2018 SYDNEY BOYS HIGH SCHOOL
YEAR 9 Yearly Examination (Stage 5.3)

General Instructions:

- All questions may be attempted.
- Write in black pen
- Marks may be deducted for careless or badly arranged work.
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of this booklet. Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed: 90 minutes

Examiner: AF

Name: _____

Class	Teacher	
9**A**	Ms Kilmore	
9**B**	Mr Gurjar	
9**C**	Ms Millar	
9**P**	Mr Choy	
9**L**	Mr Gainford	
9**U**	Mr Wang	
9**S**	Ms Ward	

Question	Marks
1	/ 15
2	/ 15
3	/ 15
4	/ 15
5	/ 15
6	/ 15
7	/ 15
Total	/ 105

Question One (15 marks)

Parts (a) – (e) are multiple choice each worth one mark. *Circle the correct answer.*

(a) The gradient of the line $2x - 3y + 7 = 0$ is

(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

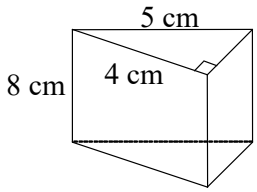
(b) If two coins are tossed, the probability of obtaining at least one head is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

(c) The local sports store advertised a 45% discount on all cricket bats. Glenn bought a cricket bat and paid \$88 after discount. How much did Glenn save?

(A) \$16.00 (B) \$39.60 (C) \$48.40 (D) \$72.00

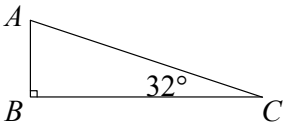
(d)



The volume of this triangular prism is

(A) 48 cm^3 (B) 60 cm^3 (C) 80 cm^3 (D) 96 cm^3

(e)



As a decimal correct to 3 decimal places, the ratio $\frac{BC}{AC}$ is

(A) 0.530 (B) 0.625
(C) 0.848 (D) unable to be calculated.

(f) Express 325 000 000 in scientific notation. [1]

(g) Simplify the following:

(i) $2\sqrt{5} \times \sqrt{5}$ [1]

(ii) -2^2 [1]

(iii) $3\sqrt{10} \times 2\sqrt{15}$ [1]

(iv) $0.\dot{3} + 1.507$ [1]

(h) State the number of significant zeros in the following:

(i) 48000 (to the nearest hundred) [1]

(ii) 0.00900 [1]

(i) Write $15^{\circ}49'35''$ in degrees correct to 2 decimal places. [1]

(j) Evaluate $\cos 27^{\circ}$ correct to 2 decimal places. [1]

(k) Convert 3500 cubic centimetres to cubic metres [1]

Question Two (15 marks)

(a) Find the acute angle α to the nearest degree if $\tan\alpha = 2$. [1]

(b) Expand and simplify the following:

(i) $\sqrt{3}(\sqrt{2}+3)-\sqrt{3}$ [1]

(ii) $(2x^3)^4$ [1]

(iii) $(3-x^2)^2$ [1]

(c) Write an algebraic expression for "*m* less than the product of *p* and *q*". [1]

(d) Determine the linear relationship for the table of values. [2]

x	1	2	3	4
y	5	2	-1	-4

(e) If $a = -3$ and $b = 2$, evaluate the following:

(i) $1 - (a + b)$ [1]

(ii) ab^a [1]

(f) Factorise the following:

(i) $8ab - 6a + 4a^2$ [1]

(ii) $1 - 4a^2$ [1]

(iii) $a^2 + 7a - 30$ [2]

(iv) $8ab - 6a - 12b + 9$ [2]

Question Three (15 marks)

(a) Solve the following equations for x:

(i) $3 - 5x = 1$ [1]

(ii) $2x^2 + 1 = 11$ [1]

(iii) $\frac{x}{3} - \frac{x+1}{6} = 5$ [2]

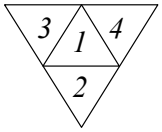
(iv) $\frac{1}{x} + \frac{1}{y} = \frac{1}{xy}$ [1]

(b) A cylinder has a height of 10 cm and a diameter of 10 cm.

(i) Calculate the volume (correct to 2 decimal places) [1]

(ii) Calculate the Surface Area (correct to 2 decimal places) [1]

(c) A die is made in the form of a tetrahedron (triangular pyramid). The net of the die is shown below.



The faces are numbered 1, 2, 3 and 4. The die is rolled twice. The number on the face that the die lands on is recorded each time.

