SYDNEY BOYS HIGH SCHOOL

2023

YEAR 9 STAGE 5.3
TERM 4
YEARLY EXAMINATION

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- **S**how ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the **simplest exact form**, unless otherwise stated.
- NESA approved calculators may be used
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: AMG

Name: _____

Class	Teacher	
9Ma**A**	A. Wang	
9Ma**B**	M. Zhang	
9Ma**C**	J. Tao	
9Ma**P**	D. Taylor/W. Beare	
9Ma**L**	R. Wang	
9Ma**U**	A. Ward	
9Ma**S**	J. Chan	

Section	Marks
A	/15
B	/15
C	/15
D	/15
E	/10
F	/10
Total	/80

Section A

Question 1 (1 mark)
Write 75.7083° in degrees and minutes (to the nearest minute). **1**

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Question 2 (1 mark) **1**
A birthday cake costs $26.40 at the checkout. How much GST is paid for the cake?

……

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Question 3 (2 marks)
Are the following True or False?
(a) Any parallelogram is a trapezium. **1**

……

……

(b) All quadrilaterals with perpendicular diagonals are rhombuses. **1**

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Question 4 (1 mark)
Andrew borrows $10 000 for three years at 4.5% per annum simple interest. **1**
How much interest does he pay?

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Question 5 (1 mark)
A photographic negative 3 cm by 2.5 cm is enlarged to print on paper with the largest side 32 cm high. How wide is the print? **1**

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Question 6 (2 marks)

In ΔMNO, $\angle N$ is a right angle, $MN = 23.1$ cm and $NO = 31.6$ cm. **2**

Find the size of $\angle MON$ to the nearest minute.

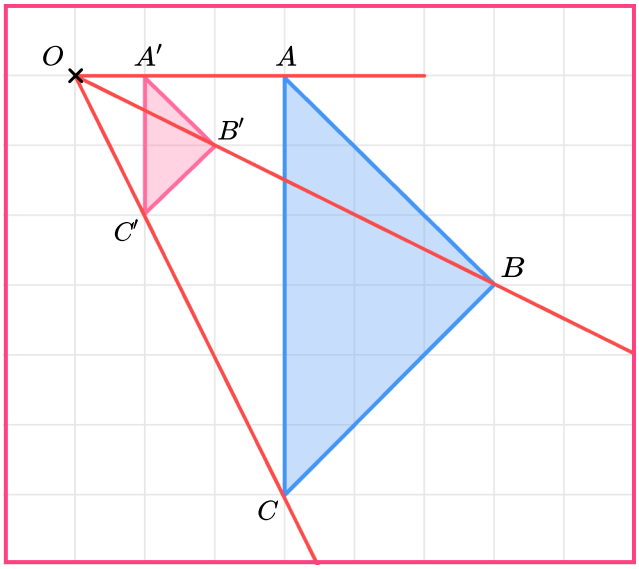
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Question 7 (1 mark)

In the diagram below, triangle ABC is an enlargement of triangle $A'B'C'$. **1**

What is the scale factor?

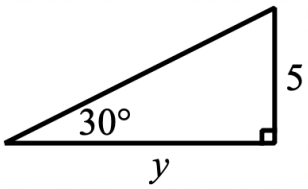


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Question 8 (1 mark)

Find y in exact form. **1**



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Question 9 (1 mark)

The point M (1, 10) is the midpoint of AB, where A has coordinates (–3, 7). **1**

What are the coordinates of B?

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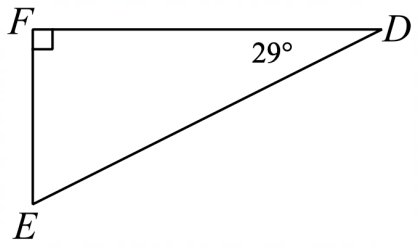
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Question 10 (1 mark)

In the diagram below, what is the ratio $\frac{FE}{FD}$, correct to 3 decimal places. **1**



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Question 11 (1 mark)

What is the supplementary angle to 84°45′ ? **1**

……

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Question 12 (2 marks)

(a) Simplify $x\left(\frac{2}{\sqrt[4]{x}}\right)^4$ **1**

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(b) Express 256×64^x in the form 8^y, stating y in terms of x. **1**

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End of Section A

Extra Working Page for Section A only

Section B

Name: _____

Class	Teacher	
9**A**	A. Wang	
9**B**	M. Zhang	
9**C**	J. Tao	
9**P**	D. Taylor/W. Beare	
9**L**	R. Wang	
9**U**	A. Ward	
9**S**	J. Chan	

Mark	**/15**

Question 13 (7 marks)
Factorise the following:

(a) $a^2 + 4a - 5$ **1**

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(b) $6x^2 + 17x - 3$ **2**

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(c) $m^4 - 16$ **2**

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(d) $y^3 + 125$ **1**

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Question 14 (2 marks)
Alan bought a radio for $900 and then sold it to Brian at a loss of 20%. **2**
At what price must Brian sell the radio in order to make a profit of 5%?

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Question 15 (3 marks)
Find the exact value of:

(a) $\sin 45° . \tan 60°$. **1**

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(b) $\tan \theta$, given that $\cos \theta = \frac{\sqrt{3}}{2}$, where θ is acute. **2**

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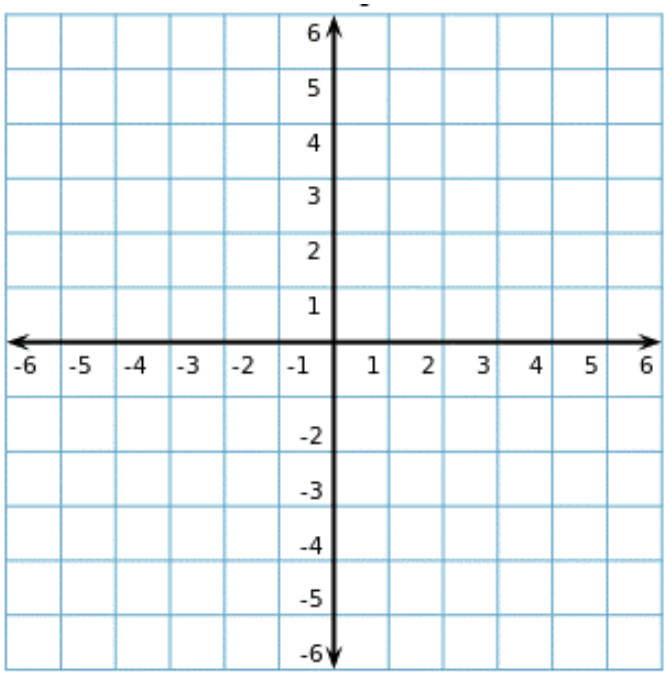
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Question 16 (3 marks)

(a) Neatly sketch $y = 2x - 4$ on the number plane, clearly showing both intercepts. **2**



(b) On the same number plane shade the region where $y - 2 \leqslant 0$ and $2x - y - 4 \leqslant 0$ hold simultaneously. **2**

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End of Section B

Extra Working Page for Section B only

Section C

Name: _____

Class	Teacher	
9**A**	A. Wang	
9**B**	M. Zhang	
9**C**	J. Tao	
9**P**	D. Taylor/W. Beare	
9**L**	R. Wang	
9**U**	A. Ward	
9**S**	J. Chan	

Mark	**/15**

Question 17 (3 marks)

A solid glass cylinder has a volume of 75 π cm^3 . **3**

The cost of polishing the surfaces of this cylinder is \$2 per square cm for the curved surface and \$3 per square cm for the circular top and base areas.

