NAME MASTER

SYDNEY GRAMMAR SCHOOL



2015 Annual Examination

FORM III

MATHEMATICS

Friday 30th October 2015

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: SO	3B: DWH	3C: PKH	3D: SG/BDD
3E: FMW	3F: LYL	3G: KWM	3H: RCF
3I: DNW/LRP	3J: MLS/BR		

Checklist

- Writing paper required.
- Candidature — 193 boys

Examiner

SG

QUESTION ONE (13 marks) Start a new page.

(a) Use your calculator to find $\sin 73^\circ$ correct to four decimal places.

(b) Simplify $27^{\frac{1}{3}}$.

(c) Find the area of a circle with radius 4 cm. Give your answer in terms of π.

(d) Factorise $2x - x^2$.

(e) For the line $y = 3x - 1$, write down:

(i) the gradient

(ii) the y-intercept

(f) Simplify $\dfrac{3k}{2} + \dfrac{k}{2}$.

(g) Express $\dfrac{3}{\sqrt{5}}$ with a rational denominator.

(h) Simplify $a^2 \times a^3$.

(i) Given $\cos\theta = 0{\cdot}809$, find θ correct to the nearest degree.

(j) Simplify $4\sqrt{7} - \sqrt{7}$.

(k) Find the midpoint M of $P(1, 5)$ and $Q(9, 3)$.

QUESTION TWO (13 marks) Start a new page.

(a) Find the gradient of the interval PQ where $P = (2, 1)$ and $Q = (3, 4)$.

(b) For the set of scores

0, 1, 1, 2, 3, 3, 3, 4, 5

find:

(i) the range

(ii) the mean

(iii) the median

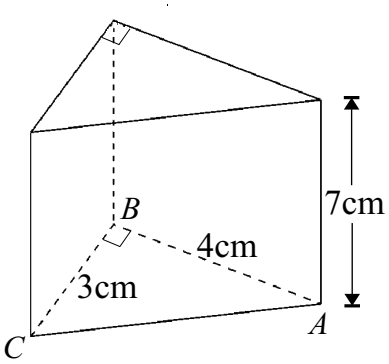
(iv) the mode

Exam continues next page ...

(c) Solve the simultaneous equations:

$$x - y = 2$$
$$x = 3y$$

(d)



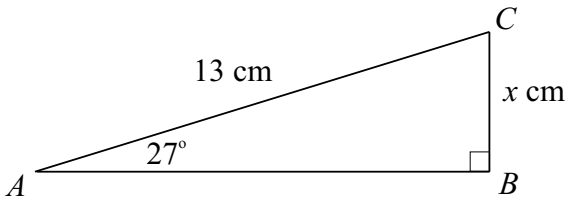
The diagram above shows a triangular prism with height 7 cm. The base ABC is a right-angled triangle with sides $BC = 3$ cm, $AB = 4$ cm and hypotenuse AC.
Find the volume of the prism.

(e) Solve $(x - 4)(x + 11) = 0$.

(f) Find the distance between the points $A = (6, 3)$ and $B(2, -3)$. Leave your answer in exact form and fully simplified.

(g) Factorise $x^2 - 9$.

(h)



In $\triangle ABC$ above, find the value of x correct to one decimal place.

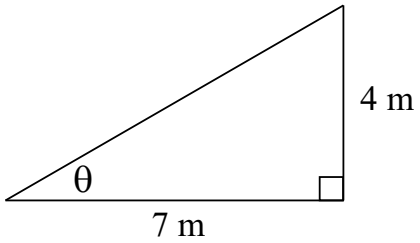
Exam continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

(a) Solve $9 - 3x < 6$.

(b) Find the equation of the line with gradient 3 and passing through the point $(1, 5)$.

(c)



In the triangle above, find angle θ correct to the nearest degree.

(d) (i) Factorise $x^2 + 5x - 24$.

(ii) Hence solve $x^2 + 5x - 24 = 0$.

(e) Solve simultaneously:

$$4x + y = 9$$
$$3x - 2y = 4$$

(f) State whether the following sentences are true or false:

(i) A square is also a rhombus.

(ii) The diagonals of a parallelogram always intersect at right-angles.

(iii) A parallelogram that is not a rectangle must be a square.

(g)

1	6	8							
2	0	2	2	4	8	8			
3	0	0	2	2	4	4	6	6	8
4	0	2	2	2	4	6	6		
5	2	4							

The stem-and-leaf plot above shows 26 results for a class test marked out of 60.

(i) Find the mode.

(ii) Find the median.

Exam continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) Express $\frac{4y}{5} - \frac{2y}{3}$ as a single fraction.