(i) Find the probability that the two numbers recorded are even [1]

(ii) Find the probability that the sum of the two numbers is 4 [2]

(d) Solve the following simultaneously:

(i) $4x - y = 3$ [2]

$2x + y = 15$

(ii) $3x + 2y = 10$ [3]

$4x + 3y = 13$

Question Four (15 marks)

(a) Rationalise the denominator of the following:

(i) $\dfrac{3}{2\sqrt{2}}$ [1]

(ii) $\dfrac{1+\sqrt{2}}{2-\sqrt{2}}$ [2]

(b) Solve $\dfrac{1+2x}{3} \geq -1$ and graph the solution on the number line. [2]

(c) Are two rhombuses which have one angle equal similar? [1]

(d) $A = \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$

Determine the following:

(i) $2C$ [1]

(ii) $A + D$ [1]

(iii) DB [1]

(iv) AC [1]

(v) A^{-1} [1]

(e) The bearing of C from A is $130°$. The bearing of B from C is $330°$.

With the aid of a diagram:

(i) Find the size of $\angle BCA$ [1]

(ii) Write down the bearing of C from B. [1]

(f) Two tanks were filled with water Tank A and Tank B. [2]

At midday a pump began drawing water at a constant rate from both tanks.

By 2:00 pm, both tanks had the same depth of water.

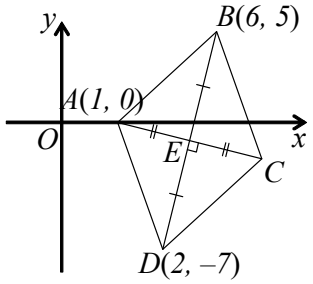
At 5:00 pm, Tank A ran dry.

At 8:00 pm, Tank B ran dry.

If Tank B is 10 metres high, how high is Tank A?

Question Five (15 marks)

(a) A, B, C and D are vertices of a rhombus.



(i) The point of intersection of the diagonals is E. [1]
Find the coordinates of E.

(ii) Show that the equation of the line BD is $3x - y - 13 = 0$. [2]

(iii) Calculate the distance BD. [1]

(iv) Calculate the area of the rhombus [1]

(v) Calculate the distance that the lines AB and CD are apart. [2]
{Do not use the formula for perpendicular distance}

(b) I have 3 tickets in a raffle which has 3 prizes. 100 tickets are sold. [2]
1st prize is drawn, then 2nd and 3rd prize with the winning tickets
discarded before the next ticket is drawn.
Which prize am I more likely to win? {Provide working to justify your response}

(c) Solve the following equations for x:

(i) $ax + b = bx$ [2]

(ii) $3 - \sqrt{x^2 + 1} = x - 1$ [2]

(iii) $\sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}}} = 3$ [2]

Question Six (15 marks)

(a) Factorise the following:

(i) $10a^2 - a - 2$ [2]

(ii) $a(1-a) - b(1-b)$ [2]

(b) Find the exact value of $\sin\theta$ where θ is acute and $\tan\theta = \frac{2}{\sqrt{3}}$. [1]

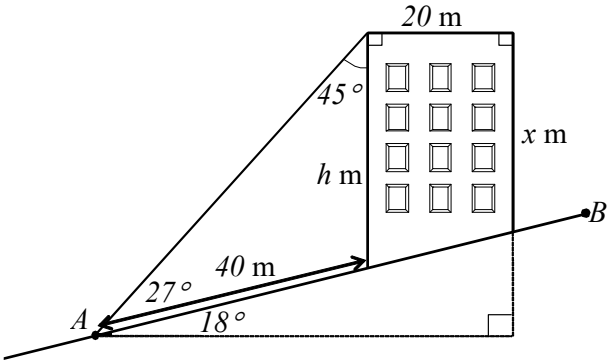
(c) If $9^x = A$. Write the following in terms of A:

(i) 9^{x+2} [1]

(ii) 9^{-x} [1]

(iii) 3^x [1]

(d)

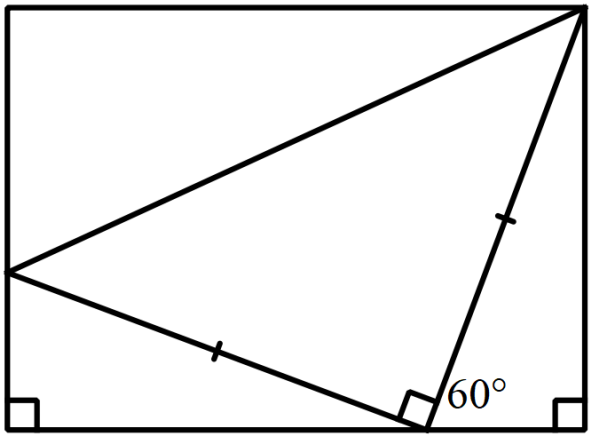


The diagram shows a building on sloping ground AB.

