SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 4
YEARLY EXAMINATION

Mathematics

General Instructions: **Time Allowed:** 70 minutes

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- **S**how ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the **simplest exact form**, unless otherwise stated.
- NESA approved calculators may be used
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: AF

Name: _____

Class	Teacher	
9Ma**A**	B. Kilmore	
9Ma**B**	J. Bui	
9Ma**C**	R. Wang	
9Ma**P**	A. Wang	
9Ma**L**	M. Gainford	
9Ma**U**	J. James	
9Ma**S**	T. Broockmann	

Section	Marks
A	/13
B	/13
C	/13
D	/13
E	/13
Total	/65

Question 1
Write 23.5236° in degrees and minutes (to the nearest minute). **1**

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Question 2 **1**
Lollies cost $2.50 at the checkout. How much GST is paid for the lollies?

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Question 3 (2 marks)
Are the following True or False?
(a) Any two equilateral triangles are similar. **1**

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(b) Any two rhombuses are similar. **1**

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Question 4
Anthony wants to invest $1000 at 5% p.a. simple interest. **1**
How many years until Anthony doubles his investment?

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Question 5 **1**
A certain map has a scale of 1 : 25 000. How many kilometres does 5 cm on this map represent?

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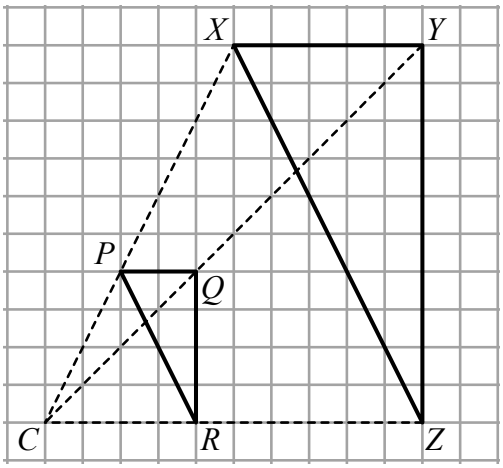
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Question 6 (2 marks)
Convert the following:

(a) 3.57 square metres = …… square centimetres. **1**

(b) 10^a cubic millimetres = ……. cubic centimetres (in index notation) **1**

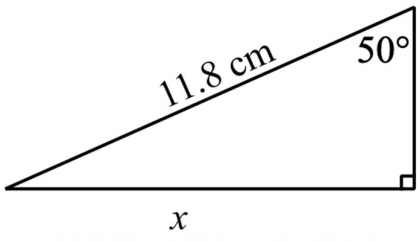
Question 7
In the diagram below, triangle *XYZ* is an enlargement of triangle *PQR*. **1**
What is the scale factor?



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Question 8
Find *x* correct to one decimal place. **1**



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Question 9

$M(2, -1)$ is the midpoint of AB, where A has coordinates $(-1, 3)$. **1**

What are the coordinates of B?

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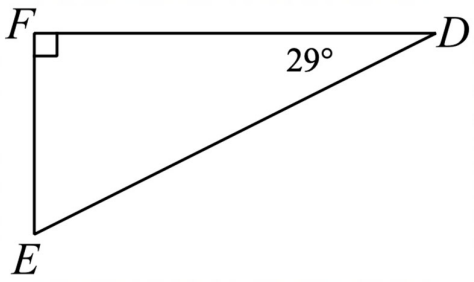
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Question 10 (1 mark)

In the diagram below, what is the ratio $\frac{FD}{ED}$, correct to 3 decimal places. **1**



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Question 11

What is the complement of $65° \; 35'$? **1**

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Extra Working Page for Section A only

Section B

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/13**

Question 12 (5 marks)

Factorise the following:

(a) $a^2 + 8a + 12$ **1**

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(b) $4a^2 - 5a - 6$ **2**

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(c) $a(a-b) - b(b-a)$ **1**

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(d) $(a-b)^2 - 4$ **1**

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Question 13 (3 marks)

(a) Prove that a triangle with sides of length 48 cm, 55 cm and 73 cm is right angled. **1**

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(b) Hence, find the smallest angle in the triangle (to the nearest degree). **2**

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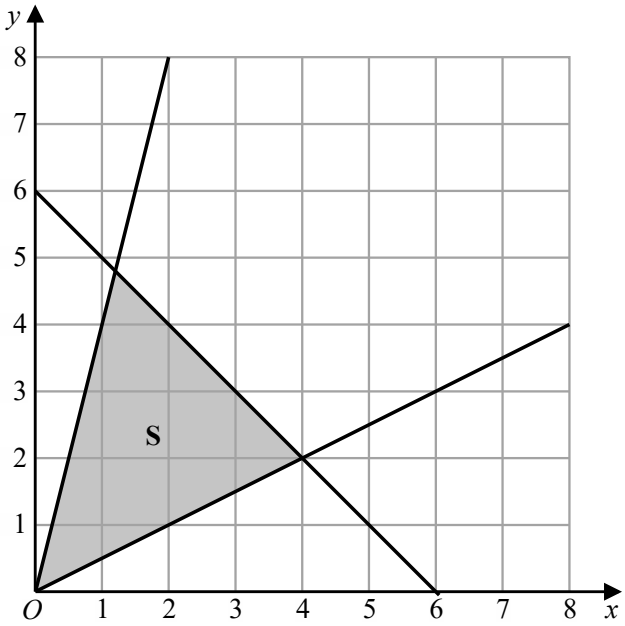
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Question 14 (2 marks)

The diagram shows the region **S** that satisfies $y \leq 4x$, $y \geq \frac{1}{2}x$, and $x + y \leq 6$. **2**



How many points with integer x and y coordinates satisfy all of the inequalities above?

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Question 15 (3 marks)

Solve the following:

(a) $5 - \frac{3x}{2} \leq 7$ **2**

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(b) $5 - 2x^2 = 1$ **1**

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Extra Working Page for Section B only

Section C

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/13**

Question 16 (3 marks)

A cylinder has a diameter of 10 cm and a height of 10 cm.

(a) Calculate the volume of the cylinder in terms of π. **1**

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(b) Calculate the surface area of the cylinder correct to one decimal place. **2**

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Question 17 (3 marks)

Find the exact value of:

(a) $\tan 30°$. **1**

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(b) $\sin\theta$, given that $\tan\theta = \frac{\sqrt{3}}{2}$. **2**

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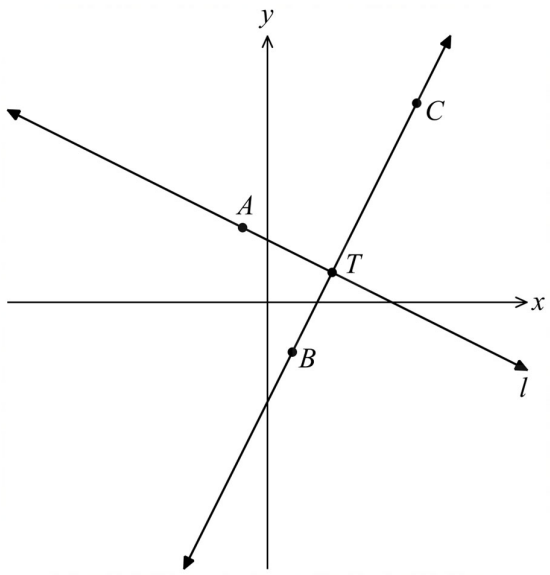
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Question 18 (7 marks)

The line l passes through $A(-1, 3)$ and has the equation $x + 2y - 5 = 0$.

The point B has coordinates $(1, -2)$ and the line CB is perpendicular to l.



(a) Find the length of interval AB. **1**

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(b) Find the gradient of interval AB. **1**

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(c) Show that the equation of the line CB is $2x - y - 4 = 0$. **2**

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Question 18 continues over the page

Question 18 (continued)

(d) Show that $(5-2a, a)$ lies on the line l. **1**

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(e) Hence, if T is the intersection of l and CB. Find the coordinates of T. **2**

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Extra Working Page for Section C only

Section D

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/13**

Question 19 (3 marks)

Simplify the following:

(a) $\dfrac{8}{x+3} \div \dfrac{2}{x^2+3x}$ **2**

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(b) $\dfrac{x}{4} - \dfrac{4-x}{8}$ **1**

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Question 20 (3 marks)

(a) Expand and simplify $(a+2)(a^2-2a+4)$. **1**

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(b) Hence, factorise $a^3+a^2+4a+12$. **2**

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Question 21 (6 marks)

5	2	4	6	3

Two of these cards are chosen at random. They are placed next to each other to form a two-digit number.

(a) Find the probability that the two-digit number is less than 30. **1**

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(b) Find the probability that the two-digit number is even. **1**

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(c) Find the probability that the two-digit number is prime. **2**

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(d) Find the probability that the two-digit number is a multiple of 4. **1**

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(e) If two of the remaining three cards are chosen at random and placed next to each other to form a second two-digit number, find the probability that the second two-digit number is greater than the first two-digit number. **1**

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Question 22

S is the solution set $(-\infty, 2]$ and T is the solution set $(-1, 5)$. **1**

What is the union of S and T? (i.e. $S \cup T$)

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Extra Working Page for Section D only

Section E

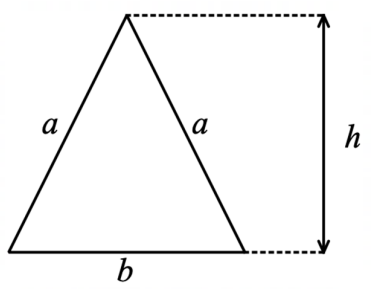
Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/13**

Question 23 (4 marks)

(a) Using the diagram below, show that $h = \frac{1}{2}\sqrt{4a^2 - b^2}$. **2**



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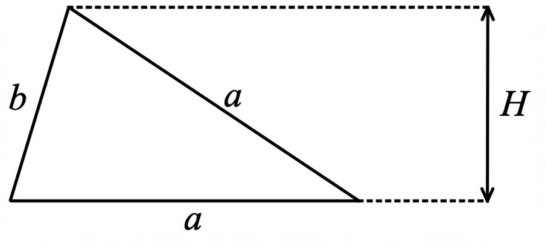
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(b) **2**



Hence, or otherwise, find H in terms of a and b.