(b) Simplify $\frac{25p^9r^{-3}}{5p^3r^{-1}}$, expressing your answer with positive indices.

(c) The formula for the area A of a trapezium is given by

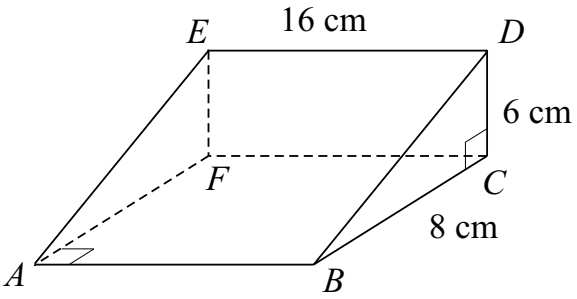
$$A = \frac{1}{2}(a+b)h$$

where a and b are the lengths of the parallel sides and h is the height.

(i) Make a the subject of the formula.

(ii) For the trapezium with $A = 72\,\text{mm}^2$, $h = 4\,\text{mm}$ and $b = 6\,\text{mm}$, find a.

(d)



For the solid above:

(i) Find BD.

(ii) Hence find the surface area of the triangular prism.

(e) Solve:

(i) $5x^2 + 7x - 12 = 0$ by factoring.

(ii) $x^2 + x - 1 = 0$ by completing the square.

(f) Simplify $\frac{4x+2}{2}$.

Exam continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Simplify $\dfrac{2x+6}{x^2+6x+9}$.

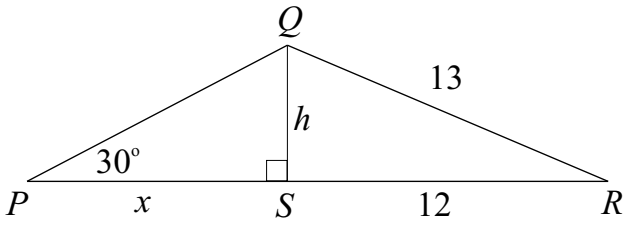
(b) Find the equation of the line passing through the points $A(5,-2)$ and $B(7,4)$.

(c) Solve:

(i) $\dfrac{2x}{3}-\dfrac{5x}{2}=1$

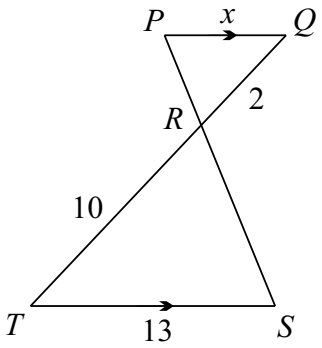
(ii) $\dfrac{x}{12}=\dfrac{5}{x-11}$

(d)



In triangle PQR, interval QS is perpendicular to PR, side $QR=13$ and $SR=12$.
Let $PS=x$ and $QS=h$.
Find the length of PS, leaving your answer in exact form.

(e)



In the figure, PQ is parallel to ST.

(i) Prove that $\triangle PQR \,|||\, \triangle STR$.

(ii) Hence find x, giving a reason.

Exam continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a) A cylinder has a volume of $108\pi\text{cm}^3$. Its height is twice its diameter. Find the diameter.

(b) Solve $\dfrac{5y-6}{6y} = \dfrac{1}{y+1}$.

(c) Two friends, Atticus and Che, spend the day at an amusement park enjoying their favourite rides: *The Claw* and *The Drop*. Their tickets are stamped each time they take one of these rides. At the end of the day, their tickets show the following:

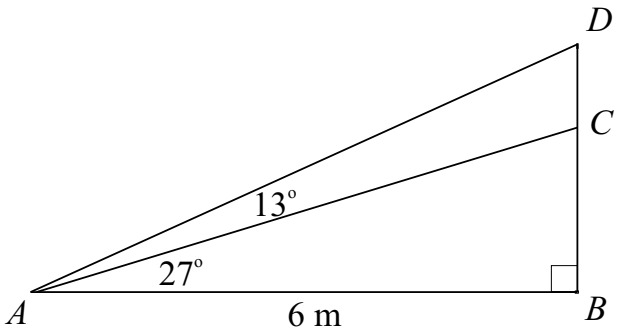
Ride	Number
The Claw	****
The Drop	*
Total($)	21.20

Ride	Number
The Claw	**
The Drop	*****
Total($)	27.70

(i) Form a pair of simultaneous equations for the cost c of one ride on *The Claw* and the cost d of one ride on *The Drop*.

(ii) Solve your equations for c and d.

(d)



Find the value of CD in the triangle above. Give your answer to the nearest whole number.