(i) Show that the height (h metres) of the taller side of the building is 25.68 metres. [2]

(ii) Find the height (x metres) of the shorter side of the building. [2]

(e) The figure below is known as the Allies Rectangle.



(i) Write in any missing angles [1]

(ii) Use this figure to find the exact value of $\cos 75°$ [2]

Question Seven (15 marks)

(a) A company's logo is made from straight lines. It is used in two main sizes which are in the ratio $2:3$.

(i) If the smaller sized logo has a height of 9 cm. [2]

What is the height of the larger sized logo?

(ii) If the smaller sized logo is printed on a page is it possible to draw the larger sized logo using only a straight edge and a pair of compasses? [3]

If so describe the steps that would be necessary.

(b) The numbers *a, b, c* are a Pythagorean triple. (ie. integers for which $a^2 + b^2 = c^2$)

(i) Show that $\left(\frac{c+b}{a}\right)\left(\frac{c-b}{a}\right) = 1$ [2]

(ii) What can be said about $\left(\frac{c+b}{a}\right)$ and $\left(\frac{c-b}{a}\right)$? [1]

(iii) Let $\left(\frac{c+b}{a}\right)=\frac{x}{y}$ and $\left(\frac{c-b}{a}\right)=\frac{y}{x}$ for positive integers x and y. [3]

Show that $a: b: c = 2xy: x^2 - y^2: x^2 + y^2$

(iv) Show that abc is divisible by $a + b + c$ [2]

(v) Find all Pythagorean triples which contain the integer 12. [2]

End of Exam

Use this space if you wish to **rewrite** any answers.

Clearly *indicate* the **Question** number.

SYDNEY
BOYS
HIGH
SCHOOL

2018

YEAR 9 YEARLY
ASSESSMENT
TASK

Mathematics (Stage 5.3)

Suggested Solutions

(Year 9) Question (1)

- (a) $3y = 2x + 7$

$\therefore y = \frac{2}{3}x + \frac{7}{3}$

(C)

- (b) $1 -$ Prob(no heads)

$1 - \frac{1}{4} = \frac{3}{4}$

(D)

- (c)

$\$88 = 55\%$

$\frac{88}{55} = 1\%$

$\therefore$ Saved $= 45\%$

ie $\frac{8}{5} \times 45 = \$72$

(J)

- (d) $A = \frac{1}{2} \times 4 \times 3$

$= 6$

$V = 6 \times 8 = 48\ cm^3$

(A)

- (e)

$\frac{BC}{AC} = \sin 58^\circ$

$= 0.848$

(C)

(f) 3.25×10^8

(g) $2\sqrt{5} \times \sqrt{5} = 10$

(i)

(ii) $-2^2 = -4$

(iii) $3\sqrt{10} \times 2\sqrt{15}$

$= 6\sqrt{150} = 6 \times \sqrt{25} \times \sqrt{6}$

$= 30\sqrt{6}$

(iv) $0.\dot{3} + 1507$

1.8403

(h) (i) 1

(ii) 2

(i) 15.83 (2 d.p.)

(j) $\cos 27^\circ = 0.89$ (2.d.p)

(k) $3500\ cm^3$

$= 0.0035\ m^3$

Question Two (15 marks)

(a) Find the acute angle α to the nearest degree if $\tan\alpha = 2$. [1]

$\text{Tan}\,\alpha = 2$

$\alpha = \text{Tan}^{-1}2 \approx 63^\circ$

(b) Expand and simplify the following:

(i) $\sqrt{3}(\sqrt{2}+3)-\sqrt{3}$ [1]

$= \sqrt{6} + 3\sqrt{3} - \sqrt{3}$

$= \sqrt{6} + 2\sqrt{3}$

(ii) $(2x^3)^4$ [1]

$16x^{12}$

(iii) $(3-x^2)^2$ [1]

$= 9 - 6x^2 + x^4$

$= x^4 - 6x^2 + 9$

(c) Write an algebraic expression for "m less than the product of p and q". [1]

$pq - m$

(Note: Significant number of students wrote $m < pq$ but the phrase is a missing a "is" to be an inequality)