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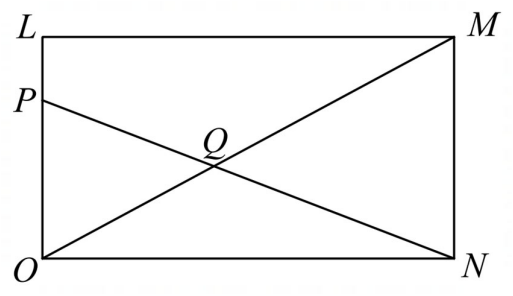
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Question 24 (6 marks)
In the diagram $LMNO$ is a rectangle and $ON = 3MN$.
The point P divides OL such that $OP = 2PL$.



(a) Show that $\triangle OQP$ is similar to $\triangle MQN$. **2**

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(b) Show that $3OM = 5QM$. **2**

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(c) Show that $2(ON)^2 = 5(QM)^2$. **2**

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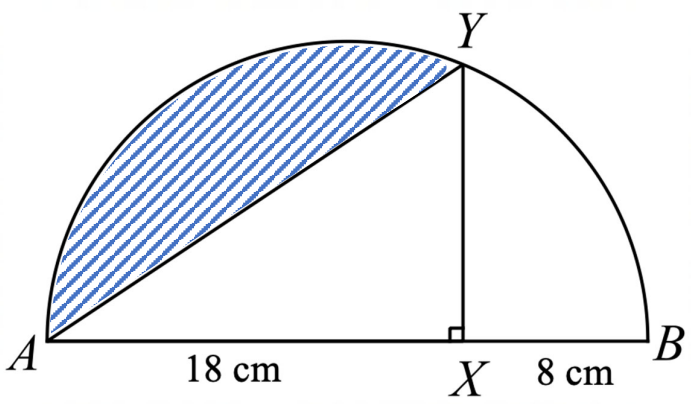
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Question 25 (3 marks)

In the semi-circle $AX = 18$ cm, $XB = 8$ cm, and $XY = 12$ cm. $XY \perp AB$. **3**

Calculate the shaded area correct to 2 decimal places.



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End of paper

Extra Working Page for Section E only

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 4
YEARLY EXAMINATION Part 2

Mathematics

General Instructions: **Time Allowed:** 30 minutes

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back pages of the booklet.
  Write 'see back' next to your crossed out attempt.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: PSP

Name: _____

Class	Teacher	
9MaA	B. Kilmore	
9MaB	J. Bui	
9MaC	R. Wang	
9MaP	A. Wang	
9MaL	A.M. Gainford	
9MaU	E. Choy	
9MaS	T. Broockmann	

Total	/19

1 An additive sequence has third term 3 and fifth term 5. **2**

Complete the sequence up to its sixth term.

____ , ____ , 3 , ____ , 5 , ____

2 On lines 17 and 18 the question was asked: **1**

"Are there any values of x for which a geometric sequence generated is also an arithmetic sequence?" $x = 1$ was found to be the only solution

Are there any other values of x for which the odd terms of the geometric sequence (with first term 1) form an arithmetic sequence?

3 An additive sequence has nth term n and $(n + 1)$th term $n + 1$. **2**
Will this sequence have any negative terms?
State any conditions, surrounding this.

4 An additive sequence has the first two terms which are odd. **2**
State, with reasoning, whether the 2022nd term will be odd or even.

5 On line 56 and 57 it is stated that the terms of the sequence **2**

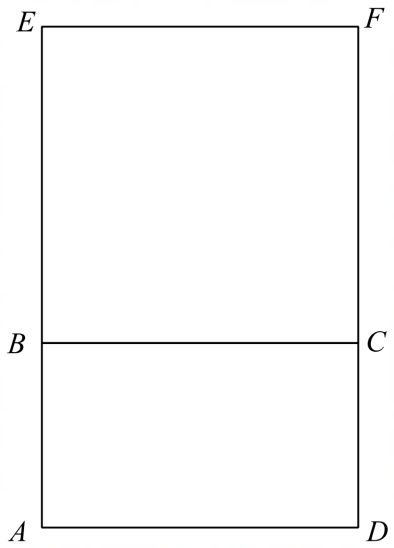
$$1, \phi, \phi^2, \phi^3, \phi^4, \phi^5, \ldots$$

can be written in the form

$$a\phi + b, \text{ where } a \text{ and } b \text{ are integers.}$$

What set of integers does a and b belong to?

6 Rectangle $ABCD$ is a golden rectangle i.e. $AB : BC = 1 : \phi$. **2**



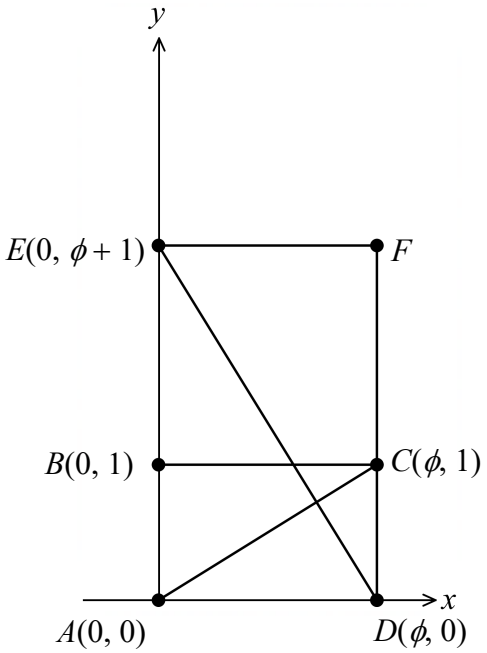
Show that by adding square $BCFE$ then rectangle $AEFD$ is a golden rectangle.
i.e. show that $AD : AE = 1 : \phi$.

7 On line 90 it is stated that the point Q has coordinates $\left(\frac{\phi+1}{2\phi-1}, \frac{\phi-1}{2\phi-1}\right)$, where $\phi = \frac{1+\sqrt{5}}{2}$. **3**

Find the y-coordinate of Q as a surd with a rational denominator.

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8



The equation of line AC is $x - \phi y = 0$

(a) Find the equation of ED in terms of ϕ. **2**

Question 8 continues on page 7

8 (continued)

(b) Find the point X on AD so that $EX \perp AC$. **1**

(c) If AE is a side of a golden rectangle, write down all possible sets of coordinates, in the first quadrant, for the other two coordinates (in terms of ϕ). **2**

End of paper

Extra Working Page

Comprehension in Applications of Advanced Mathematics

Term 4 2022

Time: 30 mins

INSTRUCTIONS TO CANDIDATES

- This is the text for use with the questions.

The Golden Ratio

Introduction

In mathematics there are many important constants. You have met some of these, such as π, the ratio of the circumference of a circle to its diameter, and e, the base of natural logarithms.

In this article, you will be introduced to another important constant and some of its properties, one of which is linked to sequences.

Geometric sequences

The sequence

$$1,\ x,\ x^2,\ x^3,\ x^4,\ x^5,\ \ldots \qquad (*)$$

is a geometric sequence with first term 1 and common ratio x. You can substitute various values for x and each one will generate a new geometric sequence.

For example, $x = 3$ generates the geometric sequence

$$1,\ 3,\ 9,\ 27,\ 81,\ 243,\ \ldots$$

and $x = -\frac{1}{2}$ generates the geometric sequence

$$1,\ -\tfrac{1}{2},\ \tfrac{1}{4},\ -\tfrac{1}{8},\ \tfrac{1}{16},\ -\tfrac{1}{32},\ \ldots\ .$$

Notice that neither of these is an arithmetic sequence, prompting the following question.

> Are there any values of x for which the geometric sequence generated is also an arithmetic sequence?

In order to answer this question, it is helpful to think about the first three terms of the sequence $(*)$.

Suppose that $1,\ x,\ x^2$ are the first three terms of an arithmetic sequence. Then the difference between the first two terms, $x - 1$, is equal to the difference between the next two terms, $x^2 - x$, so that

$$x - 1 = x^2 - x.$$

This equation can be written in the form $x^2 - 2x + 1 = 0$. This has a repeated root of $x = 1$; the only value of x for which $1,\ x,\ x^2,\ \ldots$ is both an arithmetic and a geometric sequence is $x = 1$.

You will notice that $x = 1$ leads to the rather uninteresting sequence $1,\ 1,\ 1,\ 1,\ 1,\ \ldots$.

Additive sequences

In the Fibonacci sequence, the first two terms are given as $1,\ 1$. After that, each term is the sum of the two previous terms. The Fibonacci sequence is

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ \ldots\ .$$

Another sequence with the same property, that each term is the sum of the two previous terms, is the Lucas sequence

$$2,\ 1,\ 3,\ 4,\ 7,\ 11,\ 18,\ 29,\ \ldots\ .$$

In general, you can choose any two numbers for the first two terms, call them u_1 and u_2, and then use the rule

$$u_{n+1} = u_n + u_{n-1} \text{ for } n \geqslant 2$$

to generate subsequent terms. For the purposes of this article, any sequence with this property will be called an *additive sequence*.