(e) Simplify:

(i) $\dfrac{a^2+3a+2}{2a+4} \times \dfrac{1}{a+1}$

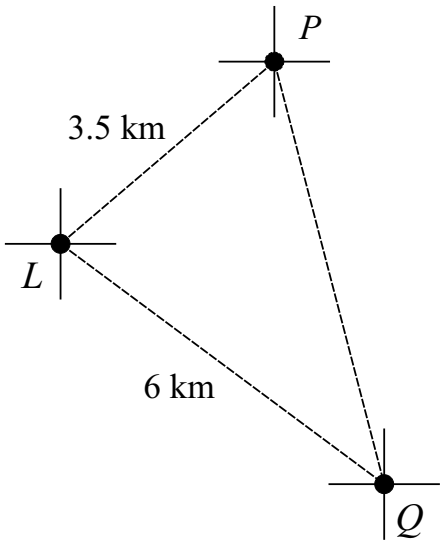
(ii) $\dfrac{3}{2x+8} \div \dfrac{9}{3x+12}$

Exam continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

(a) Factorise $6x^2y - 15y - 5x + 18xy^2$.

(b)



In the diagram above, a ship P is 3·5 km from a lighthouse L on a bearing of 031°T. A second ship Q is 6 km from L on a bearing of 121°T.

(i) Find $\angle PLQ$, showing clear reasoning.

(ii) Hence find the distance of Q from P.

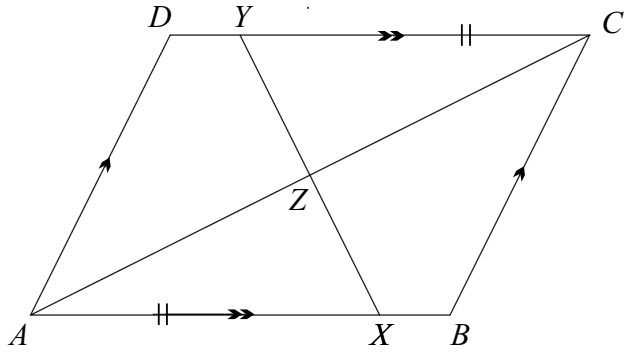
(c) Two people, A and B, took part in a race. B completed the race in 20 minutes, while A had yet to cover $\frac{3}{8}$ of the track. If A's average speed is 75 metres per minute less than B's, find:

(i) the course distance,

(ii) the average speed of A in metres per minute.

Exam continues next page ...

(d)



In the parallelogram $ABCD$ above, a point X is chosen on AB and a point Y on CD so that $AX = CY$. The interval XY meets the diagonal AC at the point Z.

(i) Prove $\triangle AXZ \equiv \triangle CYZ$.

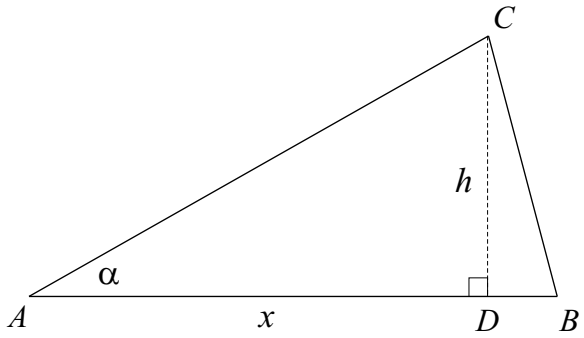
(ii) Hence show that XY, AC and BD all meet at Z, giving clear reasoning.

QUESTION EIGHT (13 marks) Start a new page.

(a) By letting $u = x^2 - x$, solve the following for x:

$$(x^2 - x)^2 - 18(x^2 - x) + 72 = 0.$$

(b)



The diagram above shows triangle ABC with $AB = 1$ and $AC = 1$. Point D is the foot of the perpendicular from C to the line AB.

Let $AD = x$, $\angle BAC = \alpha$ and $CD = h$.

(i) Using Pythagoras' theorem, show that $BC^2 = 2 - 2x$.

(ii) Hence show that $BC^2 = 2 - 2\cos\alpha$.

(iii) Let E be the point on the interval BC such that AE bisects $\angle BAC$. Show that

$$\sin\frac{\alpha}{2} = \frac{\sqrt{2 - 2\cos\alpha}}{2}.$$

(iv) Hence determine $\sin 15^\circ$ in exact form.

Exam continues overleaf ...

(c) Let t be a rational number and n be a positive integer.

(i) Solve $t + \dfrac{1}{t} = n$ for t.

(ii) Show that for $n \geq 3$:

$$(n-1)^2 < n^2 - 4 < n^2.$$

(iii) Hence find all t such that $t + \dfrac{1}{t}$ is a positive integer.