(d) Determine the linear relationship for the table of values. [2]

x	1	2	3	4
y	5	2	-1	-4

-3 -3 -3

$\therefore \quad y = -3x + 8$

7

(e) If $a = -3$ and $b = 2$, evaluate the following:

(i) $1 - (a + b)$ [1]

$= 1 - (-3 + 2)$

$= 1 - (-1)$

$= 2$

(ii) ab^a [1]

$= -3 \times (2)^{-3}$

$= -3 \times \frac{1}{8}$

$= -3/8$

(f) Factorise the following:

(i) $8ab - 6a + 4a^2$ [1]

$2a(4b - 3 + 2a)$

(ii) $1 - 4a^2$ [1]

$(1 - 2a)(1 + 2a)$

(iii) $a^2 + 7a - 30$ [2]

$= (a + 10)(a - 3)$

$P = -30$

$S = 7$

$\therefore 10, -3$

(iv) $8ab - 6a - 12b + 9$ [2]

$= 2a(4b - 3) - 3(4b - 3)$

$= (2a - 3)(4b - 3)$

8

Question Three (15 marks)

(a) Solve the following equations for x:

(i) $3 - 5x = 1$ [1]

$-5x = -2$

$x = \frac{2}{5}$

WELL DONE

SOME HAD $x = \frac{5}{2}$.

(ii) $2x^2 + 1 = 11$ [1]

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

POORLY DONE

FORGET $\pm$ $(-\frac{1}{2})$

(iii) $\frac{x}{3} - \frac{x+1}{6} = 5$ [2]

$\frac{2x}{6} - \frac{(x+1)}{6} = \frac{30}{6}$

$2x - x - 1 = 30$

$x = 31$

POORLY DONE

$x = 29$ WAS COMMON $(-\frac{1}{2})$

(iv) $\frac{1}{x} + \frac{1}{y} = \frac{1}{xy}$ [1]

$\frac{y}{xy} + \frac{x}{xy} = \frac{1}{xy}$

$y + x = 1$

$x = 1 - y$

POORLY DONE

SOME DID NOT SOLVE FOR x.

(b) A cylinder has a height of 10 cm and a diameter of 10 cm.

(i) Calculate the volume (correct to 2 decimal places) [1]

$V = \pi r^2 h$

$= \pi(5^2)(10)$

$= 250\pi$

$= 785.40 \text{cm}^3$ (2dp)

WELL DONE

SOME DID NOT HAVE WORKING $(-\frac{1}{2})$

(ii) Calculate the Surface Area (correct to 2 decimal places) [1]

$SA = 2\pi r^2 + 2\pi rh$

$= 50\pi + 100\pi$

$= 150\pi$

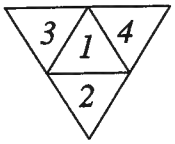
$= 471.24 \text{cm}^2$ (2dp)

GENERALLY WELL DONE

PROBLEMS WITH FORMULA

SOME HAD NO WORKING.

(c) A die is made in the form of a tetrahedron (triangular pyramid). The net of the die is shown below.



The faces are numbered 1, 2, 3 and 4. The die is rolled twice. The number on the face that the die lands on is recorded each time.

(i) Find the probability that the two numbers recorded are even [1]

	1	2	3	4
1			O	
2		⊗		X
3	O			
4		X		X

X

$P(\text{even}) = \frac{4}{16}$

$= \frac{1}{4}$

WELL DONE

(ii) Find the probability that the sum of the two numbers is 4 [2]

O

$P(\text{sum } 4) = \frac{3}{16}$

POORLY DONE
PROBLEMS WITH SAMPLE SPACE -1

(d) Solve the following simultaneously:

(i) $4x - y = 3$ ——— ①

$2x + y = 15$ ——— ②

[2]

① + ②

$6x = 18$

$x = 3$

SUB IN TO ②

$2(3) + y = 15$

$y = 9$

Solutions are $x = 3$ $y = 9$

WELL DONE
ISSUES WITH LABELLING -1
+ FINAL SUBSTITUTION -½

(ii) $3x + 2y = 10$ ——— (1) [3]

$4x + 3y = 10$ ——— (2)

$3\times(1)$ $9x + 6y = 30$ ——— (3)

$2\times(2)$ $8x + 6y = 26$ ——— (4)

(3) − (4) $x = 4$

SUB INTO (1)

$3(4) + 2y = 10$

$2y = -2$

$y = -1$

Solutions are $x = 4$ $y = -1$

FAIRLY WELL DONE

PROBLEMS WITH ARITHMETIC (-1)

+ LABELLING (-1)

+ FINDING x ONLY (-1)

Question Four (15 marks)

(a) Rationalise the denominator of the following:

(i) $\frac{3}{2\sqrt{2}}$ [1]

$\times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{2\times 2}$ $= \frac{3\sqrt{2}}{4}$

[Many lost ½ mark for not simplifying.]