Now think again about the geometric sequence $(*)$. Are there any values of x for which this sequence is an additive sequence?

To be an additive sequence, x must satisfy each of the following equations.

$$\begin{aligned} x^2 &= x + 1 \\ x^3 &= x^2 + x \\ x^4 &= x^3 + x^2 \\ &\vdots \end{aligned}$$

The first equation shows that $x \neq 0$. So each of the other equations reduces to the quadratic equation $x^2 = x + 1$. The two roots of this quadratic equation are $x = \dfrac{1+\sqrt{5}}{2}$ and $x = \dfrac{1-\sqrt{5}}{2}$. Therefore these are the only two values of x for which the sequence $(*)$ is both a geometric sequence and an additive sequence.

The Golden Ratio

The first of these numbers, $\dfrac{1+\sqrt{5}}{2}$, is called the Golden Ratio and is denoted by the Greek letter ϕ, pronounced 'phi'.

Substituting $x = \phi$ in the sequence $(*)$ gives

$$1,\ \phi,\ \phi^2,\ \phi^3,\ \phi^4,\ \phi^5,\ \ldots\ .$$

When written in this form, it is not at all obvious that it is an additive sequence. However, using the fact that $\phi^2 = \phi + 1$, the third term can be written as $\phi + 1$ and the fourth term, ϕ^3, can be written as $\phi^2 + \phi$ which simplifies to $2\phi + 1$. Continuing in this way, the sequence can be written as

$$1,\ \phi,\ \phi + 1,\ 2\phi + 1,\ 3\phi + 2,\ 5\phi + 3,\ \ldots\ ,$$

confirming the additive property $u_{n+1} = u_n + u_{n-1}$.

The Golden Rectangle

The rectangle ABCD in Fig. 1 has width $\phi = \dfrac{1+\sqrt{5}}{2}$ units and height 1 unit.

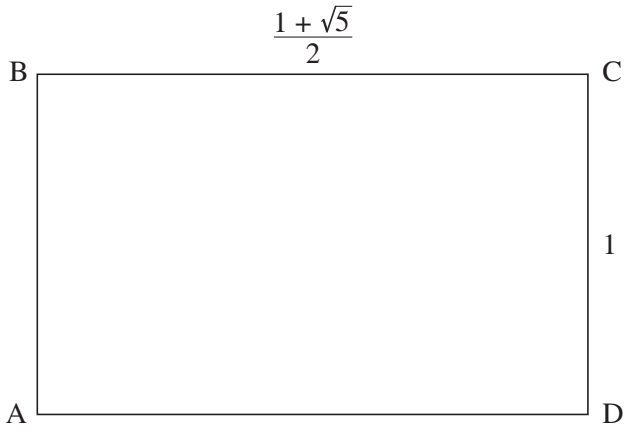


Fig. 1

The ratio of the sides of this rectangle is $\dfrac{1+\sqrt{5}}{2} : 1$.

Many artists and architects believe that, of all rectangles, ones with this ratio are particularly pleasing to the human eye. For this reason approximations to these rectangles have been used extensively in art and architecture.

Any rectangle with sides in the ratio $\dfrac{1+\sqrt{5}}{2} : 1$ is called a Golden Rectangle.

Golden Rectangles have many interesting properties, some of which will be described here.

First remove a unit square, ABEF, from a Golden Rectangle, as shown in Fig. 2.

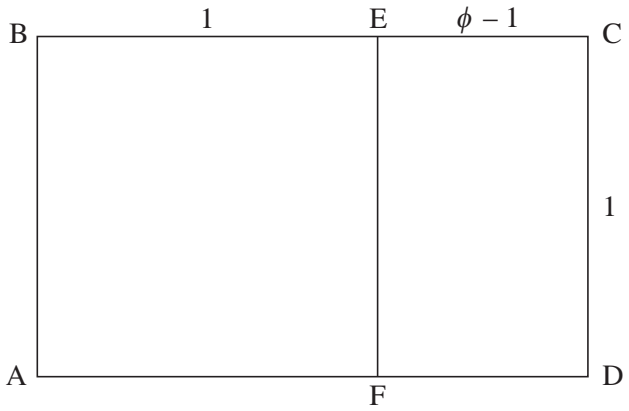


Fig. 2

Then using $\phi^2 = \phi + 1$, the ratio of the sides of rectangle ECDF may be found as follows.

$$\begin{aligned} \text{CD} : \text{DF} &= 1 : \phi - 1 \\ &= \phi : \phi^2 - \phi \\ &= \phi : 1 \end{aligned}$$

Therefore the ratio is the same for rectangle ECDF as for rectangle ABCD. So rectangle ECDF is also a Golden Rectangle.

This shows a property of a Golden Rectangle: any Golden Rectangle can be divided into a square and a smaller Golden Rectangle.

So if you remove a square ECGH from the Golden Rectangle ECDF, as shown in Fig. 3, then the ratio of the sides of rectangle FHGD is the same as that for rectangle ABCD.

This implies that the point H must lie on the line BD.

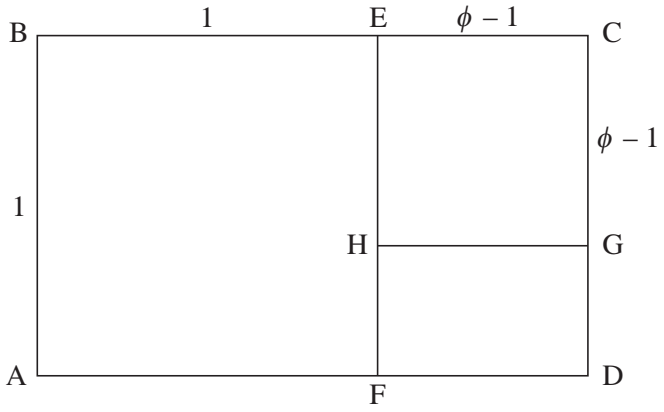


Fig. 3

Continuing in this way, removing a square from the Golden Rectangle just constructed, you can form a sequence of Golden Rectangles as shown in Fig. 4. The lines BD and CF are also shown.

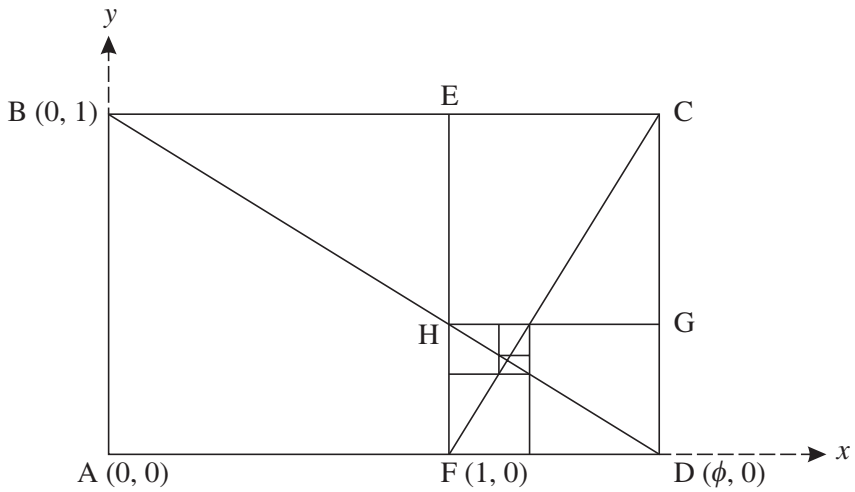


Fig. 4

This process can be continued indefinitely, each rectangle generated being similar to the original rectangle ABCD. Each square has a corner on the line BD and another corner on the line CF.

Taking A as the origin, AD as the positive x-axis and AB as the positive y-axis, as in Fig. 4, it can be shown that the equation of the line BD is

$$\phi y + x = \phi$$

and the equation of the line CF is

$$(\phi - 1)y = x - 1,$$

where $\phi = \dfrac{1 + \sqrt{5}}{2}$.

It follows that lines BD and CF are perpendicular and Q, the point of intersection of these two lines, has coordinates $\left(\frac{\phi+1}{2\phi-1}, \frac{\phi-1}{2\phi-1}\right)$.

Spirals

In Fig. 5, circular arcs have been drawn in each square. Arc AE has centre F, arc EG has centre H, and so on.

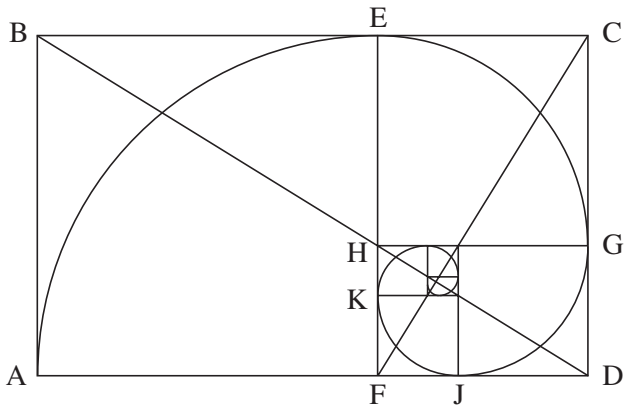


Fig. 5

In this spiral, the radius changes abruptly as it moves into each new square. For example, at the point E the radius changes from 1 unit in square ABEF to $\frac{-1+\sqrt{5}}{2}$ units in square ECGH.

It is, however, possible to construct a spiral through points A, E, G, J, K and so on (see Fig. 6) in a different way so that these 'jumps' are smoothed out.

