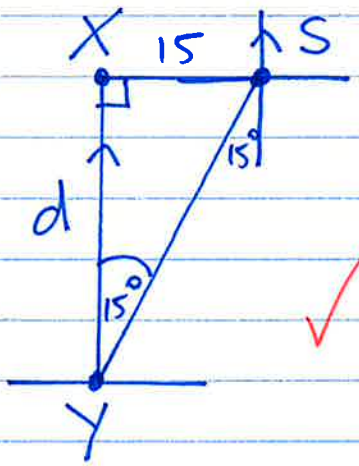
$y = 3x + 1$ ✓ (must be in gradient-intercept form)

b) $SA = 2\pi\left(\frac{20}{2}\right)^2 + 2\pi\left(\frac{20}{2}\right)(5)$

$+ 2\pi\left(\frac{8}{2}\right)(10)$ ✓

$= 380\pi \text{ cm}^2$ ✓ ✓ (units)

c) i)



✓

ii) $\tan 15° = \frac{15}{d}$ ✓

$d = \frac{15}{\tan 15°} \doteq 56 \text{ km}$ ✓

d) $\frac{7}{2x+3} + 1 = x$

$7 + 2x + 3 = x(2x+3)$

$2x + 10 = 2x^2 + 3x$ ✓

$2x^2 + x - 10 = 0$

p	-20
5	5, -4

$2x^2 + 5x - 4x - 10 = 0$

$x(2x+5) - 2(2x+5) = 0$ ✓

$(2x+5)(x-2) = 0$

$\therefore x = -\frac{5}{2}, 2$ ✓

e) $2x - 3y = 4 \quad \ldots (1)$

$4x + 5y = 30 \quad \ldots (2)$

$(1) \times 2 \Rightarrow \quad 4x - 6y = 8 \quad \ldots (3)$

$(2) - (3) \Rightarrow \quad 11y = 22$

$y = 2 \quad \ldots (4)$ ✓

sub (4) into (1) $\Rightarrow \quad 2x - 3(2) = 4$

$2x - 6 = 4$

$2x = 10$

$x = 5$ ✓

$\therefore \quad x = 5, \; y = 2$

Question 7

a) $P = 50\,000 \quad n = 4$

i) $r = 0 \cdot 12$

$\therefore I = (50\,000)(0 \cdot 12)(4)$

$= \$24\,000$ ✓

ii) $r = 0 \cdot 1$

$\therefore A = (50\,000)(1 + 0 \cdot 1)^4$

$= \$73\,205$ ✓

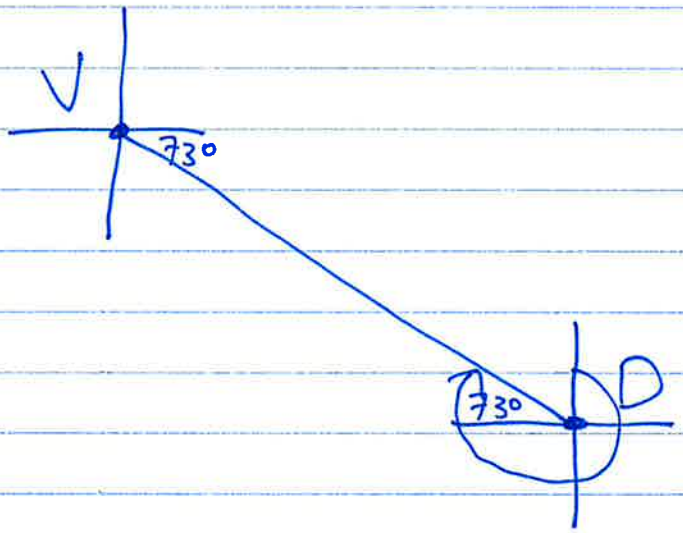
iii) $I_{simple} = 24\,000$

$I_{compound} = 73\,205 - 50\,000$

$= 23\,205$

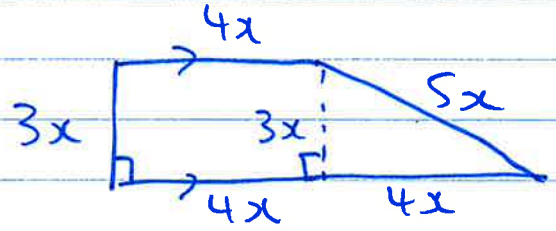
$\therefore$ St George will earn more money by $\$795$. ✓

b)