(ii) $\frac{1+\sqrt{2}}{2-\sqrt{2}}$ [2]

$\times \frac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \frac{2+\sqrt{2}+2\sqrt{2}+2}{4-2}$ $= \frac{4+3\sqrt{2}}{2}$

$= \frac{4+3\sqrt{2}}{2}$

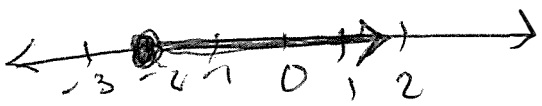
[Some were careless simplifying.]

(b) Solve $\frac{1+2x}{3} \geq -1$ and graph the solution on the number line. [2]

$1+2x \geqslant -3$

$2x \geqslant -4$

$x \geqslant -2$



(c) Are two rhombuses which have one angle equal similar? [1]

Yes

(d) $A = \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -2 & -1 \end{pmatrix}, C = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$

Determine the following:

(i) $2C$ [1]

$= 2 \times \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -8 \\ 2 & 6 \end{bmatrix}$

(ii) $A + D$ [1]

$\begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix}$

(iii) DB [1]

$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & -11 & -4 \end{bmatrix}$

$2 \times 2 \quad 2 \times 3$

[Several thought they were not matched for X]

(iv) AC [1]

$\begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(v) A^{-1} [1]

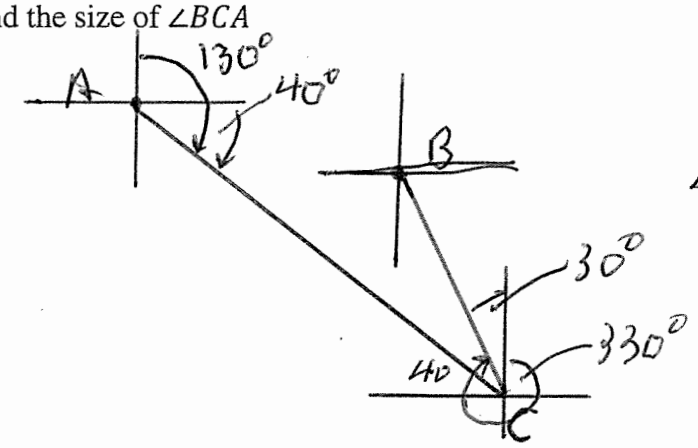
$= C = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$

[Many failed to notice part (iv), then erred calculating the determinant.]

(e) The bearing of C from A is 130°. The bearing of B from C is 330°.

With the aid of a diagram:

(i) Find the size of $\angle BCA$ [1]



$\angle BCA = 20°$

(ii) Write down the bearing of C from B. [1]

150°T

[Parts (i) and (ii) poorly understood by about half.]

(f) Two tanks were filled with water, Tank A and Tank B. [2]

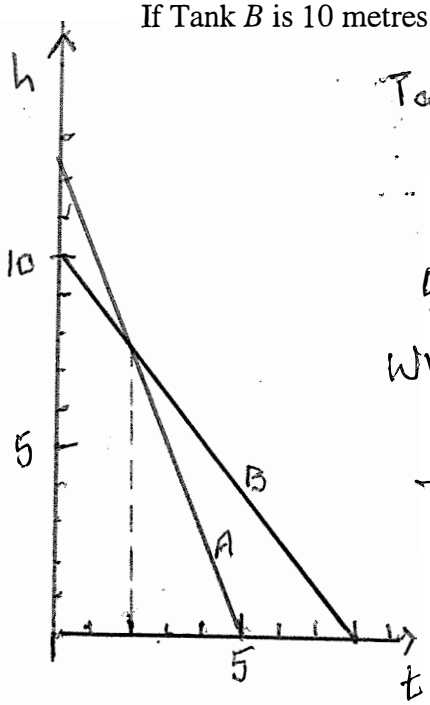
At midday a pump began drawing water at a constant rate from both tanks.

By 2:00 pm, both tanks had the same depth of water.