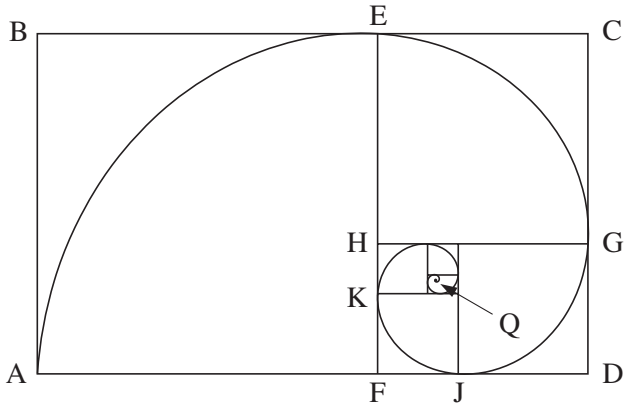


Fig. 6

As a point P moves along the resulting spiral from A towards Q, the distance QP decreases uniformly. During each full turn, the distance QP decreases by a constant factor of $\phi^4 = \left(\frac{1+\sqrt{5}}{2}\right)^4 = \frac{7+3\sqrt{5}}{2} \approx 6.9$.

For example, if you draw a straight line from Q, such as the one shown in Fig. 7, and measure the distances QP_1 and QP_2 you will find that $QP_1 \approx 6.9 \times QP_2$.

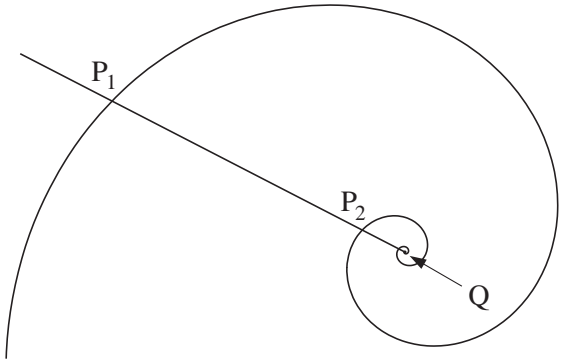


Fig. 7

This spiral is called a logarithmic spiral. It has the property that, although any two segments of the spiral differ in size, they have the same shape. If you zoomed in near the centre of the spiral and then enlarged this view, the shape would fit exactly onto the original spiral.


© iStockphoto.com / Horst Gossmann

Fig. 8

Fig. 8 is a photograph of the shell of a nautilus, a marine creature. The shape of the shell forms a logarithmic spiral. This is just one of many examples where the Golden Ratio may be found in the natural world.

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 4
YEARLY EXAMINATION PART 1

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

Section A Solutions

Question 1

Write 23.5236° in degrees and minutes (to the nearest minute). **1**

From calculator: $23.5236° = 23°31'$

[Well done by all]

Question 2 **1**

Lollies cost $2.50 at the checkout. How much GST is paid for the lollies?

$$\frac{250}{110} \times 10 = 22.73 \approx 23 \text{ cents}$$

[Quite a few students just put 25 cents thinking GST is 10% of $2.50]

Question 3 (2 marks)

Are the following True or False?

(a) Any two equilateral triangles are similar. **1**

True

[Well done]

(b) Any two rhombuses are similar. **1**

False – they are not necessarily equiangular

[Most students got this wrong]

Question 4

Anthony wants to invest $1000 at 5% p.a. simple interest. **1**

How many years until Anthony doubles his investment?

$5\% \times \$1000 = \$50$

To get another $1000 then $1000 \div 50 = 20$

i.e. it will take 20 years to double to $2000

[Well done by all]

Question 5 **1**

A certain map has a scale of 1 : 25 000. How many kilometres does 5 cm on this map represent?

$$5 \text{ cm} \leftrightarrow \frac{(25 \times 5) \times 1000}{1000} \text{ km} = 1.25 \text{ km}$$

[Well done]

Question 6 (2 marks)

Convert the following:

(a) 3.57 square metres = …… square centimetres. **1**

$$3.57 \text{ m}^2 = 3.57 \times (100)^2 = 35700 \text{ cm}^2$$

[Careless mistakes like 3570 or 3 570 000]

(b) 10^a cubic millimetres = ……. cubic centimetres (in index notation) **1**

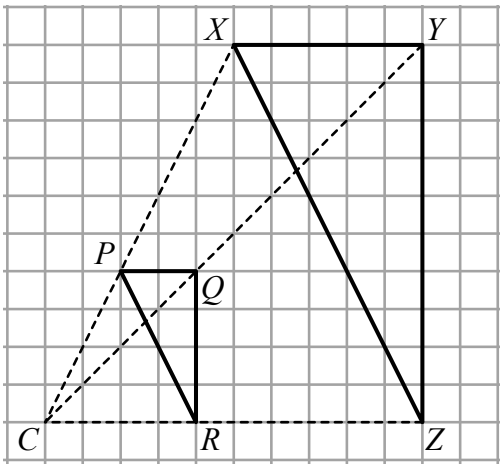
$$1 \text{ cm} = 10 \text{ mm}$$

$$10^a \text{ mm}^3 = 10^a \div (10)^3 = 10^{a-3} \text{ cm}^3$$

Question 7

In the diagram below, triangle *XYZ* is an enlargement of triangle *PQR*. **1**

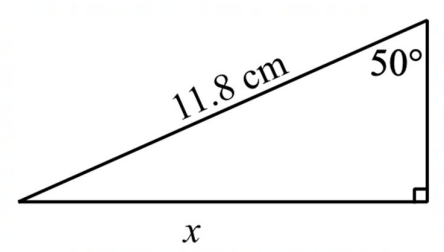
What is the scale factor?



The sides are in ratio 2 : 5, so scales factor is 2.5

Question 8

Find x correct to one decimal place. **1**



$$\frac{x}{11.8} = \sin 50°$$
$$\therefore x = 11.8 \sin 50°$$
$$\therefore x = 9.0$$

[$\tan 50°$ or $\cos 50°$ were common mistakes]

Question 9

$M(2, -1)$ is the midpoint of AB, where A has coordinates $(-1, 3)$. **1**

What are the coordinates of B?

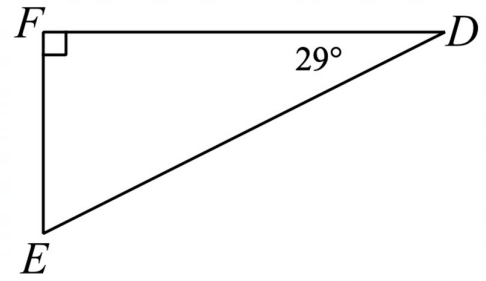
$$(-1, 3) = (x_1, y_1)$$

$$(2, -1) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \Rightarrow (x_2, y_2) = (5, -5)$$

[Well done]

Question 10 (1 mark)

In the diagram below, what is the ratio $\frac{FD}{ED}$, correct to 3 decimal places. **1**



$\angle FED = 61°$

$\frac{FD}{ED} = \sin 61° \approx 0.875$

[Quite a few students didn't read the question and just wrote $\frac{FD}{ED}$ without a numerical answer

Some students got the wrong trig ratio.]

Question 11

What is the complement of $65° \; 35'$? **1**

$90° - 65°35' = 24°25'$

[Some students wrote the supplement]

Section B Marking Scheme and Comments

Question 12 (5 marks)
Factorise the following:

(a) $a^2+8a+12$ **1**

$(a+6)(a+2)$

Marking Scheme:
1 mark for correct answer.

Marker's comment:
Majority of candidates did well in this question.

(b) $4a^2-5a-6$ **2**

$4a^2-8a+3a-6$

$=4a(a-2)+3(a-2)$

$=(4a+3)(a-2)$

Marking Scheme:
1 mark for correct working.
1 mark for correct answer.

Marker's comments:
- Significant number of candidates were not able to factorise this quadratic correctly.
- Most successful method by many candidates were splitting the middle term as shown and factorising by grouping in pairs.
- A common error was the expansion of candidates' factorisation did not give the middle term of original expression i.e. $-5a$.

(c) $a(a-b)-b(b-a)$ **1**

$=a(a-b)+b(a-b)$

$=(a+b)(a-b)$

Marking scheme:
1 mark for the correct answer

Marker's comments:
- Majority of the candidates did well in this question.
- Candidates who just expanded the expression and not factorise it correctly were not given the mark.
- Candidates should not write the expansion of the correct factorisation as their final answer as they are not answering the question.

Question 12 (continued)

(d) $(a-b)^2-4$ **1**

$(a-b)^2-2^2$

$=(a-b-2)(a-b+2)$

Marking Scheme:
1 mark for correct answer

Marker's comments

- Significant number of candidates were not able to factorise this difference of two squares correctly.
- No marks were given if expansion of the expression above was only given, and no correct factorisation was given.
- Common mistakes included $(a-b+2)(a+b-2)$ and $(a-b+4)(a-b-4)$

Question 13 (3 marks)

(a) Prove that a triangle with sides of length 48 cm, 55 cm and 73 cm is right angled. **1**

$(48)^2 + 55^2 = 5329$

$= 73^2$

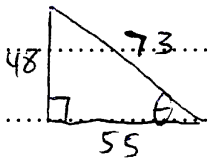
By the converse of Pythagoras' theorem, triangle is right angled.

Marking scheme:
1 mark for correctly **proving** it is right angled triangle.

Marker's comment:
- Many candidates were able to use the converse of Pythagoras's theorem (via numerical working) to state it is a right-angled triangle. However, many candidates should state **explicitly** that they are using the converse of Pythagoras theorem as a reason in their proof, to give meaning of their numerical working.