Given that the radius of the base is r cm, show that the cost of polishing, \$$C$, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

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Question 18 (3 marks)

Solve the following:

(a) $x + 3 = 2 + \frac{x-3}{5}$ **1**

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(b) $x - 3 = \frac{16}{x-3}$ **2**

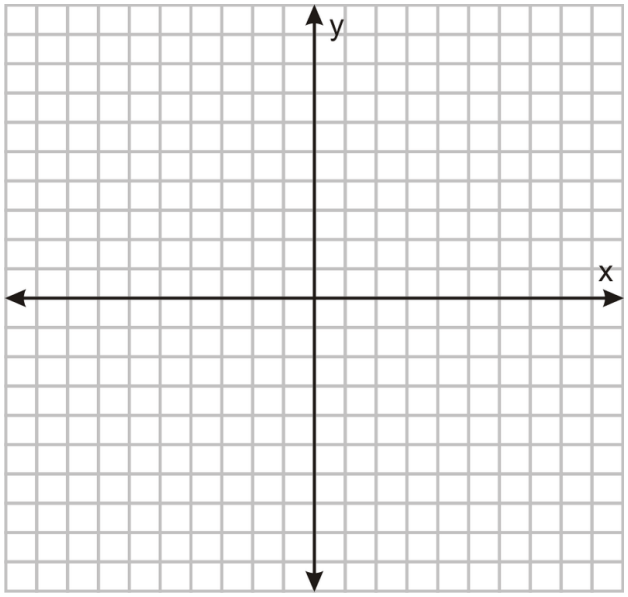
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Question 19 (7 marks)
The line straight line L has gradient 3 and passes through the point $A(-6, 4)$.



(a) Find an equation for L in the form $y = mx + c$. **1**

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(b) The straight line K has equation $x - 7y + 14 = 0$.
Given that K crosses the y-axis at the point B and intersects L at C,
find the coordinates of B and C. **2**

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(c) Show that $\angle BAC = 90°$. **2**

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(d) Find the area of ΔABC. **2**

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Question 20 (1 mark)

Evaluate $\dfrac{5.6\times 10^4+7\times 10^3}{2.8\times 10^{-3}}$ **1**

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Question 21 (1 mark)

Two inequalities have solution sets P $(-\infty, 0)$ and Q $[0, 10]$. **1**

What is the union of P and Q? (i.e. $P \cup Q$)

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End of Section C

Extra Working Page for Section C only

Section D

Name: _____

Class	Teacher	
9**A**	A. Wang	
9**B**	M. Zhang	
9**C**	J. Tao	
9**P**	D. Taylor/W. Beare	
9**L**	R. Wang	
9**U**	A. Ward	
9**S**	J. Chan	

Mark	**/15**

Question 22 (3 marks)

(a) Simplify $\frac{5}{x+2} \div \frac{5x-10}{x^2-4}$ **2**

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(b) Expand and simplify $(a-3)(a^2+3a+9)$ **1**

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Question 23 (4 marks)
In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are NOT acceptable.

(a) Simplify $\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$, giving the answer in the form $b\sqrt{2}+c$, where $b, c \in \mathbb{Z}$. **2**

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(b) Express the following in the form $p + q\sqrt{2}$ where p and q are rational. **2**

$$\frac{2}{2+\sqrt{2}}$$

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Question 24 (6 marks)

(a) Four friends each throw a biased coin a number of times.
The table shows the number of heads and the number of tails each friend got.

	Ben	**Helen**	**Paul**	**Sharif**
Heads	34	66	80	120
Tails	8	12	40	40

(a) Which of the friends' results will give the best estimate for the probability that the coin will land heads? **2**

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(b) Paul says "With this coin you are twice as likely to get heads as to get tails." **2**
Is Paul correct?
Justify your answer.

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(c) The coin is to be thrown twice more. Use all the results in the table to work out an estimate for the probability that the coin will land heads both times. **2**

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Question 25 (2 marks)

Let $P = \dfrac{2a - c}{d}$. **2**

If a equals 58.4 correct to 3 significant figures, c equals 20 correct to 2 significant figures, and d equals 3.6 correct to 2 significant figures, then what is the upper bound for the value of P?

Show your working clearly, and answer correct to 2 decimal places.

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End of Section D

Extra Working Page for Section D only

Section E

Name: _____

Class	Teacher	
9**A**	A. Wang	
9**B**	M. Zhang	
9**C**	J. Tao	
9**P**	D. Taylor/W. Beare	
9**L**	R. Wang	
9**U**	A. Ward	
9**S**	J. Chan	

Mark	**/10**

Question 26 (3 marks)

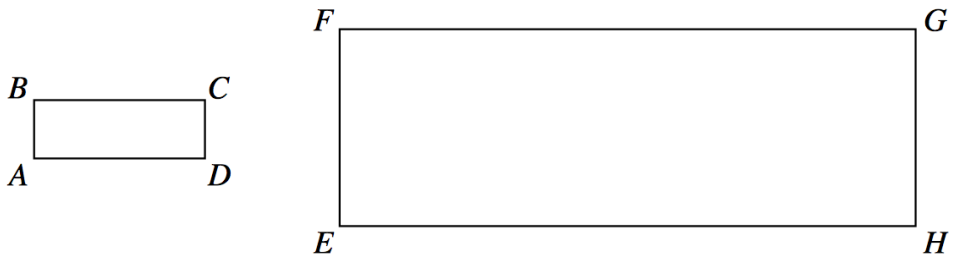


Figure 1

Figure 1 shows the rectangles *ABCD* and *EFGH* which are similar.

Given that $AB = (3 - \sqrt{5})$ cm, $AD = \sqrt{5}$ cm and $EF = (1 + \sqrt{5})$ cm, find the length *EH* in cm, giving your answer in the form $a + b\sqrt{5}$ where *a* and *b* are integers.

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Question 27 (4 marks)

The table shows information about the weights, in kilograms, of 40 babies.