The bearing of Vishal from Devon is $343°\,T$. ✓

c)

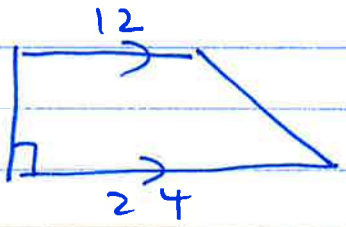


$60 = 3x + 4x + 5x + 8x$

$60 = 20x$

$\therefore x = 3$ ✓

$\therefore$



$A = \frac{1}{2}(9)(12+24)$

$= 162 \text{ cm}^2$ ✓

d) $v^2 = u^2 + 2as$

$u^2 = v^2 - 2as$

$\therefore u = \pm\sqrt{v^2 - 2as}$ ✓✓ 1 mark for $u = \sqrt{v^2-2as}$

e) Let Hugo have h dollars

Let Jamie have j dollars

$h = 2j$

$j = \frac{1}{2}h \quad \ldots (1)$

$3(h - 10.25) = j + 10.25 \quad \ldots (2)$ ✓

sub (1) into (2) $3h - 30.75 = \frac{1}{2}h + 10.25$

$2\frac{1}{2}h = 41$

$h = 16.4$

$\therefore$ Hugo has \$16.40 ✓

f) $4x^2 - x - 5$

P	-20
S	-5, 4

$= 4x^2 - 5x + 4x - 5$

$= x(4x-5) + 1(4x-5)$

$= (4x-5)(x+1)$ ✓

$x^2 - 4x - 5$

$= (x+1)(x-5)$

$3x^2 - 75 = 3(x^2 - 25)$

$= 3(x+5)(x-5)$

$4x^2 - 5x = x(4x-5)$

$\therefore \frac{4x^2 - x - 5}{x^2 - 4x - 5} \times \frac{3x^2 - 75}{4x^2 - 5x}$ ✓

$= \frac{(4x-5)(x+1)}{(x+1)(x-5)} \times \frac{3(x+5)(x-5)}{x(4x-5)}$

$= \frac{3(x+5)}{x}$ ✓

OR

$\frac{3x+15}{x}$

Question 8

a) $\frac{5}{x+2} - \frac{8}{x-7} + \frac{3x+2}{(x+2)(x-7)}$

$= \frac{5(x-7) - 8(x+2) + 3x + 2}{(x+2)(x-7)}$ ✓

$= \frac{5x - 35 - 8x - 16 + 3x + 2}{(x+2)(x-7)}$

$= \frac{-49}{(x+2)(x-7)}$ ✓

b) $x^2 - 3x - 8 = 0$

$x^2 - 3x = 8$

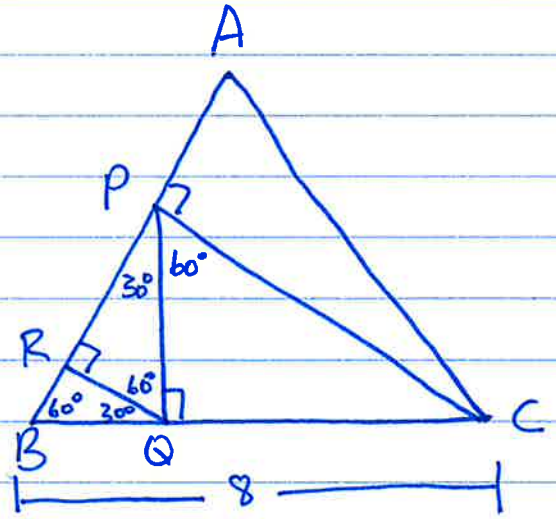
$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 8 + \left(\frac{3}{2}\right)^2$

$\left(x - \frac{3}{2}\right)^2 = \frac{41}{4}$ ✓

$x - \frac{3}{2} = \pm \frac{\sqrt{41}}{2}$

$\therefore x = \frac{3 \pm \sqrt{41}}{2}$ ✓

c)



$\angle BQR = 30°$ [angle sum of $\triangle RBQ$]
$\angle RQP = 60°$ [adjacent angles]
$\angle QPR = 30°$ [angle sum of $\triangle QPR$]
$\angle CPQ = 60°$ [adjacent angles]