END OF EXAMINATION

QUESTION ONE

(a) $\sin 73^\circ \doteq 0.9563$ ✓ (four d.p.)

(b) $27^{1/3} = 3$ ✓

(c) $A = \pi r^2$

$= \pi (4\text{ cm})^2$

$= 16\pi \text{ cm}^2$ ✓ (award mark if units are missing).

(d) $2x - x^2 = x(2 - x)$ ✓

(e) (i) 3 ✓

(ii) -1 OR $(0, -1)$ (either accepted) ✓

(f) $2k$ ✓

(g) $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{3\sqrt{5}}{5}$ ✓

(h) a^5 ✓

(i) $\theta \doteq 36^\circ$ (nearest degree) ✓

(j) $3\sqrt{7}$ ✓

(k) $M = \left(\frac{1+9}{2}, \frac{5+3}{2}\right)$ ✓

$= (5, 4)$ ✓

QUESTION TWO

(a) $P(2,1)$ $Q(3,4)$

$$m = \frac{4-1}{3-2}$$

$$= 3$$ ✓

(b) (i) 5 ✓
(ii) 22/9 ✓ (or $2.\dot{4}$)
(iii) 3 ✓
(iv) 3 ✓

(c) $x - y = 2$ ___①

$x = 3y$ ___②

Substitute eqn ② into eqn ① for x:

$3y - y = 2$

$2y = 2$

$y = 1$ ___③ ✓

& by ② & ③ $x = 3$ ___④ ✓

(d) Volume $V = \frac{1}{2} 3 \cdot 4 \cdot 7$

$= 42 \text{ cm}^3$ ✓

(e) $(x-4)(x+11) = 0$

So $x = 4$ or $x = -11$. ✓

(f) $AB^2 = (2-6)^2 + (-3-3)^2$ ✓

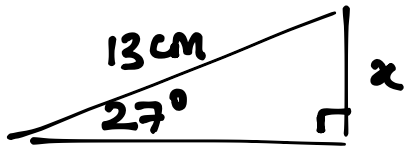
$= 16 + 36$

$= 52$

So $AB = \sqrt{52}$ ✓

$= 2\sqrt{13}$

(g) $x^2 - 9 = (x-3)(x+3)$ ✓

(h) 

$\sin 27° = \frac{x}{13\text{cm}}$

so $x = 13\sin 27°$ cm

$= 5.9$ cm ✓ (one d.p.) ✓

13

QUESTION THREE

(a) $9 - 3x < 6$

$3 < 3x$

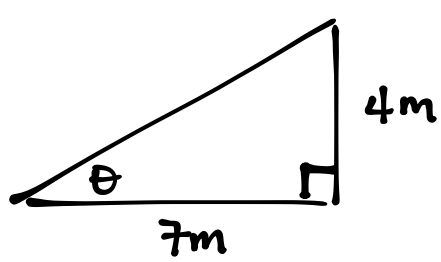
$x > 1$ ✓

(b) $y - 5 = 3(x - 1)$ ✓

$y - 5 = 3x - 3$

$y = 3x + 2$ ✓ (general solution accepted too).

(c)



$\tan\theta = \frac{4}{7}$ so $\theta = \tan^{-1}\frac{4}{7}$

$\div 30°$ (nearest degree) ✓

(d) (i) $x^2 + 5x - 24 = x^2 + 8x - 3x - 24$

$= (x - 3)(x + 8)$ ✓

(ii) $x^2 + 5x - 24 = 0$

so $(x - 3)(x + 8) = 0$

giving $x = 3$, $x = -8$ ✓

(d) $4x + y = 9$ —— ①

$3x - 2y = 4$ —— ②

$2 \times$ ①: $8x + 2y = 18$ —— ③

② + ③: $11x = 22$

$\boxed{x = 2}$ —— ④ ✓

Substitute ④ into ①: $4(2) + y = 9$

so $\boxed{y = 1}$ ✓

(f) (i) true ✓

(ii) false ✓

(iii) false ✓

(g) (i) mode: the score with greatest frequency is $\boxed{42}$. ✓

(ii) median: # results = 26 so median will be the mean of the 13^{th} & 14^{th} scores; that is,