Weight (w kg)	**Frequency**
$2 < w \leq 3$	12
$3 < w \leq 4$	16
$4 < w \leq 5$	9
$5 < w \leq 6$	2
$6 < w \leq 7$	1

(a) Write down the modal class. **1**

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(b) Work out an estimate for the mean weight of the 40 babies. **1**

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(c) One of the babies is chosen at random. **2**
Find the probability that this baby has a weight of more than 5 kg.

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Question 28 (3 marks)

Solve the following pair of simultaneous equations **3**

$$x + 2y + 4 = 0$$

$$3x - y - 3 = 0$$

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End of Section E

Extra Working Page for Section E only

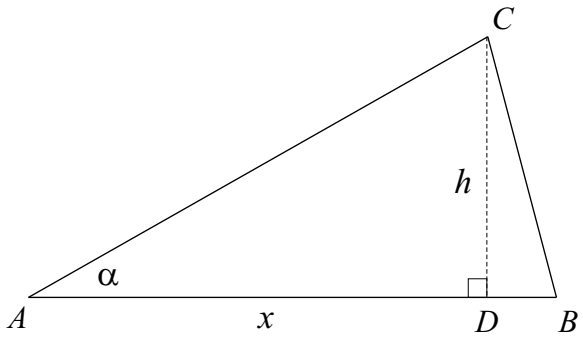
Section F

Name: _____

Class	Teacher	
9**A**	A. Wang	
9**B**	M. Zhang	
9**C**	J. Tao	
9**P**	D. Taylor/W. Beare	
9**L**	R. Wang	
9**U**	A. Ward	
9**S**	J. Chan	

Mark	**/10**

Question 29 (7 marks)



The diagram above shows ΔABC with $AB = 1$ and $AC = 1$.
Point D is on AB such that $CD \perp AB$. Let $AD = x$, $\angle BAC = \alpha$, and $CD = h$.

(a) Show that $BC^2 = 2 - 2x$. **2**

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(b) Hence show that $BC^2 = 2 - 2\cos\alpha$. **1**

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Question 29 is continued on the next page

Question 29 (continued)

(iii) By considering $\angle BCD$, or otherwise, show that $\sin\frac{\alpha}{2} = \frac{\sqrt{2 - 2\cos\alpha}}{2}$. **3**

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(iv) Hence determine $\sin 15°$ in exact form. **1**

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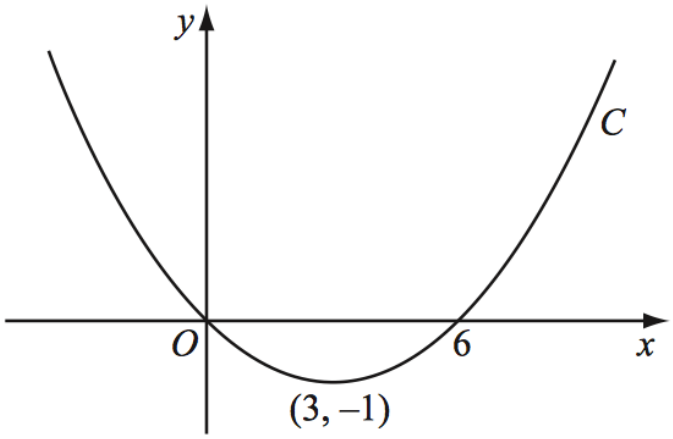
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Question 30 is on the next page

Question 30 (3 marks)

The graph below shows $y = \frac{x}{9}(x - 6)$. **3**



On the graph above, sketch the graph of $y = \frac{x}{9}(6 - x) - 1$.

Note: You must indicate the intercepts with the coordinate axes.

End of paper

Extra Working Page for Section F only

SYDNEY BOYS HIGH SCHOOL

2023

YEAR 9 STAGE 5

YEARLY

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen, and/or whiteout, may not be able to appeal.

Question 1

Extremely well done, though some rounded incorrectly as no rounding was necessary.

Question 2

About 80% got this correct.
The main error was students calculating the GST on the amount given.
The amount stated is GST inclusive.
Many students did not work to 2 decimal places, though no marks were deducted for this particular error.

Question 3(a)

Very poorly done.

Question 3(b)

Poorly done.
Students who got this wrong did not consider a kite.

Question 4

Reasonably well done.
Some students worked out the compound interest.
This was not the skill being tested as the question clearly states simple interest.
Students need to read the question carefully.

Question 5

This was generally well done, but the degree of accuracy of the answers varied.
The test paper requires “simplest exact form”, that said stating the degree of accuracy was accepted.
Approximations with no stated degree of accuracy had marks deducted.

Question 6

Surprisingly well done.
Students took care in drawing a diagram and thus thinking about the question.
Marks were deducted for incorrect diagrams and using the wrong ratio.

Question 7

Poorly done.
Only answers which included numerical values obtained marks.
Various forms of answers were given especially ratios, which was incorrect.

Question 8

Not well done.
Most students got the equation from the diagram correctly.
Also, many knew the exact value for tan 30°.
The final solution, however, showed a variety of manipulations, the answer $\sqrt{75}$ was accepted.

Question 9

Very well done.
The detail in the working varied widely.
Students should be careful as other exams or tests may have more marks assigned and so more working will be expected.

Question 10

Extremely well done.

Question 11

Although some found the complement, this was generally well done.

Question 12 (a)

Reasonably well done.
The main problem was working out the powers of x and simplifying.
Marks were deducted for completing this incorrectly.

Question 12(b)

Poorly done.
Only a few got this correct.
Marks were awarded for simplifying the righthand side of the expression.
Only a few got further than that.

Section A

Question 1 (1 mark)
Write 75.7083° in degrees and minutes (to the nearest minute). **1**

75° 42'

Question 2 (1 mark) **1**
A birthday cake costs $26.40 at the checkout. How much GST is paid for the cake?

26.40 / 11 = $2.40

Question 3 (2 marks)
Are the following True or False?
(a) Any parallelogram is a trapezium. **1**

True

(b) All quadrilaterals with perpendicular diagonals are rhombuses. **1**

False

Question 4 (1 mark)
Andrew borrows $10 000 for three years at 4.5% per annum simple interest. **1**
How much interest does he pay?

$I = PRT$
$= 10,000 \times 0.045 \times 3$
$= \$1350$

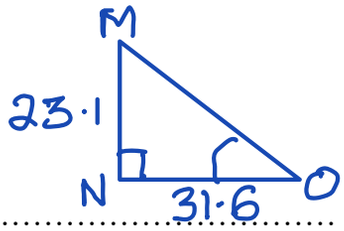
Question 5 (1 mark)
A photographic negative 3 cm by 2.5 cm is enlarged to print on paper with the largest side 32 cm high. How wide is the print? **1**

$3 : 32 = 1 : 10\frac{2}{3}$ ∴ $2.5 : 26\frac{2}{3}$

Question 6 (2 marks)

In ΔMNO, $\angle N$ is a right angle, $MN = 23.1$ cm and $NO = 31.6$ cm. **2**

Find the size of $\angle MON$ to the nearest minute.