In $\triangle QPR$, $\cos 60° = \frac{QR}{QP}$

$QR = QP \cos 60°$

$\therefore QR = \frac{1}{2} QP$ ✓

In $\triangle PQC$, $\cos 60° = \frac{QP}{PC}$

$QP = PC \cos 60°$

$\therefore QP = \frac{1}{2} PC$ ✓

In $\triangle PBC$, $\sin 60° = \frac{PC}{8}$

$PC = 8 \sin 60°$

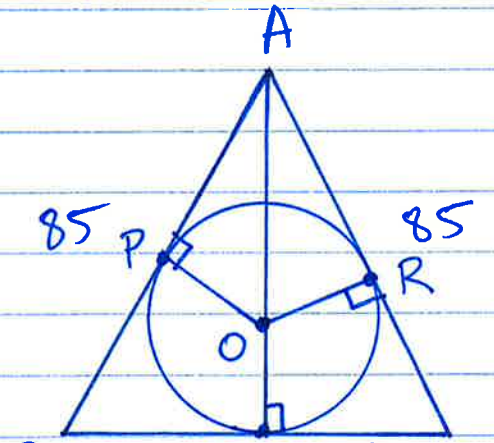
$PC = 8 \cdot \frac{\sqrt{3}}{2}$

$\therefore PC = 4\sqrt{3}$

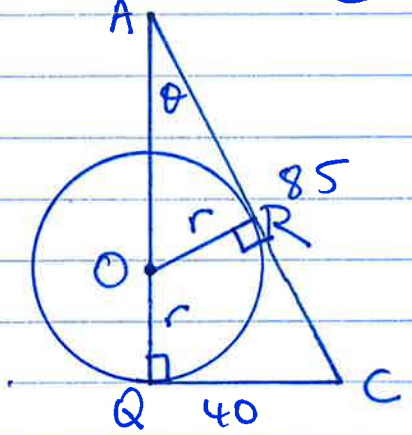
$\therefore QP = 2\sqrt{3}$

$\therefore QR = \sqrt{3}$ cm ✓

d)

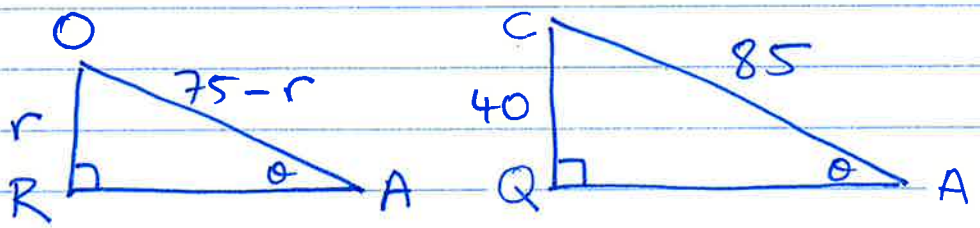


Recall; altitude of isosceles triangle bisects the base.



$AQ^2 = 85^2 - 40^2$

$\therefore AQ = 75$ ✓



$\triangle ORA \,|||\, \triangle CQA$ [AA] ✓

$\therefore \dfrac{75-r}{85} = \dfrac{r}{40}$ [matching sides of similar triangles]

$40(75-r) = 85r$

$3000 - 40r = 85r$

$125r = 3000$

$r = 24$

$\therefore$ The radius is 24 cm ✓

e) Let $x^2 - 19x + 94 = b^2$ where b is an integer ✓

$\therefore x^2 - 19x + \left(\frac{19}{2}\right)^2 + 94 - \left(\frac{19}{2}\right)^2 = b^2$

$\left(x - \frac{19}{2}\right)^2 + \frac{15}{4} = b^2$

$(2x - 19)^2 + 15 = 4b^2$

$4b^2 - (2x-19)^2 = 15$

$(2b + 2x - 19)(2b - 2x + 19) = 15$ ✓ ...①

$\therefore 2b + 2x - 19 = 1, 3, 5, 15, -1, -3, -5, -15$

$2b - 2x + 19 = 15, 5, 3, 1, -15, -5, -3, -1$ ...②

① - ② $\Rightarrow 4x - 38 = -14, -2, 2, 14, 14, 2, -2, -14$

$4x - 38 = -14, -2, 2, 14$

$4x = 24, 36, 40, 52$

$\therefore x = 6, 9, 10, 13$

$\therefore$ The largest x is 13. ✓