$$\frac{34+34}{2} = 34.$$

The median is $\boxed{34}$ ✓

QUESTION FOUR

(a) $\frac{4y}{5} - \frac{2y}{3} = \frac{12y}{15} - \frac{10y}{15}$

$= \frac{2y}{15}$ ✓

(b) $\frac{25p^9r^{-3}}{5p^3r^{-1}} = 5p^6r^{-2}$ ✓

$= \frac{5p^6}{r^2}$ ✓

(c)(i) $A = \frac{1}{2}(a+b)h$

$2A = (a+b)h$ ✓

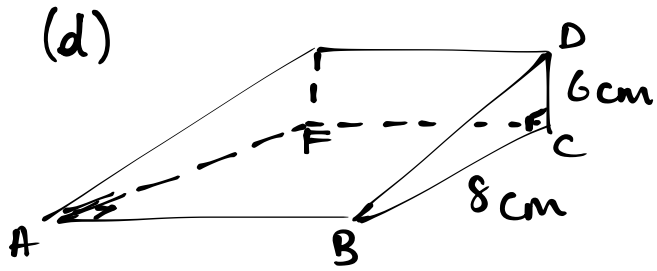
$\frac{2A}{h} = a+b$

$\therefore a = \frac{2A}{h} - b$ ✓ (or equivalent)

(ii) $a = \frac{2(72)}{4} - 6$

$= 30\text{ mm}$ ✓

(d)



(i) By Pyth. th'm: $BD^2 = 64 + 36\ cm^2 \Rightarrow BD = 10cm$. ✓

(ii) Surface area: $(2 \times \frac{1}{2} \times 8 \times 6) + (6 \times 16) + (16 \times 8) + (BD \times 16) = 432\ cm^2$. ✓

(e) (i) $5x^2 + 7x - 12 = 0$

$5x^2 - 5x + 12x - 12 = 0$

$5x(x-1) + 12(x-1) = 0$

$(5x+12)(x-1) = 0$

One mark if at least 12 & -5 identified.

& $x = 1$ or $x = -\frac{12}{5}$.

(ii) $x^2 + x - 1 = 0$

$x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 1 = 0$

$\left(x + \frac{1}{2}\right)^2 - \frac{5}{4} = 0$

$\left(x + \frac{1}{2}\right)^2 = \frac{5}{4}$

so $x = \frac{-1 \pm \sqrt{5}}{2}$

(f) $\frac{4x+2}{2} = 2x+1$

13

QUESTION FIVE

(a) $\frac{2x+6}{x^2+6x+9} = \frac{2(x+3)}{(x+3)^2}$ ✓

$= \frac{2}{x+3}$. ✓

(b) $y - y_1 = m(x - x_1)$

$= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

so $y - 4 = \frac{-2-4}{5-7}(x-7)$ ✓

$= 3(x-7)$

so $y = 3x - 21 + 4$

$\therefore y = 3x - 17$. ✓

First mark awarded for correct approach (e.g. calculating gradient first, then using point-gradient formula).

(or general form)

(c) (i) $\frac{2x}{3} - \frac{5x}{2} = 1$

Multiply through by six:

$2 \cdot 2x - 3 \cdot 5x = 6$ ✓

$-11x = 6$

$\therefore x = -\frac{6}{11}$. ✓

(ii) $\frac{x}{12} = \frac{5}{x-11}$

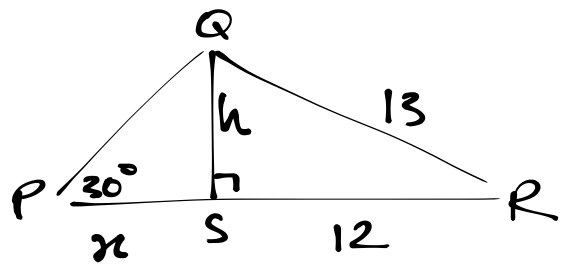
so $x(x-11) = 60$ ✓

$x^2 - 11x - 60 = 0$

$(x-15)(x+4) = 0$

$\therefore x = 15, -4$. ✓

(d)



$h^2 = 13^2 - 12^2$ by Pythagoras' th'm,

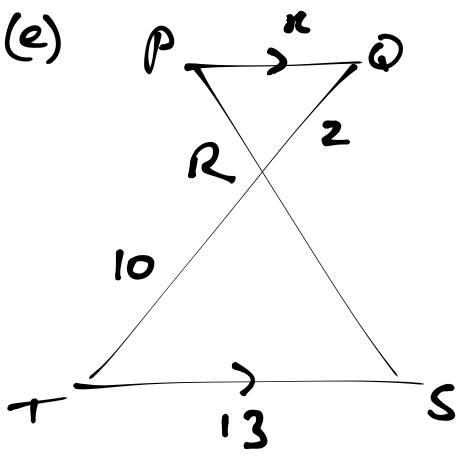
so $h = 5$ u ✓

Now, $\tan 30° = \frac{h}{x}$ & $\tan 30° = \frac{1}{\sqrt{3}}$.