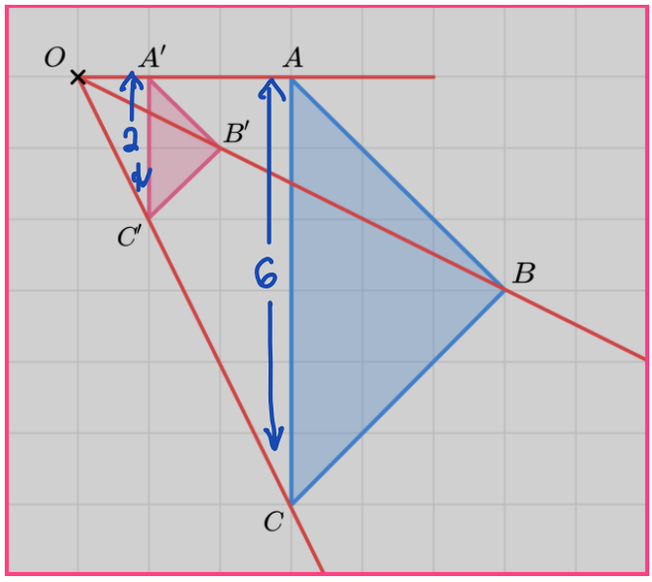
$\tan(\angle MON) = \frac{23.1}{31.6}$ $\angle MON = \tan^{-1}\frac{23.1}{31.6}$

$\angle MON \approx 36°10'$

Question 7 (1 mark)

In the diagram below, triangle ABC is an enlargement of triangle $A'B'C'$. **1**

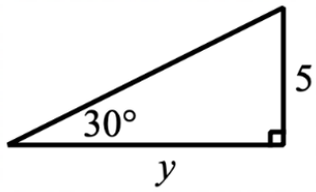
What is the scale factor?



Scale factor $= \frac{6}{2} = 3$

Question 8 (1 mark)

Find y in exact form. **1**



$\tan 30 = \frac{5}{y}$ $y = \frac{5}{\tan 30} = \frac{5}{1/\sqrt{3}} = 5\sqrt{3}$

Question 9 (1 mark)

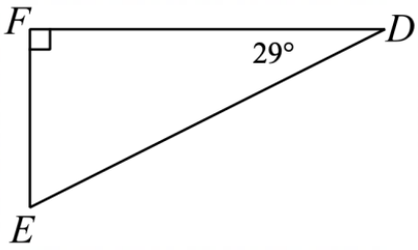
The point M (1, 10) is the midpoint of AB, where A has coordinates (–3, 7). **1**

What are the coordinates of B?

$M = (1, 10) = \left(\frac{-3+x}{2}, \frac{7+y}{2}\right) \Rightarrow B(x, y) = (5, 13)$

Question 10 (1 mark)

In the diagram below, what is the ratio $\frac{FE}{FD}$, correct to 3 decimal places. **1**



$\frac{FE}{FD} = \tan 29° = 0.554$ (3 dp)

Question 11 (1 mark)

What is the supplementary angle to 84°45′ ? **1**

$180° - 84°45' = 95°15'$

or $95.25°$

Question 12 (2 marks)

(a) Simplify $x\left(\frac{2}{\sqrt[4]{x}}\right)^4$ **1**

$= x\left(\frac{16}{(x^{1/4})4}\right) = x\left(\frac{16}{x}\right) = 16$

(b) Express 256×64^x in the form 8^y, stating y in terms of x. **1**

$\frac{8^3}{2} \times 8^{2x} = \frac{8^3}{8^{1/3}} \times 8^{2x}$

$= 8^{8/3} \times 8^{2x} = 8^{2x + \frac{8}{3}}$

End of Section A

Section B Solutions

Question 13 (6 marks)
Factorise the following:

(a) $a^2 + 4a - 5$ **1**

$$a^2 + 4a - 5 = (a-1)(a+5)$$

Well done.
Common mistakes: $(a-4)(a-1)$, $(a+1)(a-5)$, $(a+1)(a+5)$

(b) $6x^2 + 17x - 3$ **2**

$$6x^2 + 17x - 3 = (6x-1)(x+3)$$

Generally well done.
Common mistakes:

- *Partial factorisation:* $6x(x+3) - (x+3)$
- *Incorrect factorisation:* $(6x+1)(x-3)$; $(6x+3)(x-1)$

(c) $m^4 - 16$ **2**

$$m^4 - 16 = (m^2 - 4)(m^2 + 4) = (m+2)(m-2)(m^2 + 4)$$

Generally well done.
Common mistakes:

- *Partial factorisation:* $(m^2 - 4)(m^2 + 4)$
- *Incorrect factorisation:* $(m-2)(m+2)(m^2 + 4m + 4)$; $(m-2)(m+2)(m-2)(m+2)$
- *Careless mistakes:* $(m-2)(m+2)(m+4)$, $(m+4)(m-4)(m^2 + 4)$

If students wrote only $(m^2)^2 - (2^2)^2$, this was *not sufficient to gain any mark.*

(d) $y^3 + 125$ **1**

$$y^3 + 125 = y^3 + 5^3 = (y+5)(y^2 - 5y + 25)$$

Poorly done.
Common mistakes: $(y+5)(y^2 + 5y + 25)$; $(y+5)(y^2 - 10y + 25)$; $(y-5)(y^2 + 5y + 25)$

If students wrote only $(y+5)$, this was *not sufficient to gain any mark.*

Question 14 (2 marks)

Alan bought a radio for $900 and then sold it to Brian at a loss of 20%. **2**
At what price must Brian sell the radio in order to make a profit of 5%?

The price Alan sold the radio to Brian: $\$900\times(1-20\%)=\$720$

The price Brian sells the radio: $\$720\times(1+5\%)=\$756$

Extremely well done.
In order to get 2 marks, students needed to

- *include correct numerical working out,*
- *have the correct price.*

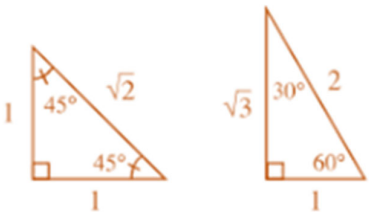
Some students provided incorrect numerical working out: $\$900-20\%=\$720$; $\$720+5\%=\$756$

Question 15 (3 marks)

Find the exact value of:

(a) $\sin 45°.\tan 60°$ **1**

$$\text{p}\sin 45°.\tan 60° = \sin 45°\times\tan 60° = \frac{1}{\sqrt{2}}\times\sqrt{3} = \frac{\sqrt{2}}{2}\times\sqrt{3} = \frac{\sqrt{6}}{2}$$



Poorly done.
Most students only provided the exact value of $\sin 45°$ *and* $\tan 60°$, *but did not calculate the product of them.*