(b) Hence, find the smallest angle in the triangle (to the nearest degree). **2**

Smallest angle is opposite smallest side.



let θ be the smallest angle

$\sin\theta = \frac{48}{73}$

$\theta = \sin^{-1}\left(\frac{48}{73}\right)$

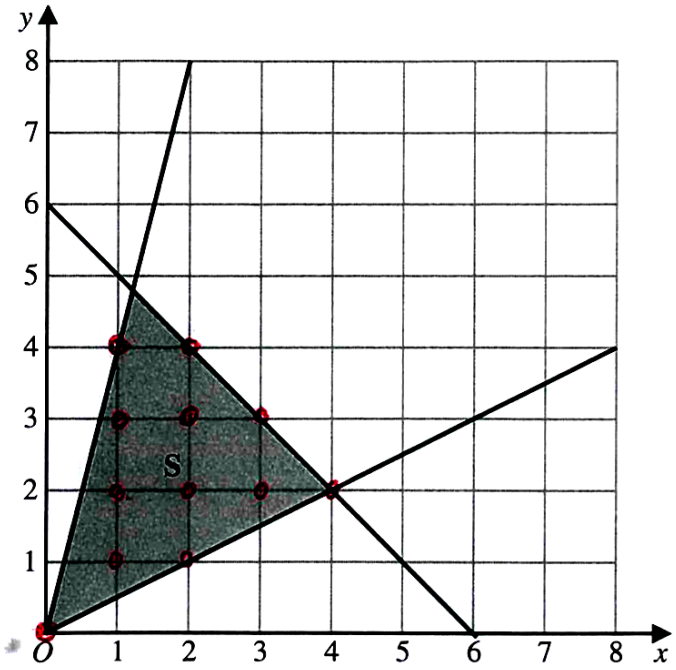
$\doteq 41^\circ$ (nearest degree)

Marking scheme:
1 mark for correct working out.
1 mark for correct answer (41°)

Marker's comments:
- Many candidates were able to do this question correctly.
- Many candidates found both angles smaller than 90° through trigonometry, to compare and determine the smallest angle. Candidates could have saved some time if they use the fact that the smallest angle is always opposite to the smallest side in a triangle and found that angle directly.

Question 14 (2 marks)

The diagram shows the region **S** that satisfies $y \leq 4x$, $y \geq \frac{1}{2}x$, and $x + y \leq 6$. **2**



How many points with integer x and y coordinates satisfy all of the inequalities above?

12 points. (From counting above)

Marking Scheme:

2 marks for stating 12 points (correct answer)

1 mark only for any other reasonable number of points.

0 mark only for no response or not answering what the question is asking.

Marker's comments:

- Many candidates were able to get this question correctly.
- Many candidates counted 11 and most of them missed the point (0,0).

Question 15 (3 marks)
Solve the following:

(a) $5-\frac{3x}{2}\leq 7$ **2**

$-\frac{3x}{2} \leq 2$

$-3x \leq 4$

$x \geq -4/3$

Marking Scheme:
1 mark for correct working.
1 mark for correct answer.

Marker's comments:

- Many candidates were **not** able to answer this question correctly.
- Candidates who tried to get rid of the denominator of 2 were more likely to make mistake of not multiplying every term with 2 (i.e. multiply 2 with either 5 or 7 but not both) or multiply the numerator of the fraction by 2.
- Significant number of candidates did not switch the inequality sign when multiplying or dividing by a negative number. A mark was penalised for this error.
- Significant number of candidates made silly arithmetic errors which they need to take more care of.

(b) $5-2x^2=1$ **1**

$-2x^2 = -4$

$x^2 = 2$

$x = \pm\sqrt{2}$

Marking scheme:
1 mark for the correct answer

Marker's comment:

- Many candidates were penalised a half mark for not writing $\pm\sqrt{2}$ as there are two correct answers i.e. $\sqrt{2}$ and $-\sqrt{2}$, not just 1 answer. Candidates need to be more aware of this common error.
- Some candidates need to take more care with silly arithmetic errors.

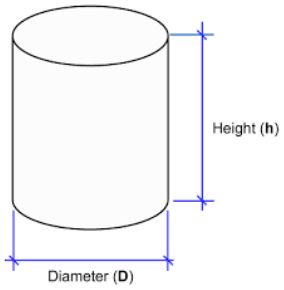
Section C

Several candidates failed to read the whole section, and were put off by the "Blank Page" in this section.

Question 16 (3 marks)

A cylinder has a diameter of 10 cm and a height of 10 cm.

(a) Calculate the volume of the cylinder in terms of π. **1**



$V = \pi r^2 h$

$= \pi(5^2).10$

$= 250\pi$

[This part was very well answered.]

(b) Calculate the surface area of the cylinder correct to one decimal place. **2**

$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi(5^2) + 2\pi \times 5 \times 10$

$= 150\pi$

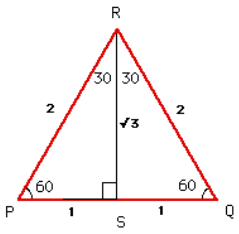
$\approx 471.2 \text{ cm}^2$

[Some candidates failed to calculate the approximation.]

Question 17 (3 marks)

Find the exact value of:

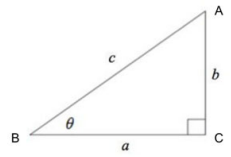
(a) $\tan 30°$. **1**



$\tan 30° = \frac{1}{\sqrt{3}}$

[Quite well answered. Some had $\sqrt{3}$ instead of $\frac{1}{\sqrt{3}}$]

(b) $\sin\theta$, given that $\tan\theta = \frac{\sqrt{3}}{2}$. **2**



$b = \sqrt{3}, a = 2, c^2 = 2^2 + \left(\sqrt{3}\right)^2$

$\therefore c = \sqrt{7}$

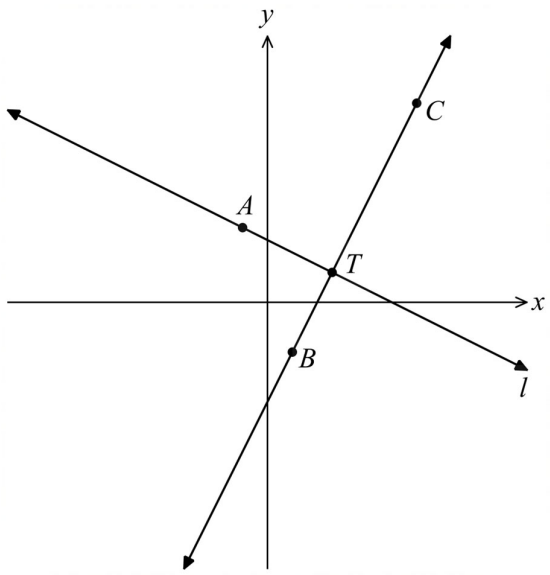
$\therefore \sin\theta = \frac{\sqrt{3}}{\sqrt{7}}$

[Well answered by most. Some thought $\frac{\sqrt{3}}{\sqrt{7}} = \frac{3}{7}$ (subsequent error).]

Question 18 (7 marks)

The line l passes through $A(-1, 3)$ and has the equation $x + 2y - 5 = 0$.

The point B has coordinates $(1, -2)$ and the line CB is perpendicular to l.



(a) Find the length of interval AB. **1**

$d^2 = (-1-1)^2 + (3+2)^2 = 4 + 25$

$\therefore d = \sqrt{29}$ *[Very well answered by most candidates.]*

(b) Find the gradient of interval AB. **1**

$m = \frac{-2-3}{1+1} = -\frac{5}{2}$ *[Mostly well done, but some had the opposite, or the reciprocal]*

(c) Show that the equation of the line CB is $2x - y - 4 = 0$. **2**

$B(1, -2)$, $m_{AT} = -A/B = -\frac{1}{2}$, $m_{CB} = 2$. ($\perp$)

$y + 2 = 2(x - 1)$, and so $2x - y - 4 = 0$.

[About half managed this, by one means or another. Some only earned one mark.]

Question 18 solutions continues over the page

Question 18 (continued)

(d) Show that $(5-2a, a)$ lies on the line l. **1**

l: $x + 2y - 5 = 0$

LHS = $(5 - 2a) + 2a - 5$
= 0
= RHS $\therefore$ The point lies on l. [*Well answered.*]

(e) Hence, if T is the intersection of l and CB. Find the coordinates of T. **2**

$x + 2y - 5 = 0$ ——(1)
$2x - y - 4 = 0$ ——(2)
(2)×2: $4x - 2y - 8 = 0$ ——(3)
(1)+(3): $5x - 13 = 0$
$x = 13/5$

In (2): $13/5 + 2y - 5 = 0$
$y = 6/5$ $\therefore$ $T(13/5, 6/5)$

[*About 30% of candidates pushed this through to the correct answer.*]

2022 YR9 YE Section D Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 19

Simplify the following:

A. $\dfrac{8}{x+3} \div \dfrac{2}{x^2+3x}$ 2

Solution	Comment(s)
$\dfrac{8}{x+3} \div \dfrac{2}{x^2+3x} = \dfrac{8}{x+3} \times \dfrac{x^2+3x}{2}$ $= \dfrac{8}{x+3} \times \dfrac{x(x+3)}{2}$ $= \dfrac{4}{1} \times \dfrac{x}{1}$ $= 4x$	Many students didn't simplify completely because they expanded $4(x^2+3x)$ instead of factorising x^2+3x.

B. $\dfrac{x}{4} - \dfrac{4-x}{8}$ 1

Solution	Comment(s)
$\dfrac{x}{4} - \dfrac{4-x}{8} = \dfrac{2x}{8} - \dfrac{4-x}{8}$ $= \dfrac{2x-4+x}{8}$ $= \dfrac{3x-4}{8}$	Common error(s): • Incorrectly expanding $-(4-x)$ into $-4-x$. • Forgetting about the 8 on the denominator.