At 5:00 pm, Tank A ran dry.

At 8:00 pm, Tank B ran dry.

If Tank B is 10 metres high, how high is Tank A?



Tank B: (0,10), (8,0)

$\frac{t}{8} + \frac{h}{10} = 1$

$5t + 4h = 40$

When $t = 2$ (2pm)

$h = 7.5$

Tank A:

$h - 7.5 = \frac{0 - 7.5}{5 - 2}(t - 2)$

$5t + 2h = 25$

When $t = 0$, $h = 12\frac{1}{2}$

$\therefore$ Tank A is $12\frac{1}{2}$m high

[Very few answered this correctly. Some used a proportional approach successfully]

5. (a) (i) $MP_{BD} = \left(\frac{6+2}{2}, \frac{5+(-7)}{2}\right)$

$= \left(\frac{8}{2}, \frac{-2}{2}\right)$

$= (4, -1)$ [1]

(ii) $m_{BD} = \frac{5-(-7)}{6-2}$

$= \frac{12}{4}$

$= 3$

$\therefore y - y_1 = m(x - x_1)$

$y - 5 = 3(x - 6)$

$y - 5 = 3x - 18$

$\therefore 3x - y - 13 = 0$ [2]

(iii) $d_{BD} = \sqrt{(6-2)^2 + (5-(-7))^2}$

$= \sqrt{16 + 144}$

$= \sqrt{160}$

$= 4\sqrt{10}$ [1]

(iv) $d_{AE} = \sqrt{(4-1)^2 + (-1-0)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$\therefore A = \frac{1}{2} \times 4\sqrt{10} \times (2 \times \sqrt{10})$

$= 40 \quad u^2$ [1]

(v) $d_{AB} = \sqrt{(6-1)^2 + (5-0)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$\therefore \quad A = b \times h$

$40 = 5\sqrt{2} \times h$

$h = \frac{40}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{8\sqrt{2}}{2}$

$= 4\sqrt{2}$ units [2]

(a) (i) General done well.

(ii) Most students didn't understand the 'show that'. You still needed to show that the gradient of BD was 3, then correct substitute in either B or D.

(iii) General done well.

(iv) Very poorly done.

(v) Done extremely poorly.

Note: If there are only 3 tickets (1, 2, 3) in a raffle each ticket is just as likely to win each prize

1st	2nd	3rd
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

If there are 100 tickets in a raffle each ticket is just as likely to win each prize. It won't matter how many tickets I have.

$$P(\text{win 1st prize}) = \frac{3}{100}$$

$$P(\text{win 2nd prize}) = \frac{3}{100} \times \frac{2}{99} + \frac{97}{100} \times \frac{3}{99}$$

$$= \frac{3}{100}$$

$$P(\text{win 3rd prize}) = \frac{3}{100} \times \frac{2}{99} \times \frac{1}{98} + \frac{3}{100} \times \frac{97}{99} \times \frac{2}{98} + \frac{97}{100} \times \frac{3}{99} \times \frac{2}{98}$$

$$+ \frac{97}{100} \times \frac{96}{99} \times \frac{3}{98}$$

$$= \frac{3}{100}$$

Therefore, all prizes equally likely.

(c) (i) $ax + b = bx$ [1]

$b = bx - ax$

$b = x(b - a)$

$\therefore x = \frac{b}{b-a}$ [2]

or $ax - bx = -b$

$x(a - b) = -b$

$x = \frac{-b}{a-b}$

(ii) $3 - \sqrt{x^2+1} = x - 1$

$-\sqrt{x^2+1} = x - 4$

$\sqrt{x^2+1} = 4 - x$

$x^2 + 1 = (4 - x)^2$

$x^2 + 1 = 16 - 8x + x^2$

$8x = 15$

$x = \frac{15}{8}$ [2]

(iii) $\sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}}} = 3$

$x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}} = 3^2$

$x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}} = 9$

$\therefore x - 3 = 9$

$x = 12$ [2]

All equations were done very poorly.