(b) $\tan\theta$, given that $\cos\theta=\frac{\sqrt{3}}{2}$, where θ is acute. **2**

Method 1:



$$\tan\theta = \frac{\sqrt{2^2-(\sqrt{3})^2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Method 2:

$\because \cos\theta = \frac{\sqrt{3}}{2}$ and θ is acute,

$\therefore \theta = \cos^{-1}\frac{\sqrt{3}}{2} = 30°$

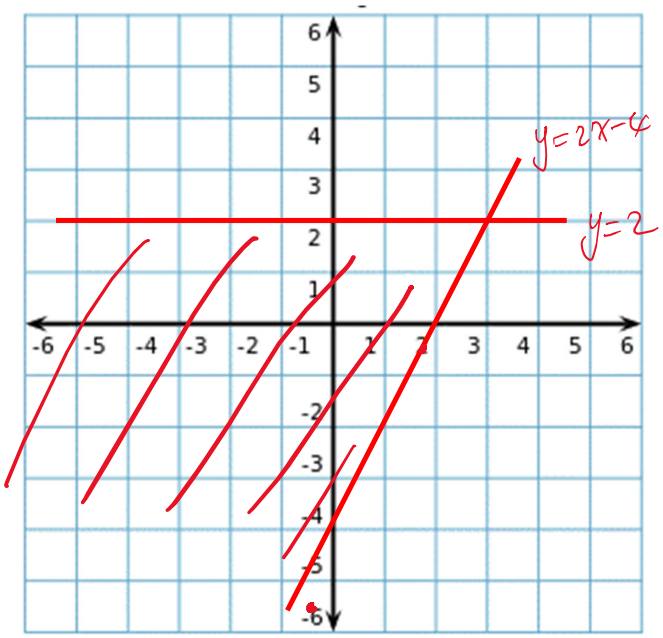
$\therefore \tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Well done.

Common mistake $\tan\theta=\sqrt{3}$; $\tan\theta=\frac{7}{\sqrt{3}}$

Question 16 (4 marks)

(a) Neatly sketch $y = 2x - 4$ on the number plane, clearly showing both intercepts. **2**



Well done.
Common mistakes:
- *x -intercept: 4, –4*
- *y-intercept : –2, 2*

(b) On the same number plane shade the region where $y - 2 \leq 0$ and $2 - y - 4 \leq 0$ hold simultaneously. **2**

See the above number plane for the answer.

Poorly done. Some students did not attempt this question.

To get 2 marks, students needed to
- *Sketch* $y = 2$,
- *Shade the region below* $y = 2$ *and above* $y = 2x - 4$

Common mistakes:
- *Shaded the region below* $y = 2x - 4$ *or above* $y = 2$
- *Did not clearly sketch* $y = 2$
- *Sketched* $x = 2$

End of Section B Solutions

Section C Solutions

General comments:

Overall, this section was done somewhat poorly- scores above 10/15 were in the minority.

Question 17 (3 marks)

A solid glass cylinder has a volume of 75π cm^3 . **3**

The cost of polishing the surfaces of this cylinder is \$2 per square cm for the curved surface and \$3 per square cm for the circular top and base areas.

Given that the radius of the base is r cm, show that the cost of polishing, \$$C$, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

$$V_{\text{cylinder}} = \pi r^2 h = 75\pi \rightarrow h = \frac{75}{r^2}$$

$$\therefore\ C = 3 \times \left(2\pi r^2\right) + 2 \times (2\pi r h)$$

$$= 6\pi r^2 + 4\pi r h$$

$$= 6\pi r^2 + 4\pi r\left(\frac{75}{r^2}\right)$$

$$= 6\pi r^2 + \frac{300\pi}{r} \quad \text{as required.}$$

Most students correctly recalled the surface area formula for a cylinder, but didn't use the given volume to find 'h' in terms of 'r'.

Many students did not read the question closely enough and tried to calculate the cost. The question was asking to show that the cost formula was correct, given the other information.

Question 18 (3 marks)
Solve the following:

(a) $x + 3 = 2 + \frac{x-3}{5}$ **1**

$$x+3=2+\frac{x-3}{5}$$
$$5(x+3)=5\times 2+(x-3)$$
$$5x+15=10+x-3$$
$$5x-x=10-3-15$$
$$4x=-8$$
$$x=-2$$

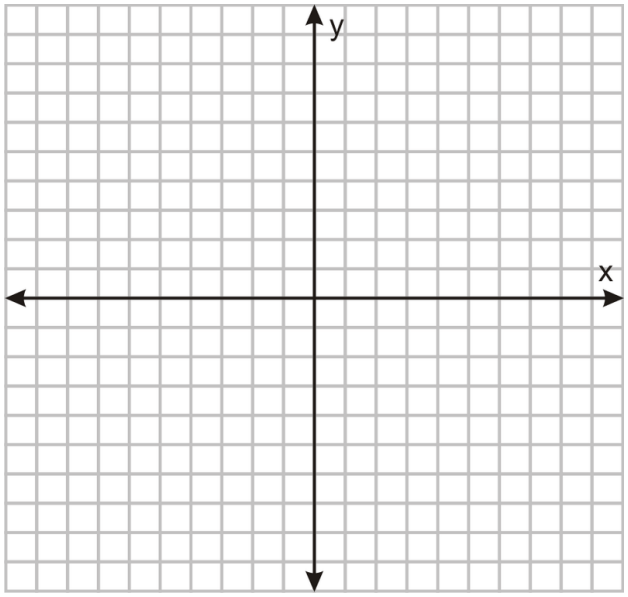
(b) $x - 3 = \frac{16}{x-3}$ **2**

$$x-3=\frac{16}{x-3}$$
$$(x-3)^2=16$$
$$x-3=\pm 4$$
$$x=\pm 4+3$$
$$x=7,-1$$

Part (a) was generally done well. A common error was not multiplying **every** term by 5 when multiplying out the denominator.

Part (b) was done ok. Most students correctly multiplied the denominator of (x-3) to the left side of the equation. Many students did not realise that they could rearrange the equation into a quadratic that they could solve, and many other students forgot to consider both the positive and negative roots of 16.

Question 19 (7 marks)
The line straight line L has gradient 3 and passes through the point $A(-6, 4)$.