Question 20

A. Expand and simplify $(a+2)(a^2-2a+4)$. 1

Solution	Comment(s)
$(a+2)(a^2-2a+4)$ $= a^3-2a^2+4a+2a^2-4a+8$ $= a^3+8$	Students generally performed well on this question.

B. Hence, factorise $a^3+a^2+4a+12$. 2

Solution	Comment(s)
$a^3+a^2+4a+12$ $= a^3+8+a^2+4a+4$ $= (a+2)(a^2-2a+4)+(a+2)^2$ $= (a+2)\big(a^2-2a+4+(a+2)\big)$ $= (a+2)(a^2-a+6)$	Most students were penalised for failing to make use of their answer in Part A as indicated by the 'hence' in the question, regardless of whether their answer was correct.

2022 YR9 YE Section D Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 21

5	2	4	6	3

Two of these cards are chosen at random.

They are placed next to each other to form a two digit number.

A. Find the probability that the two digit number is less than 30. 1

Solution	Comment(s)
For a two digit number x to be less than 30, the first digit must be 2 and the second digit is irrelevant. Hence: $$P(x<30)=\frac{1}{5}$$	Some students took the less efficient approach of listing out 23, 24, 25 and 26 as the four two digit numbers out of the possible twenty that are less than 30 in value. Common error(s): • Miscounting the number of numbers less than 30.

B. Find the probability that the two digit number is even. 1

Solution	Comment(s)
For the two digit number x to be even, the first digit is irrelevant and the second digit must be either 2, 4 or 6. Hence: $$P(x \text{ is even})=\frac{3}{5}$$	As with Part A, some students took the less efficient approach of listing out all the possibilities. Common error(s): • Miscounting the number of even numbers.

C. Find the probability that the two digit number is prime. 2

Solution	Comment(s)
There are five possible choices for the first digit and four for the second. Hence, it is possible to form $5 \times 4 = 20$ two digit numbers. Among the twenty, there are three prime numbers: 23, 43 and 53. Hence: $$P(x \text{ is prime})=\frac{3}{20}$$	Common error(s): • Misinterpreting the question as one involving replacement. • Miscounting the number of prime numbers.

2022 YR9 YE Section D Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 21

5	2	4	6	3

D. Find the probability that the two digit number is a multiple of 4. 1

Solution	Comment(s)
Among the twenty, there are six numbers that are multiples of 4: 24, 32, 36, 52, 56 and 64. Hence: $P(x \text{ is a multiple of } 4) = \frac{6}{20}$ $= \frac{3}{10}$	Common error(s): • Misinterpreting the question as one involving replacement. • Miscounting the number of numbers which are a multiple of 4.

E. If two of the remaining three cards are chosen at random 1
and placed next to each other to form a second two digit number,
find the probability that the second two digit number is greater than the first two digit number.

Solution	Comment(s)
Consider the case where the two digit numbers chosen are 52 and 46. If 52 is chosen first and 46 is chosen second, then the second isn't greater than the first. If 46 is chosen first and 52 is chosen second, then the second is greater than the first. Hence, for the pair of 52 and 46, there is a 50% chance that the second number is greater than the first. Therefore, for any pair of two numbers x and y, there is a 50% chance that the second number is greater than the first, regardless of what the numbers are. Hence: $P(x < y) = \frac{1}{2}$	While some students gave the correct answer, most students were unable to use the method shown, instead opting to brute force their way to the answer. Considering that this is a one mark question, they have potentially wasted time and not maximised their exam mark.

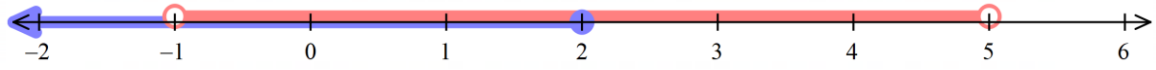
2022 YR9 YE Section D Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 22

S is the solution set $(-\infty, 2]$ and T is the solution set $(-1, 5)$. 1

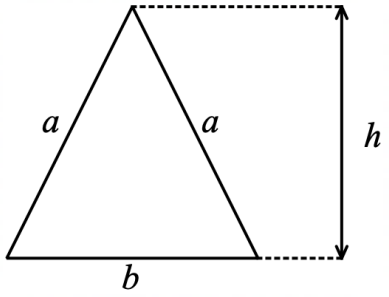
What is the union of S and T? (i.e. $S \cup T$)

Solution
Below are the intervals $(-\infty, 2]$ and $(-1, 5)$ plotted on the same number line:  The solution set of $S \cup T$ contains all the solutions that satisfy S, T or both. Hence, the solution set of $S \cup T$ is $(-\infty, 5)$.
Comment(s)
Common error(s): • Misinterpreting this question as a set theory problem rather than a number line problem. • Finding the solution set of $S \cap T$, which is $(-1, 2]$, rather than $S \cup T$.

Part E Solutions

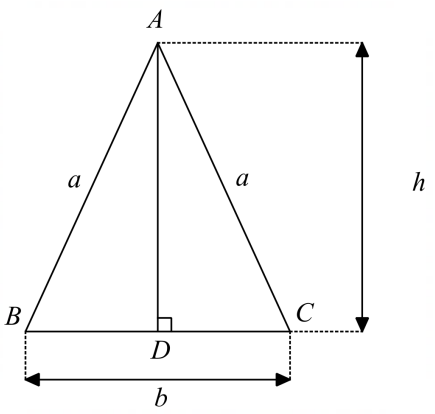
Question 23

(a) Using the diagram below, show that $h = \frac{1}{2}\sqrt{4a^2 - b^2}$. **2**



Answer:

We label the triangle ABC and construct the height AD to the base BC as the following diagram:



Since $AB = AC = a$, ΔABC is isosceles.
In ΔABC, the altitude AD drawn to the base is also the median of BC.

$$\text{Hence, } BD = \frac{1}{2}BC$$

$$= \frac{b}{2}$$

Using Pythagoras' theorem in ΔABD, we have:

$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{a^2 - \frac{b^2}{4}}$$

$$= \sqrt{\frac{4a^2 - b^2}{4}}$$

$$= \frac{\sqrt{4a^2 - b^2}}{2}$$

$$\therefore h = \frac{1}{2}\sqrt{4a^2 - b^2}$$

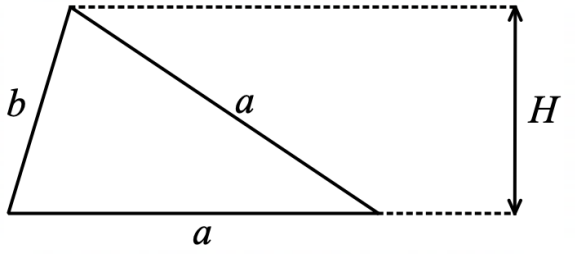
Question 23 (continued)

(a) **Comment:** Majority of students recognised the Pythagoras' theorem and re-arranged the formula to find h in terms of a and b.

The common error was writing $h = \sqrt{a^2 - \frac{1}{2}b^2}$, then students assumed that this formula resulted in $h = \frac{1}{2}\sqrt{4a^2 - b^2}$.

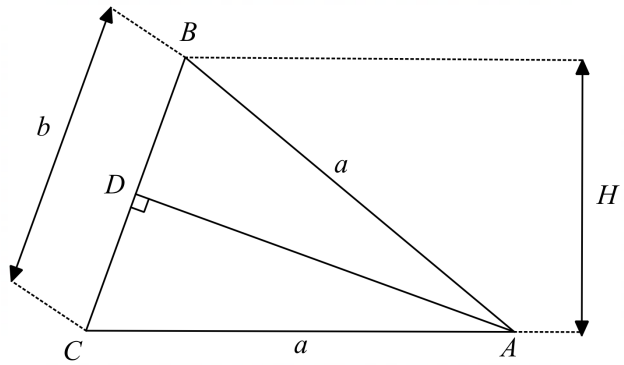
A few students attempted to find h by applying the area of a triangle formula or the difference of two squares.
However, these approaches did not lead to the final answer.

(b) **2**



Hence, or otherwise, find H in terms of a and b.

Answer:



Using the result of AD from part (a), we have:

$$A_{\Delta ABC} = \frac{1}{2} \times b \times \frac{1}{2}\sqrt{4a^2 - b^2}$$

$$= \frac{b\sqrt{4a^2 - b^2}}{4}$$

Looking at the same area: $A_{\Delta ABC} = \frac{aH}{2}$

Question 23 (continued)

(b)

$$\frac{aH}{2} = \frac{b\sqrt{4a^2 - b^2}}{4}$$

$$\therefore H = \frac{b\sqrt{4a^2 - b^2}}{2a}$$

Comment: This question was poorly done by many students.

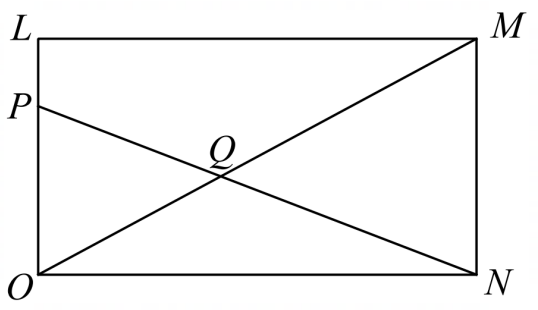
Some candidates applied Pythagoras' theorem to find H in a similar approach as part (a) or used trigonometry to find $\angle BCA$ and could not progress further.