So $x = \sqrt{3}h$

Hence $x = PS = 5\sqrt{3}$ u. ✓

(e)



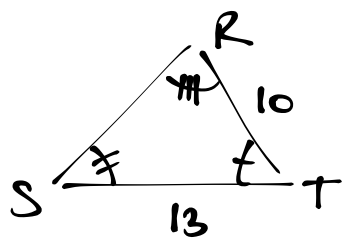
(i) In $\triangle PQR$ & $\triangle STR$:

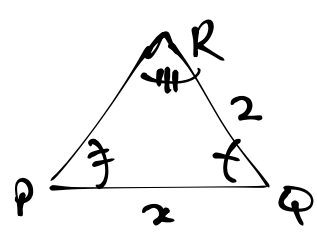
(1) $\angle QPS = \angle TSP$ (alt. $\angle$s $PQ \parallel TS$) ✓

(2) $\angle PRQ = \angle SRT$ (vert. opp. $\angle$s) ✓

(3) By AAA test, $\triangle PQR \,|||\, \triangle STR$. ✓

(ii) Since $\triangle QPR \,|||\, \triangle STR$, we have ratios of matching sides being equal. So:





$\frac{x}{13} = \frac{2}{10}$

So $x = \frac{13}{5}$. ✓

13

QUESTION SIX

(a) $V = \pi r^2 h$ $\quad V = 108\pi \text{ cm}^3$

$= \frac{\pi d^2 h}{4}$

Now $h = 2d$ so $V = \frac{\pi}{4} \cdot d^2 \cdot (2d)$ ✓

$= \frac{\pi}{2} d^3$

that is: $108\pi = \frac{\pi}{2} d^3$

$216 = d^3$

so $d = 6 \text{ cm}$. ✓

(b) $\frac{5y-6}{6y} = \frac{1}{y+1}$

so $(y+1)(5y-6) = 6y$

$5y^2 - y - 6 = 6y$

$5y^2 - 7y - 6 = 0$ ✓

$5y^2 - 10y + 3y - 6 = 0$

$5y(y-2) + 3(y-2) = 0$

$(5y+3)(y-2) = 0$

so $y = 2$ or $y = -\frac{3}{5}$. ✓

(c) (i) $4c + d = 21.20$ — ①

$2c + 5d = 27.70$ — ②

] ✓ (or equivalent)

(ii) By ①: $d = 21.20 - 4c$ — ③

Substitute ③ in ② for d:

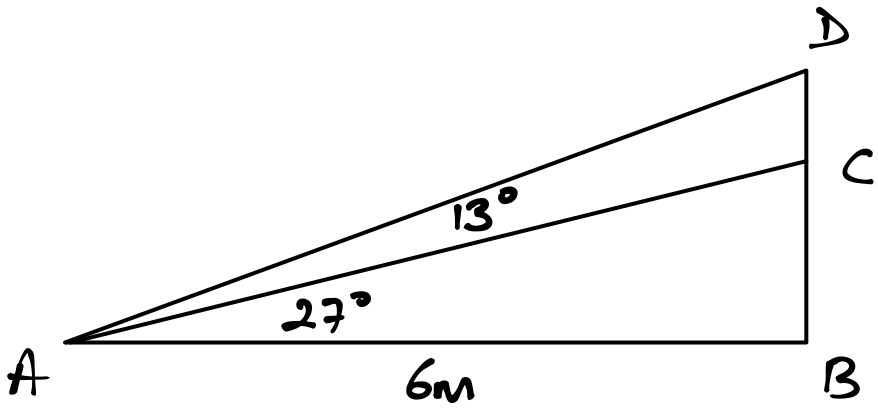
$$2c + 5(21.20 - 4c) = 27.70$$

$$-18c = 27.70 - 106.00$$

$$\text{so } c = \$4.35 \quad ✓$$

$$\& \therefore \text{by } ③: d = \$3.80 \quad ✓$$

(d)



We have: ① $\tan(27+13)^\circ = \tan 40^\circ$

$$= \frac{BD}{AB}$$

$$= \frac{BD}{6}$$

so $6\tan 40^\circ = BD$. ✓

② $\tan 27^\circ = \frac{BC}{AB}$

$$= \frac{BC}{6}$$

so $6\tan 27^\circ = BC$

Hence $CD = BD - BC$

$= 6\tan 40^\circ - 6\tan 27^\circ$

$\doteq 2$ m (nearest whole number). ✓

(2) (i) $\frac{a^2+3a+2}{2a+4} \times \frac{1}{a+1} = \frac{(a+2)(a+1)}{2(a+2)} \times \frac{1}{(a+1)}$ ✓

$= \frac{1}{2} \times 1$

$= \frac{1}{2}$ ✓

(ii) $\frac{3}{2x+8} \div \frac{9}{3x+12} = \frac{3}{2(x+4)} \div \frac{9}{3(x+4)}$