(a) Find an equation for L in the form $y = mx + c$. **1**

$L: y-4=3(x-(-6))$ using point-gradient form

$y-4=3x+18$

$y=3x+22$

(b) The straight line K has equation $x - 7y + 14 = 0$.
Given that K crosses the y-axis at the point B and intersects L at C,
find the coordinates of B and C. **2**

Consider, K: $x-7y+14=0$ (1)

$y-$axis: $x=0$ (2)

L: $y=3x+22$ (3)

To find B, substitute

$(2) \rightarrow (1): 0-7y+14=0 \quad \rightarrow \quad y=2,\ x=0$

To find C, substitute

$(3) \rightarrow (1): x-7(3x+22)+14=0$

$x-21x-154+14=0$

$-20x-140=0 \quad \rightarrow \quad x=-7 \quad (4)$

$(4) \rightarrow (3): y=3(-7)+22 \quad \rightarrow \quad y=1$

$\therefore B(0,2)$ and $C(-7,1)$

Question 19 (continued)

(c) Show that $\angle BAC = 90°$. **2**

To show that $\angle BAC = 90°$, we can show that AB and AC are perpendicular if the product of their gradients is -1.

$$m(AB) = \frac{4-2}{-6-0} = -\frac{1}{3} \quad , \quad m(AC) = \frac{4-1}{-6-(-7)} = 3$$

Since $m(AB) \times m(AC) = -1$, therefore $\angle BAC = 90°$.

(d) Find the area of ΔABC. **2**

Since $\angle BAC = 90°$, we can let the base of the triangle be $\overline{AB}$ and the perpendicular height be $\overline{AC}$.

$$\overline{AB} = \sqrt{(-6-0)^2 + (4-2)^2} = \sqrt{40}$$

$$\overline{AC} = \sqrt{(-6--7)^2 + (4-1)^2} = \sqrt{10}$$

$$\therefore \text{Area} = \frac{1}{2} \times \overline{AB} \times \overline{AC}$$

$$= \frac{1}{2}\sqrt{40}\sqrt{10}$$

$$= \frac{1}{2}\sqrt{400}$$

$$= 10 \quad \text{square units}$$

Part (a) was done very well.

Part (b) was done fairly well. The students that used the graph to find points B and C did very well. Those that opted to use simultaneous equations did ok, but tended to make algebraic errors here and there.

Part (c) was done ok, when attempted. Most students recognised that they could use either gradients (to show that AB and AC are perpendicular) or distances (to show that triangle ABC satisfies Pythagoras' theorem). Working out was often very messy and/or incomplete.

Part (d) was also done ok, when attempted. Most students that attempted this part correctly identified that the base and height of the triangle were the sides AB and AC. A common error was incorrectly simplifying square roots.

Overall, parts (c) and (d) were often not attempted by students, often because they did not complete part (b). A reminder for students that "error carried forward" will allow them to demonstrate their understanding to earn marks, even if the results from a previous part are incorrect.

Question 20 (1 mark)

Evaluate $\dfrac{5.6\times 10^4+7\times 10^3}{2.8\times 10^{-3}}$ **1**

$$\begin{aligned}\frac{5.6\times 10^4+7\times 10^3}{2.8\times 10^{-3}} &= \frac{5.6\times 10^4}{2.8\times 10^{-3}}+\frac{7\times 10^3}{2.8\times 10^{-3}}\\ &= 2\times 10^7 + 2.5\times 10^6\\ &= 22,500,000\end{aligned}$$

Done very well. Most students correctly evaluated the expression. Incorrect solutions came from improper simplifying of the fraction (remember that common factors need to be factored out of the whole numerator before being cancelled).

Question 21 (1 mark)

Two inequalities have solution sets $P\ (-\infty, 0)$ and $Q\ [0, 10]$. **1**

What is the union of P and Q? (i.e. $P \cup Q$)

$(-\infty, 0) \cup \left[0, 10\right] = (-\infty, 10]$

Done somewhat poorly. Students are advised to revise interval notation, including the difference between ‘union’ and ‘intersection’.

End of Section C Solutions

Section D Solutions

Question 22 (3 marks)

(a) Simplify $\frac{5}{x+2} \div \frac{5x-10}{x^2-4}$ Generally well done. **2**

$= \frac{5}{x+2} \times \frac{x^2-4}{5x-10}$

$= \frac{5}{x+2} \times \frac{(x-2)(x+2)}{5(x-2)}$

$= 1$

(b) Expand and simplify $(a-3)(a^2+3a+9)$ Generally well done. **1**

$= a^3 + 3a^2 + 9a - 3a^2 - 9a - 27$

$= a^3 - 27$

Question 23 (4 marks)
In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are NOT acceptable.

(a) Simplify $\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$, giving the answer in the form $b\sqrt{2}+c$, where $b, c \in \mathbb{Z}$. **2**

$= \frac{4\sqrt{2}+3\sqrt{2}}{3+\sqrt{2}}$

$= \frac{7\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{21\sqrt{2}-14}{9-2}$

$= \frac{21\sqrt{2}-14}{7}$

$= 3\sqrt{2}-2$

$(\sqrt{2})^2 = 2$

common error : $(3+\sqrt{2})(3-\sqrt{2}) = 3^2 - 2^2 = 9-4 = 5$

- Poorly done, students need to revise how to rationalise the denominator.
- Students who simplified the expression before rationalising were more successful.

(b) Express the following in the form $p + q\sqrt{2}$ where p and q are rational. **2**

$$\frac{2}{2+\sqrt{2}}$$

$\frac{2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{4-2\sqrt{2}}{4-2}$

$= \frac{4-2\sqrt{2}}{2}$

$= 2-\sqrt{2}$

Always leave your answer in simplest form.

- students need to revise how to rationalise the denominator.

Question 24 (6 marks)

(a) Four friends each throw a biased coin a number of times.
The table shows the number of heads and the number of tails each friend got.

	Ben	Helen	Paul	Sharif	
Heads	34	66	80	120	300
Tails	8	12	40	40	100

(a) Which of the friends' results will give the best estimate for the probability that the coin will land heads? **2**

Sharif, since he has rolled it the most number of times.

• Best probability ≠ highest probability

(b) Paul says "With this coin you are twice as likely to get heads as to get tails." **2**
Is Paul correct?
Justify your answer.

Paul: $P(\text{head}) = \frac{80}{120} = \frac{2}{3}$ $P(\text{tail}) = \frac{40}{120} = \frac{1}{3}$

Overall: $P(\text{head}) = \frac{300}{400} = \frac{3}{4}$ $P(\text{tail}) = \frac{100}{400} = \frac{1}{4}$

According to Paul's results alone it is true, however when everyone's results are considered, you are three times as likely to get heads. So, Paul is incorrect.

• Many students only considered Paul's results.
• Ambiguous responses (e.g. maybe correct, possibly incorrect etc.) were not accepted.

(c) The coin is to be thrown twice more. Use all the results in the table to work out an estimate for the probability that the coin will land heads both times. **2**

$P(\text{head}) = \frac{300}{400} = \frac{3}{4}$

$\therefore P(\text{head, head}) = \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

• Many students considered the individual probabilities and then took the average.
• While some students got the answer $\frac{9}{16}$ (from using Sharif's results only), without correct working, marks were not awarded.