A few students recognised the area of triangle formula and were able to re-arrange the equation to find H.

The common mistake was not simplifying their answers.
Therefore, their marks were deducted.

Question 24

In the diagram $LMNO$ is a rectangle and $ON = 3MN$.
The point P divides OL such that $OP = 2PL$.



(a) Show that $\triangle OQP$ is similar to $\triangle MQN$. **2**

Answer:

$OL \parallel MN$ (properties of rectangle $LMNO$)

In ΔOPQ and ΔMQN:

$\angle PQO = \angle NQM$ (vertically opposite angles)
$\angle OPQ = \angle MNQ$ (alternate angles, $OP \parallel MN$)

$\therefore\ \Delta OPQ \,|||\, \Delta MQN$ (AA or equiangular)

Comment: This question was very well done by most students.

The common errors were $\angle PQO = \angle NQM$ (sharing angles) or
$\angle OPQ = \angle MNQ$ (corresponding angles).

A few students assumed that since $LMNO$ was a rectangle,
$\angle OPQ = \angle MNQ = 45°$ (angle bisector).
However, this is not true. Students need to revise the properties of rectangle again.

Question 24 (continued)

(b) Show that $3OM = 5QM$. **2**

Answer:

From part (a), $\Delta OPQ \,|||\, \Delta MQN$.

Hence, $\frac{QM}{QO} = \frac{MN}{OP}$ (corresponding sides of similar triangles have the same ratio)

$OP = 2PL$ (given)

$OL = OP + PL$

Therefore, $\frac{OL}{OP} = \frac{3}{2}$

$MN = OL$ (properties of rectangle $LMNO$)

$\frac{QM}{QO} = \frac{MN}{OP} = \frac{3}{2}$

$OM = QO + QM$

$\therefore \frac{QM}{OM} = \frac{3}{5}$

$\therefore 3OM = 5QM$

Comment: This question was well done by some students.

A significant number of candidates were able to use the result from part (a) to find $\frac{QM}{QO} = \frac{MN}{OP} = \frac{3}{2}$, then concluded that $\frac{QM}{OM} = \frac{3}{5}$.

Question 24 (continued)

(c) Show that $2(ON)^2 = 5(QM)^2$. **2**

Answer:

Method 1:

$ON = 3MN$ (given)

$MN = \frac{1}{3}ON$

$3OM = 5QM$ (from part b)

$OM = \frac{5}{3}QM$

Using Pythagoras's theorem in ΔOMN:

$ON^2 + MN^2 = OM^2$

$ON^2 + \frac{1}{9}ON^2 = \frac{25}{9}QM^2$

$\frac{10}{9}ON^2 = \frac{25}{9}QM^2$

$\therefore\ 2ON^2 = 5QM^2$

Question 24 (continued)

(c) **Method 2:**

$ON = 3MN$ (given)

$MN^2 = \frac{1}{9}ON^2$

Using Pythagoras's theorem in ΔOMN:

$OM^2 = ON^2 + MN^2$

$= \frac{10}{9}ON^2$

$3OM = 5QM$ (from part b)

$QM = \frac{3}{5}OM$

$5QM^2 = \frac{9}{5}OM^2$

$= \frac{9}{5} \times \frac{10}{9}ON^2$

$= 2ON^2$

$\therefore\ 2ON^2 = 5QM^2$

Question 24 (continued)

(c) Method 3:

$3OM = 5QM$ (from part b)

$$QM = \frac{3}{5}OM$$

$ON = 3MN$ (given)

$ON^2 = 9MN^2$

Using Pythagoras's theorem in ΔOMN:

$$\begin{aligned} OM^2 &= ON^2 + MN^2 \\ &= 9MN^2 + MN^2 \\ &= 10MN^2 \end{aligned}$$

$$\begin{aligned} 5QM^2 &= 5 \times \frac{9}{25}OM^2 \\ &= \frac{9}{5}OM^2 \\ &= \frac{9}{5} \times 10MN^2 \\ &= 18MN^2 \\ &= 2 \times 9MN^2 \\ &= 2ON^2 \end{aligned}$$

$\therefore 2ON^2 = 5QM^2$

Comment: This question was poorly done by most candidates.

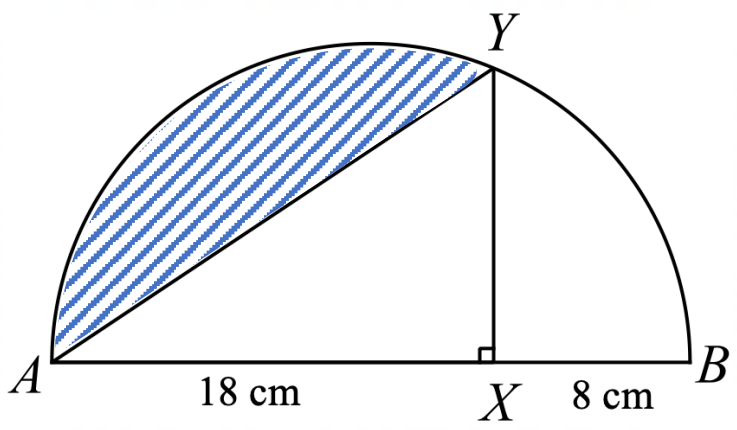
Students attempted many approaches to solve the problem. Majority of them used the Pythagoras's theorem in ΔOMN, but could not progress further to prove the result.

The most common error was starting their proofs with $2ON^2 = 5QM^2$, then students concluded that the result was true. However, the question asked them to show this. They cannot assume that $LHS = RHS$ from the beginning.

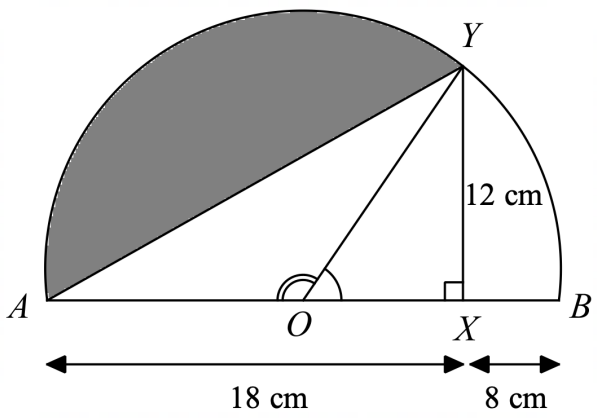
Question 25

In the semi-circle $AX = 18$ cm, $XB = 8$ cm, and $XY = 12$ cm. $XY \perp AB$. **3**

Calculate the shaded area correct to 2 decimal places.



Answer:



Let O be the centre of the semicircle and draw the line OY as the diagram above.

Since AB is the diameter, $OA = OB = \frac{1}{2}AB$.

$AB = AX + BX$

$= 26$ cm

Thus, $OA = OB = 13$ cm

$OX = OB - XB$

$= 5$ cm

In ΔOXY:

$$\tan\angle XOY = \frac{12}{5}$$

$$\angle XOY = \tan^{-1}\left(\frac{12}{5}\right)$$

Question 25 (continued)

Method 1:

$$\angle AOY + \angle XOY = 180° \text{ (straight angle)}$$

$$\angle AOY = 180° - \tan^{-1}\left(\frac{12}{5}\right)$$

$$A_{\text{sector } AOY} = \frac{180° - \tan^{-1}\left(\frac{12}{5}\right)}{360°} \times 169\pi$$

$$= 166.092 \text{ cm}^2$$

$$A_{\Delta AOY} = \frac{1}{2} \times 13 \times 12$$

$$= 78 \text{ cm}^2$$

$$A_{\text{shaded region}} = 166.092 - 78$$

$$= 88.09 \text{ cm}^2$$

Question 25 (continued)

Method 2:

$$A_{\text{semicircle}} = \frac{1}{2} \times \pi \times 13^2$$

$$= \frac{169\pi}{2} \text{ cm}^2$$

$$A_{\text{sector } BOY} = \frac{\tan^{-1}\left(\frac{12}{5}\right)}{360°} \times 169\pi$$

$$= 99.372 \text{ cm}^2$$

$$A_{\Delta AOY} = \frac{1}{2} \times 13 \times 12$$

$$= 78 \text{ cm}^2$$

$$A_{\text{shaded region}} = \frac{169\pi}{2} - 99.372 - 78$$

$$= 88.09 \text{ cm}^2$$

Comment: This question was poorly done by many pupils.

Significant number of students were able to find $r = 13$ cm and the area of semicircle correctly. Their common approach was calculating the area of semicircle and $\triangle AXY$, but they were not successful since they could not find the area of XYB to subtract from the semicircle.

A few students assumed that AYB was a sector and applied the area of sector formula. However, this is incorrect as a sector is made of an arc and two radii.
AY is the chord and AB is the diameter.

A few students applied the area of segment formula $A = \frac{\theta}{360°}\pi r^2 - \frac{1}{2}r^2 \sin\theta$, but they made mistakes when finding the angle θ.
In this case, θ is $\angle AOY$.
However, they assumed that θ is either $\angle YAX$ or $\angle AXY$.

There were rounding errors such as 88.10 cm^2 since students did not write the answers of the previous calculations as exact values or three to four decimal places.

End of solutions

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 4
YEARLY EXAMINATION PART 2

Sample Solutions

1 An additive sequence has third term 3 and fifth term 5. **2**

Complete the sequence up to its sixth term.