Question 6 Solutions (1 mark questions that are wrong will only receive half a mark if there is enough working to show significant progress to the correct answer)

a) i)

$5 \times 2 = 10$

$2 \times -1 = -2$

This gives us: $(5a+2)(2a-1)$

lose 1/2 a mark for signs swapped
lose 1 mark for pairing numbers incorrectly

ii) expanding gives:

$$a - a^2 - b + b^2$$
$$= b^2 - a^2 - b + a$$
$$= (b-a)(b+a) - (b-a)$$
$$= (b-a)(b+a-1) \quad \text{or} \quad (a-b)(1-a-b)$$

1 mark given for expansion & 1 mark for correct factorisation.

b) $\tan\theta = \frac{2}{\sqrt{3}}$ gives us:



solve for the hypotenuse: $h = \sqrt{2^2 + (\sqrt{3})^2} = \sqrt{7}$

Therefore $\sin\theta = \frac{2}{\sqrt{7}}$

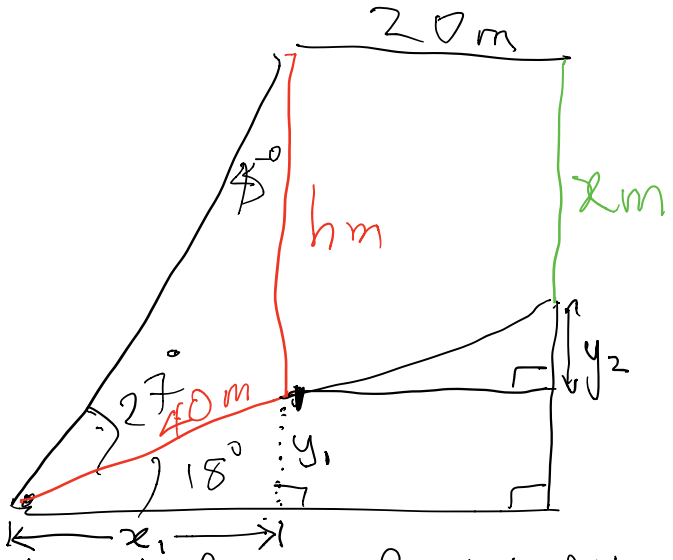
c) $9^x = A$

i) $9^{x+2} = 9^x \times 9^2 = A \times 9^2$

ii) $9^{-x} = \frac{1}{9^x} = \frac{1}{A}$

iii) $3^x = (9^{1/2})^x = 9^{x/2} = (9^x)^{1/2} = \sqrt{A}$

d)



i) Since the angle of depression from the top of the building is 45°, $x_1 = h + y_1$ or the length of the opposite side to the angle is equal to the length of the adjacent side.

$$\cos 18^\circ = \frac{x_1}{40} \Rightarrow x_1 = 40\cos 18^\circ \; ; \quad \sin 18^\circ = \frac{y_1}{40} \Rightarrow y_1 = 40\sin 18^\circ$$

Now $h = x_1 - y_1$

$= 40\cos 18^\circ - 40\sin 18^\circ$

$= 25.68$ m to 2 decimal places

1 mark for finding expressions for y_1 & $h + y_1$ (or x_1)
1 mark for evaluating h using the above expressions.

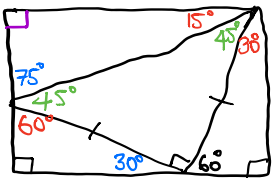
ii) $x = h - y_2$, $\tan 18^\circ = \frac{y^2}{20} \Rightarrow y^2 = 20\tan 18^\circ$

So $x = 25.68 - 20\tan 18^\circ$

$= 19.18$ m to 2 decimal places

1 mark for finding an expression for y_2
1 mark for using the expression to evaluate x.

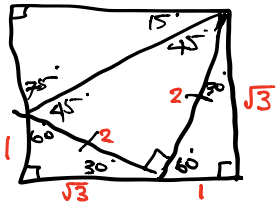
e) i)



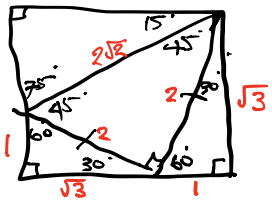
The sum of angles in a triangle is 180°
The sum of angles on a line is 180°
Angles opposite equal lengths in an Isosceles triangle are equal.
Rectangle - given

Lose half a mark if 1 angle is missing, no marks given for 2 or more angles missing.

ii) Start with standard values for a right-angled triangle with a 60° angle in it:



The hypotenuse for the isosceles triangle is thus:
$h = \sqrt{2^2+2^2}$
$= 2\sqrt{2}$ so ...



1 mark for the lengths added appropriately (or values directly proportional to these).

The rectangle has dimensions $\sqrt{3}$ and $\sqrt{3}+1$. The adjacent length to the 75° angle is $\sqrt{3}-1$
Hence $\cos 75° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{2}(\sqrt{3}-1)}{4}$

1 mark for using the correct values to find the exact value for $\cos 75°$

Question Seven (15 marks)

(a) A company's logo is made from straight lines. It is used in two main sizes which are in the ratio 2: 3.