Question 25 (2 marks)

Let $P = \dfrac{2a - c}{d}$. **2**

If a equals 58.4 correct to 3 significant figures, c equals 20 correct to 2 significant figures, and d equals 3.6 correct to 2 significant figures, then what is the upper bound for the value of P?

Show your working clearly, and answer correct to 2 decimal places.

$58.35 \leq a < 58.45$

$19.5 \leq b < 20.5$

$3.55 \leq c < 3.65$

To find the upper bound of P, we want largest 'a' and smallest 'b' and 'c'.

$\therefore P = \dfrac{2(58.45) - 19.5}{3.55}$

$= 27.44 \quad (2dp)$

∴ Very poorly done.

∴ Note: upper bound is a number that is not actually achieved.

End of Section D

Section E Solutions

Question 26 (3 marks)

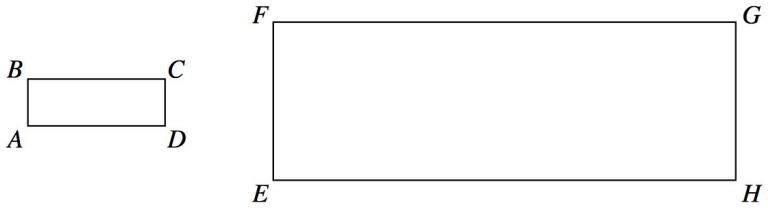


Figure 1

Figure 1 shows the rectangles *ABCD* and *EFGH* which are similar.

Given that $AB = (3 - \sqrt{5})$ cm, $AD = \sqrt{5}$ cm and $EF = (1 + \sqrt{5})$ cm, find the length *EH* in cm, giving your answer in the form $a + b\sqrt{5}$ where *a* and *b* are integers.

3

$$\frac{AB}{EF} = \frac{AD}{EH}$$

$$\therefore EH = \frac{AD \times EF}{AB}$$

$$EH = \frac{\sqrt{5}\left(1+\sqrt{5}\right)}{3-\sqrt{5}}$$

$$= \frac{\sqrt{5}\left(1+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)} \times \frac{\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)}$$

$$= \frac{3\sqrt{5}+5+15+5\sqrt{5}}{9-5}$$

$$= \frac{20+8\sqrt{5}}{4}$$

$$= \frac{4\left(5+2\sqrt{5}\right)}{4}$$

$$\therefore EH = 5+2\sqrt{5}$$

Marking Criteria	**Mark**
Correct answer with correct working out	3
Simplify the numerator OR denominator correctly for the expression of *EH*	2.5
Correct process of rationalising the denominator	2
Correct expression for *EH* using the correct proportions from the corresponding sides	1

Markers Comment:

Some students are making mistakes by multiplying $\sqrt{5}$ to the numerator AND denominator for the expression of *EH*, and some have difficulty with rationalising denominator.

Question 27 (4 marks)

The table shows information about the weights, in kilograms, of 40 babies.

Weight (w kg)	Frequency
$2 < w \leq 3$	12
$3 < w \leq 4$	16
$4 < w \leq 5$	9
$5 < w \leq 6$	2
$6 < w \leq 7$	1

(a) Write down the modal class. **1**

$3 < w \leq 4$

(b) Work out an estimate for the mean weight of the 40 babies. **1**

$$\bar{x} = \frac{2.5\times12 + 3.5\times16 + 4.5\times9 + 5.5\times2 + 6.5\times1}{40}$$

$$= \frac{144}{40}$$

$$= \frac{18}{5} \quad (\approx 3.6)$$

(c) One of the babies is chosen at random. **2**
Find the probability that this baby has a weight of more than 5 kg.

$$P(x > 5) = \frac{3}{40}$$

Marking Criteria	Mark
Correct solution.	2
3 or $\frac{12}{40}$	1

Markers Comment:

Generally, well done.

Question 28 (3 marks)

Solve the following pair of simultaneous equations **3**

$$x + 2y + 4 = 0 \qquad ①$$

$$3x - y - 3 = 0 \qquad ②$$

Elimination

$x+2y+4=0$ $\quad$ $x+2y+4=0$ ① $3x-y-3+0$ $\quad$ $\times 2 \therefore 6x-2y-6=0$ ② ① + ② $7x-2=0$ $\therefore\ x=\frac{2}{7}$ Sub into ① $\frac{2}{7}+2y+4=0$ $\therefore\ y=-\frac{15}{7}$	$x+2y+4=0$ $\quad \times 3 \therefore\ 3x+6y+12=0$ ① $3x-y-3=0$ $\quad$ $3x-y-3=0$ ② ① - ② $7y+15=0$ $\therefore\ y=-\frac{15}{7}$ Sub into ① $x-\frac{30}{7}+4=0$ $\therefore\ x=\frac{2}{7}$

Substitution

From ②, $3x-3=y$ sub into ①

$x+2(3x-3)+4=0$

$x+6x-6+4=0$

$7x-2=0$

$\therefore\ x=\frac{7}{2}$

Sub into $y=3x-3$

$\therefore y=3\times\frac{2}{7}-3$

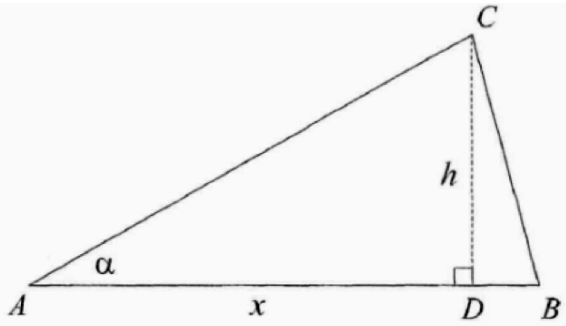
$y=-\frac{15}{7}$

Marking Criteria	**Mark**
Correct solutions	3
Using the elimination or substitution method to find the values of x or y.	2
Find values of x or y by substitution from first value	1

Markers Comments:
Generally, well done.
Most students understand how to apply the elimination or substitution method.
Other had difficulties with the algebraic manipulation.

Part F Marking Scheme

Question 29



The diagram above shows ΔABC with $AB = 1$ and $AC = 1$.
Point D is on AB such that $CD \perp AB$. Let $AD = x$, $\angle BAC = \alpha$, and $CD = h$.

(a) Show that $BC^2 = 2 - 2x$. **2**

Since $AB = 1$, $DB = AB - AD = 1 - x$ and $AC = 1$

In ΔACD

$AD^2 + DC^2 = AC^2$ (Pythagoras' theorem)

$x^2 + h^2 = 1^2$ $\therefore$ $h^2 = 1 - x^2$ (1)

In ΔDCB

$BC^2 = DC^2 + DB^2$ (Pythagoras' theorem)

$BC^2 = h^2 + (1-x)^2$ (2)

$BC^2 = 1 - x^2 + 1^2 - 2x + x^2$ (Sub (1) into (2))

$\therefore$ $BC^2 = 2 - 2x$

Marking guidelines	Marker's Comments
2 marks – Correct Solution with correct expression for *DB, DC* (in terms of *x*) and substitution. **1 mark** – Some valid working towards the correct solution.	- Many candidates did not receive full marks due to errors in finding correct expressions for *DB*, *DC*, or both. - In "SHOW" questions, it is crucial to provide all steps of your solution, as markers will not make assumptions beyond what is explicitly written on the page. **Marks are deducted for missing steps.** - A common error made by some candidates assumes that ΔCAB is right angled (no information is given) and fails to prove it is. Marks were deducted for the assumption. - Some candidates tried to use similarity with no success.