$= \frac{3}{2(x+4)} \times \frac{3(x+4)}{9\ 3}$ ✓

$= \frac{3}{2} \times \frac{1}{5}$

$= \frac{1}{2}$ ✓

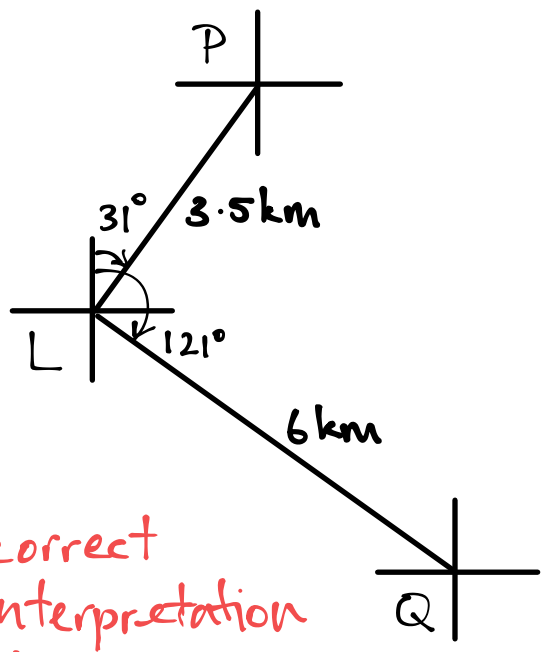
QUESTION SEVEN

(a) $6x^2y - 15y - 5x + 18xy^2 = 6xy(x+3y) - 5(x+3y)$ ✓ *

$= (6xy - 5)(x + 3y)$ ✓

* Award one mark for a reasonable attempt to factorise.

(b)



(correct interpretation of bearings) ✓

(i) $\angle PLQ = 121° - 31°$

$= 90°$ ✓

(ii) By Pythagoras' theorem,

$PQ^2 = LP^2 + LQ^2$

$= (3{\cdot}5^2 + 6^2)\ km^2$

$= 48{\cdot}25\ km^2$

So $PQ = \sqrt{48{\cdot}25}$ km

$\doteq$ 7km (nearest km). *

* Award mark for exact, approx. decimal or rounded. ✓

(c) (i) Let the distance be D & the average speeds of A & B
So PQ ≈ 6.95 km.

v_A & v_B respectively.

Then $v_B = \frac{D}{20}$ — ①

& $v_A = \frac{\frac{5}{8}D}{20}$ (A covers $\frac{5}{8}$ of track in 20 min.)

$= \frac{D}{32}$ — ②

✓

Also, $v_A = v_B - 75$ m/min. — ③

Substitute ① & ② into ③ for v_A & v_B:

$\frac{D}{32} = \frac{D}{20} - 75$

$75 = D\left(\frac{1}{20} - \frac{1}{32}\right)$

$= \frac{12}{640} D$

$= \frac{3}{160} D$

✓ Award for valid attempt at simultaneous solution.

Hence $D = \frac{75 \times 160}{3}$ metres

$= 4000$ metres. ✓

(ii) So $v_A = \frac{D}{32}$

$= \frac{4000}{32}$

$= 125$ m/min. ✓

(d) (i) In $\triangle AXZ$ & $\triangle CYZ$:

(i) $AX = CY$ (construction)
(ii) $\angle ZAX = \angle ZCY$ (alternate angles: AB // CD)
(iii) $\angle AZX = \angle CZY$ (vertically opposite angles). ✓

Hence by AAS, $\triangle AXZ \equiv \triangle CYZ$. ✓

(ii) We already have that AC & XY meet at Z.

Now, since $\triangle AXZ \equiv \triangle CYZ$, we have that matching sides are equal. Hence $AZ = CZ$ & so the point Z bisects the diagonal AC. ✓

But the diagonal BD must also then pass through Z since the diagonals of a parallelogram bisect each other. ✓ Hence XY, AC, BD all meet at Z.

QUESTION EIGHT

(a) Let $u = x^2 - x$.

Then $u^2 - 18u + 72 = 0$.