4, −1, 3, 2, 5, 7 ✓✓

2 On lines 17 and 18 the question was asked: **1**

"Are there any values of x for which a geometric sequence generated is also an arithmetic sequence?" $x = 1$ was found to be the only solution

Are there any other values of x for which the odd terms of the geometric sequence (with first term 1) form an arithmetic sequence?

odd terms: $1, x^2, x^4, \ldots$

$\Rightarrow x = \pm 1$ ∴ $x = -1$ is other soln. ✓

3 An additive sequence has nth term n and $(n + 1)$th term $n + 1$. **2**
Will this sequence have any negative terms?
State any conditions, surrounding this.

Negative terms? $T_{n+1} = n+1$
$T_n = n$
$T_{n-1} = 1$
$T_{n-2} = n-1$
$T_{n-3} = 2-n$

T_{n-3} will be negative if $n \geq 3$
But $n=3$ won't have an $(n-3)^{rd}$ term
∴ $n = 4$ ✓

ie. negative terms if there are 4 or more terms.

4 An additive sequence has the first two terms which are odd. **2**
State, with reasoning, whether the 2022nd term will be odd or even.

Let 2 terms be $2n+1$ and $2n+3$.
then $t_3 = 4n+4$ - even $t_5 = 10n+11$ - odd
$t_4 = 6n+7$ - odd $t_6 = 16n+18$ - even
⇒ every 3rd term is even and every other term is odd
∴ 2022nd term is even ✓✓

ie. term no's divisible by 3 are even. (divis by 3)

5 On line 56 and 57 it is stated that the terms of the sequence **2**

$$1, \phi, \phi^2, \phi^3, \phi^4, \phi^5, \ldots$$

can be written in the form

$$a\phi + b, \text{ where } a \text{ and } b \text{ are integers.}$$

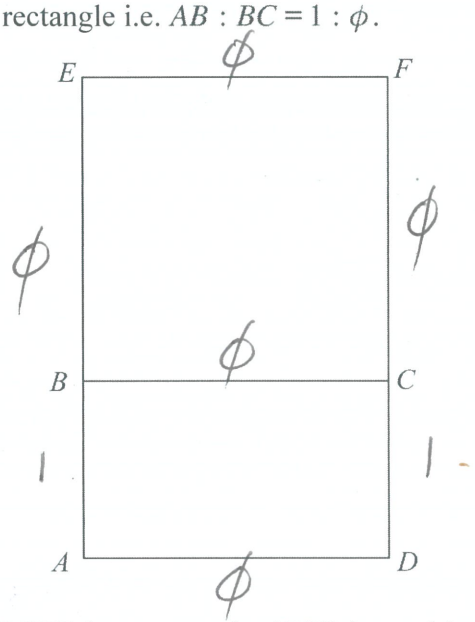
What set of integers does a and b belong to?

$a = 0, 1, 1, 2, 3, 5, \ldots$ ✓

$b = 1, 0, 1, 1, 2, 3, \ldots$

Fibonacci numbers. ✓

6 Rectangle $ABCD$ is a golden rectangle i.e. $AB : BC = 1 : \phi$. **2**



Show that by adding square $BCFE$ then rectangle $AEFD$ is a golden rectangle.
i.e. show that $AD : AE = 1 : \phi$.

$AD : AE = \phi : 1+\phi$

$= 1 : \frac{1+\phi}{\phi}$

$= 1 : 1+\frac{1}{\phi}$

Now $1+\frac{1}{\phi} = 1+\frac{2}{1+\sqrt{5}} = \frac{1+\sqrt{5}+2}{1+\sqrt{3}}$ ✓

$= \frac{3+\sqrt{5}}{1+\sqrt{5}} = \frac{(3+\sqrt{5})(1-\sqrt{5})}{1-5} = \frac{3-2\sqrt{5}-5}{-4}$

$= \frac{-2-2\sqrt{5}}{-4} = \frac{1+\sqrt{5}}{2} = \phi$ ✓

ie. $AD : AE = 1 : \phi$

Or

$AD : AE$

$= \phi : 1+\phi$ ✓

But $1+\phi = \phi^2$

$\Rightarrow AD : AE = \phi : \phi^2$ ✓

$= 1 : \phi$

7 On line 90 it is stated that the point Q has coordinates $\left(\frac{\phi+1}{2\phi-1}, \frac{\phi-1}{2\phi-1}\right)$, where $\phi = \frac{1+\sqrt{5}}{2}$. **3**

Find the y-coordinate of Q as a surd with a rational denominator.

$$y = \frac{\phi-1}{2\phi-1} = \frac{\left(\frac{1+\sqrt{5}}{2}-1\right)}{\left(\frac{2+2\sqrt{5}}{2}-1\right)}$$

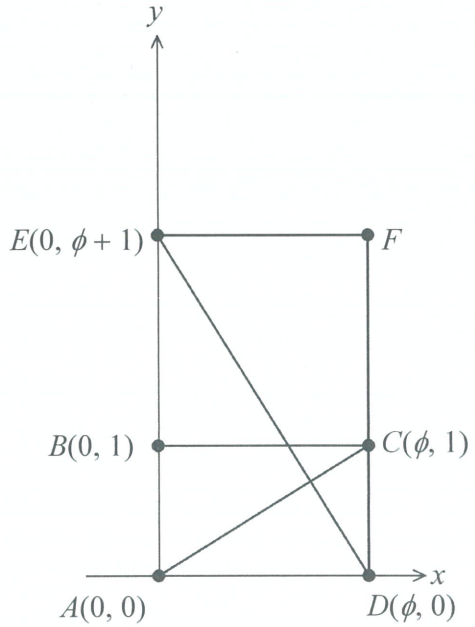
$$= \frac{-1+\sqrt{5}}{2} \div \frac{2\sqrt{5}}{2}$$

$$= \frac{-1+\sqrt{5}}{2} \times \frac{1}{\sqrt{5}} = \frac{-1+\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{-\sqrt{5}+5}{10}$$

$$y = \frac{5-\sqrt{5}}{10}$$

8



The equation of line AC is $x - \phi y = 0$

(a) Find the equation of ED in terms of ϕ. $E = (0, \phi+1)$ $D = (\phi, 0)$ **2**

$m = \frac{\phi+1}{-\phi}$

$y - 0 = \frac{\phi+1}{-\phi}(x - \phi)$ ✓

$-\phi y = (\phi+1)x - (\phi^2 + \phi)$ ✓

or $(\phi+1)x + \phi y - (\phi^2 + \phi) = 0$

OR $m = \frac{\phi+1}{-\phi}$ and $y\text{-int} = \phi+1$

$\Rightarrow y = -\frac{(\phi+1)}{\phi}x + (\phi+1)$ ✓

Further, $-\frac{(\phi+1)}{\phi} = -\frac{\phi^2}{\phi} = -\phi$

and $\phi + 1 = \phi^2$

$\Rightarrow y = -\phi x + \phi^2$

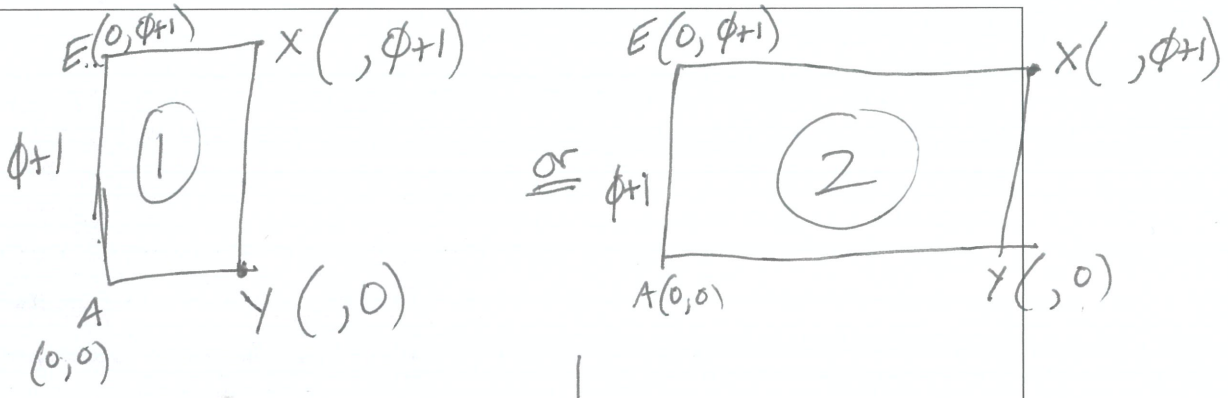
Question 8 continues on page 7

8 (continued)

(b) Find the point X on AD so that $EX \perp AC$. **1**

$X = D(\phi, 0)$

(c) If AE is a side of a golden rectangle, write down all possible sets of coordinates, in the first quadrant, for the other two coordinates (in terms of ϕ). **2**



Case 1 $\overline{EX} : \overline{EA} = 1 : \phi$

$\Rightarrow \dfrac{\overline{EX}}{\phi+1} = \dfrac{1}{\phi}$

$\Rightarrow EX = \dfrac{\phi+1}{\phi}$

$\Rightarrow X = \left(\dfrac{\phi+1}{\phi}, \phi+1\right)$

$Y = \left(\dfrac{\phi+1}{\phi}, 0\right)$

using $\phi+1 = \phi^2$

$\Rightarrow X = (\phi, \phi+1)$

$Y = (\phi, 0)$

Case 2 $\overline{AE} : \overline{EX} = 1 : \phi$

$\Rightarrow \dfrac{\phi+1}{\overline{EX}} = \dfrac{1}{\phi}$

$\overline{EX} = \phi(\phi+1)$

$\Rightarrow X = (\phi^2+\phi, \phi+1)$

$Y = (\phi^2+\phi, 0)$

and using $\phi^2 = \phi+1$

$X = (2\phi+1, \phi+1)$

$Y = (2\phi+1, 0)$

End of paper