(i) If the smaller sized logo has a height of 9 cm. [2]

What is the height of the larger sized logo?

$2:3 = 9:x$

$\frac{2}{3} = \frac{9}{x}$

$2x = 27$

$x = 13{\cdot}5$ cm

(ii) If the smaller sized logo is printed on a page is it possible to draw the larger sized logo using only a straight edge and a pair of compasses? [3]

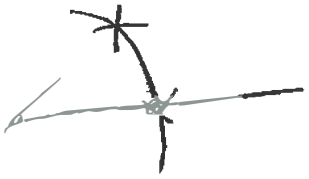
If so describe the steps that would be necessary.

Each line is 1·5 times the smaller one.

Yes

(1) For each line, open compass to length of a line on small logo. Draw a line with straight edge. With compass mark out 3 of these lengths. Then find midpoint by opening compass to $> \frac{1}{2}$ length. Make an arc from each vertex. ✓

(2) For each angle, move compass to vertex of large logo and draw an arc. Mark the point where arc crosses the line



then open compass to size of angle on small logo + draw a similar arc at the point marked on line. Connect the next line through the point where the two arcs cross.

(b) The numbers a, b, c are a Pythagorean triple. (ie. integers for which $a^2 + b^2 = c^2$)

(i) Show that $\left(\frac{c+b}{a}\right)\left(\frac{c-b}{a}\right) = 1$ [2]

c a b

$\frac{c^2 - b^2}{a^2} = 1$ ✓

$\Rightarrow c^2 - b^2 = a^2$

$c^2 = a^2 + b^2$ ✓

2

(ii) What can be said about $\left(\frac{c+b}{a}\right)$ and $\left(\frac{c-b}{a}\right)$? [1]

They are reciprocals. 1/1

(iii) Let $\left(\frac{c+b}{a}\right)=\frac{x}{y}$ and $\left(\frac{c-b}{a}\right)=\frac{y}{x}$ for positive integers x and y. [3]

Show that $a:b:c = 2xy:x^2-y^2:x^2+y^2$

$\frac{c}{a}+\frac{b}{a}=\frac{x}{y}$ (1)

$\frac{c}{a}-\frac{b}{a}=\frac{y}{x}$ (2)

(1)+(2) $\Rightarrow 2\left(\frac{c}{a}\right)=\frac{x}{y}+\frac{y}{x}$

$\frac{c}{a}=\frac{x^2+y^2}{2xy}$

Also (1)−(2) $\Rightarrow 2\left(\frac{b}{a}\right)=\frac{x}{y}-\frac{y}{x}$

$\frac{b}{a}=\frac{x^2-y^2}{2xy}$

Then $a:b = 2xy:x^2-y^2$

$a:c = 2xy:x^2+y^2$

$\Rightarrow a:b:c = 2xy:x^2-y^2:x^2+y^2$ ✓

(iv) Show that abc is divisible by $a+b+c$ [2]

$a+b+c = 2xy+x^2-y^2+x^2+y^2$

$= 2xy+2x^2$

$= 2x(y+x)$

Also, $abc = 2xy(x^2-y^2)(x^2+y^2)$

$= 2xy(x-y)(x+y)(x^2+y^2)$

Then $\frac{abc}{a+b+c} = \frac{2xy(x-y)(x+y)(x^2+y^2)}{2x(x+y)}$

$= y(x-y)(x^2+y^2) \in \mathbb{Z}$

$\therefore$ divisible ✓

(v) Find all Pythagorean triples which contain the integer 12. [2]

Let $b = 2xy = 12 \Rightarrow xy = 6$ $\Rightarrow x=6, y=1$ or $x=3, y=2$

For $x=3, y=2$

Then $b = x^2 - y^2 \Rightarrow b = 9-4 = 5$

$c = x^2 + y^2 \quad c = 9+4 = 13$

$\Rightarrow a,b,c = 5, 12, 13$ ✓ ($\frac{1}{2}$ m)

or $x=6, y=1 \Rightarrow b = 36-1 = 35$.

$c = 36+1 = 37$

$\Rightarrow a,b,c = 12, 35, 37$ ($\frac{1}{2}$ m)

Also,

$3,4,5 \Rightarrow 9, 12, 15$

$\Rightarrow 12, 16, 20$. ✓ ($\frac{1}{2}$ m each)