So $(u-6)(u-12) = 0$

$\rightarrow u = 6, 12$. ✓

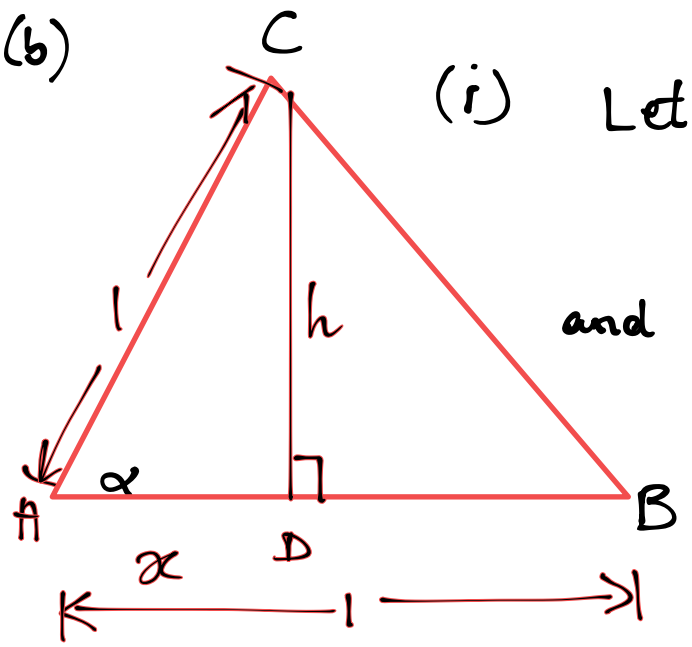
So $x^2 - x = 6$, $x^2 - x = 12$

$x^2 - x - 6 = 0$, $x^2 - x - 12 = 0$

$(x-3)(x+2) = 0$, $(x-4)(x+3) = 0$

Solutions: $x = \pm 3$ ✓, $-2, 4$. ✓

(b)



(i) Let $CD = h$. Then noting $BD = 1-x$, we have:

$BC^2 = h^2 + (1-x)^2$ by Pythagoras' theorem on ΔBCD

and $AC^2 = h^2 + x^2$ ✓

$1 = h^2 + x^2$ by ΔACD. ✓

Eliminating h^2, we have

$$BC^2 - 1 = (1-x)^2 - x^2$$

$$= 1 - 2x^2 + x^2 - x^2$$

i.e. $BC^2 = 2 - 2x$, as required. ✓

(ii) We see that in ΔACD, $\cos\alpha = \frac{x}{1} \rightarrow x = \cos\alpha$. ✓

Hence by (i), $BC^2 = 2 - 2\cos\alpha$.

(iii) Since ΔABC is isosceles, the angle bisector at $\angle BAC$ is also a perpendicular bisector of the interval, BC.

We have then: $\sin\frac{\alpha}{2} = \frac{1}{2}BC$

i.e. $\sin\frac{\alpha}{2} = \frac{\sqrt{2-2\cos\alpha}}{2}$ ✓

Hence for $\alpha = 30°$,

$$\sin 15° = \frac{\sqrt{2-2\cos 30°}}{2}$$

$$= \frac{\sqrt{2-2\sqrt{3}/2}}{2}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2} \quad . \quad ✓$$

(c) (i) $t + \frac{1}{t} = n$

so $t^2 + 1 = nt$

$t^2 - nt = -1$

$\left(t - \frac{n}{2}\right)^2 = \frac{n^2}{4} - 1$

$= \frac{n^2-4}{4}$

so $t - \frac{n}{2} = \pm\frac{\sqrt{n^2-4}}{2}$

$t = \frac{n \pm \sqrt{n^2-4}}{2}$ ✓

(ii) Aim to show $(n-1)^2 < n^2-4 < n^2$ for $n \geq 3$.

Consider that for $(n-1)^2$ we have

$$(n-1)^2 = ((n-2)+1)((n+2)-3)$$
$$= (n-2)(n+2) - 2n + 5$$
$$= (n^2-4) - (2n-5)$$

But $n \geq 3$ so $2n-5 \geq 6-5 = 1 > 0$

so $(n^2-4) - (2n-5) < n^2-4$ ✓

Clearly, also, $n^2-4 < n^2$ for $n \geq 3$. ✓

Hence $(n-1)^2 < n^2-4 < n^2$ for n integral & $n \geq 3$.

(iii) By part (i) we have

$$t = \frac{n \pm \sqrt{n^2-4}}{2}$$

We note that $(n-1)^2$ & n^2 are consecutive perfect squares so that since $(n-1)^2 < n^2-4 < n^2$, it follows that n^2-4 cannot be a perfect square. Hence $\sqrt{n^2-4}$ is irrational & therefore $\frac{1}{2}(n \pm \sqrt{n^2-4})$ is irrational also. ✓

This leaves only the cases of $n=1$ & $n=2$ to check.

For $n=1$, $\sqrt{n^2-4} = \sqrt{-3}$ $\longrightarrow$ not rational.

For $n=2$, $\sqrt{n^2-4} = 0$ & so $t = \frac{2 \pm 0}{2} = 1$.

Hence only in this case do we have a value for t such that $t + \frac{1}{t} = 1 + \frac{1}{1} = 2$ is rational. ✓