(b) Hence show that $BC^2 = 2 - 2\cos\alpha$. **1**

From $\triangle ACD$

$\cos\alpha = \frac{AD}{AC} = \frac{x}{1} = x$ (1)

Sub (1) into result in (a)

$\therefore BC^2 = 2 - 2\cos\alpha$

Marking guidelines	Marker's Comments
1 mark – Correct solution.	- Significant number of candidates were penalised for this question as they wrote down $x \cos\alpha$ without explicitly stating this was derived from ΔACD or simply just matching the two results from the questions (a) and (b). The correct solution involves the former and no marks were given for the latter. - Candidates are advised that to receive full marks, they must make use of their previous result to derive the correct solution, as indicated by the term "HENCE." Candidates must explicitly show the substitution into previous question to gain the full mark.

Question 29 (continued)

(iii) By considering $\angle BCD$, or otherwise, show that $\sin\frac{\alpha}{2}=\frac{\sqrt{2-2\cos\alpha}}{2}$. **3**

$AB = AC = 1$ $\therefore \Delta ABC$ is an isosceles triangle (2 equal sic

$\therefore \angle ACB = \angle ABC$ (base angles of isosceles ΔABC)

$\angle ACB + \angle ABC + \angle CAB = 180^\circ$ (angle sum of ΔABC)

$2\angle ACB + \alpha = 180^\circ$

$\therefore \angle ACB = \frac{180-\alpha}{2} = 90^\circ - \frac{\alpha}{2}^\circ$

$\angle ACD + \angle CDA + \angle DAC = 180^\circ$ (angle sum of ΔABC)

$\angle ACD + 90^\circ + \alpha = 180^\circ$

$\therefore \angle ACD = 90^\circ - \alpha^\circ$

Since $\angle ACD + \angle BCD = \angle ACB$

$90 - \alpha + \angle BCD = 90 - \frac{\alpha}{2}$

$\therefore \angle BCD = \frac{\alpha}{2}$

In ΔBCD: $\sin\frac{\alpha}{2} = \frac{DB}{BC}$

$= \frac{1-\cos\alpha}{\sqrt{2-2\cos\alpha}} \times \frac{\sqrt{2-2\cos\alpha}}{\sqrt{2-2\cos\alpha}}$

$= \frac{(1-\cos\alpha)\sqrt{2-2\cos\alpha}}{2(1-\cos\alpha)}$

$\therefore \sin\frac{\alpha}{2} = \frac{\sqrt{2-2\cos\alpha}}{2}$ as ($x = \cos\alpha$ from (b))

Marking guidelines	Marker's Comments
3 marks – Correct solution with ALL required steps and reasoning shown. **2 marks** – Mostly provides relevant steps and reasoning. **1 mark** – Some valid working towards the correct solution.	- This was poorly done by majority of the candidates. Many did not attempt this question. - Many candidates who attempted this question made the false assumption that $\angle BCD = \frac{\alpha}{2}$. Marks were deducted for using this false assertion. - Marks were penalised if candidates started from the result given in the question, to prove the result is true. This is because it assumes the conclusion is true without providing a rigorous logical foundation. Candidates for this question needed to start from the hint or otherwise, to show why RHS is true. - Other false assertions include ΔCAB is right angled (no information is given in the question or enough to prove it) and candidates drawing a line from A to bisect CB also bisect CD (which is false). Candidates need to take more care. - Candidates needed to justify why $\angle BCD = \frac{\alpha}{2}$ as shown in the solution above otherwise marks were deducted.

(iv) Hence determine sin 15° in exact form. **1**

From (iii) and let $x = 30°$

$$\sin\left(\frac{30}{2}\right) = \frac{\sqrt{2 - 2\cos 30}}{2}$$

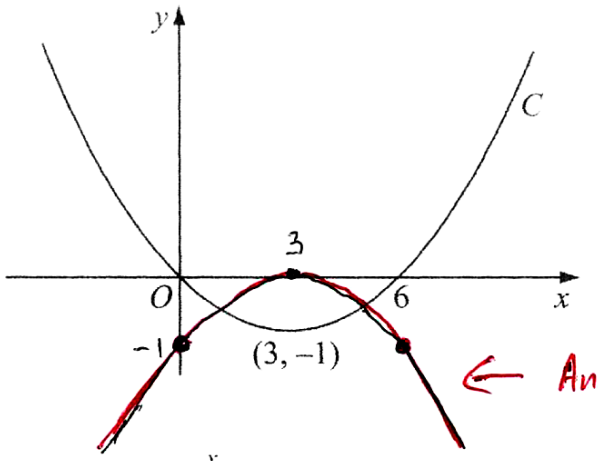
$$\sin 15 = \frac{\sqrt{2 - 2 \times \frac{\sqrt{3}}{2}}}{2}$$

$$= \frac{\sqrt{2 - \sqrt{3}}}{2}$$

Marking guidelines	Marker's Comments
1 mark – Correct answer.	- This was poorly done by majority of the candidates. Many did not attempt this question. - Calculator answer is not exact, and no marks were awarded. - Similarly, to the comment made in question (b), a 'Hence' question means candidates must show explicitly in their working how they applied the result from the previous question to achieve the correct solution to gain full marks. - Marks were penalised if candidates did not simplify cos 30°

Question 30 (3 marks)

The graph below shows $y = \frac{x}{9}(x-6)$. **3**



← Answer in RED

On the graph above, sketch the graph of $y = \frac{x}{9}(6-x) - 1$.

Note: You must indicate the intercepts with the coordinate axes.

$y = \frac{x}{9}(6-x) = -\frac{x}{9}(x-6)$

$y = \frac{x}{9}(6-x) - 1 = -\frac{x}{9}(x-6) - 1$

Graphically

Reflect along the x-axis AND THEN Translate down by 1 unit.

Marking guidelines	Marker's Comments
3 marks – Correct solution with correct graph drawn, correct x-intercept and correct y-intercept (all labelled). **2 marks** – Only 2 of the above shown correctly. **1 mark** – Only 1 of the above shown correctly.	- This was poorly done by majority of the candidates. - Candidates need to revise their graph transformation notes for the parabola. - Candidates who expanded the equation above and tried to graph it were less successful in drawing the correct graph. - No half marks were awarded.