Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 9 ADVANCED MATHEMATICS

Yearly Examination 2017

General Instructions:

- All questions may be attempted.
- Write in black pen
- Marks may be deducted for careless or badly arranged work.
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of this booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed: 90 minutes

Examiner: PSP

Name: _____

Class	Teacher	
9**A**	Mr Wang	
9**B**	Ms Stephens	
9**C**	Mr Kwong	
9**P**	Mr Fuller	
9**L**	Ms Evans/Mr Elliott	
9**U**	Ms Millar	
9**S**	Mr Choy	

Question	Marks
1	/ 15
2	/ 15
3	/ 15
4	/ 15
5	/ 15
6	/ 15
7	/ 15
Total	/ 105

Section 1 (15 Marks)

For Questions 1 – 10 and 12 circle the correct response.

1 What is 12.0957 correct to three significant figures?

(A) 12.095 (B) 12.096

(C) 12.1 (D) 12.0

2 What is the gradient of the line passing through $A(-1, 6)$ and $B(4, -4)$?

(A) -2 (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{3}{2}$

3 Which of the following is rational?

(A) $\sqrt[3]{18}$ (B) $\sqrt{8}$ (C) $0.\dot{4}\dot{9}$ (D) π

4 Which of the following shows the factors of $3x^2 - 10x - 8$?

(A) $(3x + 4)(x - 2)$ (B) $(3x - 4)(x + 2)$

(C) $(3x - 2)(x + 4)$ (D) $(3x + 2)(x - 4)$

5 Which of the following lines is perpendicular to the line $2x - 3y + 3 = 0$?

(A) $2x + 3y + 3 = 0$ (B) $2x - 3y - 3 = 0$

(C) $3x + 2y - 3 = 0$ (D) $3x - 2y + 3 = 0$

6

Taxable Income $	Tax
$1 – 5099	Nil
$5100 – 12 599	0 plus 24c for each $1 over $5100
$12 600 – 19 499	$1800.00 plus 29c for each $1 over $12 600
$19 500 – 34 999	$3801.00 plus 40c for each $1 over $19 500

Relluf earns $20 500 in one year. His allowable deductions total $1500.
From the above table, which of the following expressions represents his total tax?

(A) $1800 + 6400 \times 29c$ (B) $1800 + 6400 \times 40c$

(C) $3801 + 1000 \times 29c$ (D) $3801 + 1000 \times 40c$

7 Which of the following statements is true?

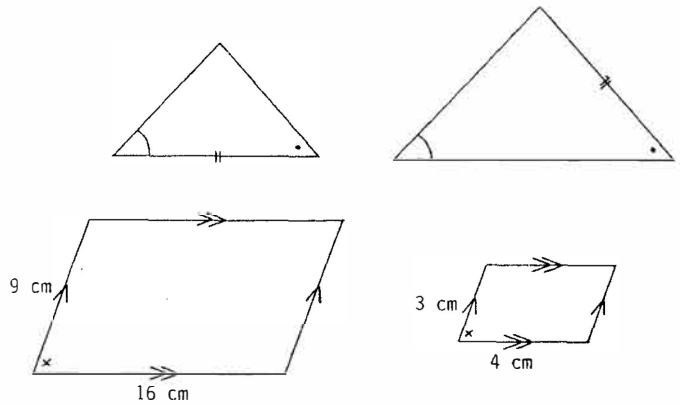
(A) $\sin 40° = 2\sin 20°$ (B) $\sin 40° = \cos 50°$

(C) $\frac{\sin 60°}{2} = \sin 30°$ (D) $\tan 60° = \frac{\cos 60^o}{\sin 60^o}$

8 Which of the following is the correct simplification of $\frac{a^6 + a^3}{a^3}$?

(A) a^6 (B) $2a^3$ (C) $a^3 + 1$ (D) $a^2 + 1$

9 Consider the diagrams and statements below.



I: The two triangles above are similar.

II: The two parallelograms above are similar.

Which statement is true?

(A) I only is true (B) II only is true

(C) Both I and II are true (D) Neither I nor II is true

10 Rallim wants some orange juice, but it is sold in 4 different packs.
Which of the following represents the best buy?

(A) 250 ml of juice for 45 cents (B) 375 ml of juice for 65 cents

(C) 1 litre of juice for \$1.85 (D) 1.5 litres of juice for \$2.75

11 Express $\frac{2}{\sqrt{5}}$ with a rational denominator. **1**

12 Which of the following is equivalent to $\dfrac{2\sqrt{3}+3\sqrt{2}}{3}$?

(A) $\dfrac{2}{2\sqrt{3}-3\sqrt{2}}$ (B) $\dfrac{-2}{2\sqrt{3}+3\sqrt{2}}$ (C) $\dfrac{-2}{2\sqrt{3}-3\sqrt{2}}$ (D) $\dfrac{2}{2\sqrt{3}+3\sqrt{2}}$

13 If $\tan 87° = \dfrac{y}{7}$, find y correct to two decimal places. **1**

14 A car is advertised for a cash price of \$8490. Yohc buys the car on the following terms: **2**

* No deposit
* 16% per annum simple interest
* Equal monthly repayments over 5 years.

Find the amount of each monthly repayment.

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Section 2 (15 Marks)

1 Simplify $4\sqrt{2} \times 3\sqrt{5}$ **1**

2 Express 4^{-3} as a fraction with no indices. **1**

3 Write down the equation of the vertical line through the point (1, –3). **1**

4 Write down the coordinates of the midpoint of the interval joining the points $A(8, 3)$ and $B(2, -1)$. **1**

5 Factorise fully

(i) $14x^2 - 7x$ **1**

(ii) $x^2 - 36$ **1**

(iii) $x^2 + 10x + 9$ **1**

(iv) $x^2 + 5x - 24$ **1**

6 Find the distance between the points $E(-2, 7)$ and $F(1, 3)$ **1**

7 Consider the formula $y = mx + b$

(i) Make m the subject of the formula **1**

(ii) Hence find the value of m when $y = 20$, $x = 8$ and $b = -12$ **1**

8 If $\sin 87° = \frac{7}{x}$, find x to two decimal places. **1**

9 Solve the following pair of simultaneous equations. **1**

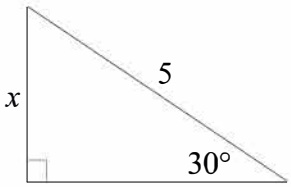
$$y = 2x - 3$$
$$x = 4$$

10 Sneh Pets tossed a coin three times. **1**
What is the probability she got three tails in a row?

11 Write down the gradient of a line perpendicular to $y = \frac{1}{3}x - 2$. **1**

Section 3 (15 Marks)

1 Find the value of x in the triangle below. **2**



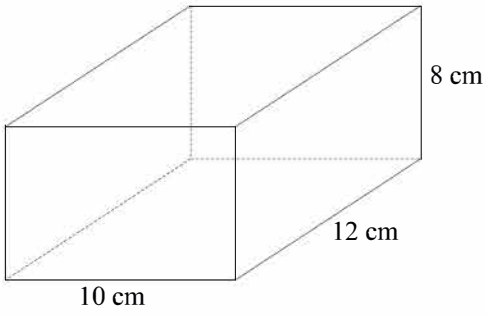
2 Given $x = 999\ 999\ 999\ 999$, determine the exact value of $\frac{x^2-4}{x-2}$. **2**

Show all your calculations.

3 Solve the simultaneous equations **2**

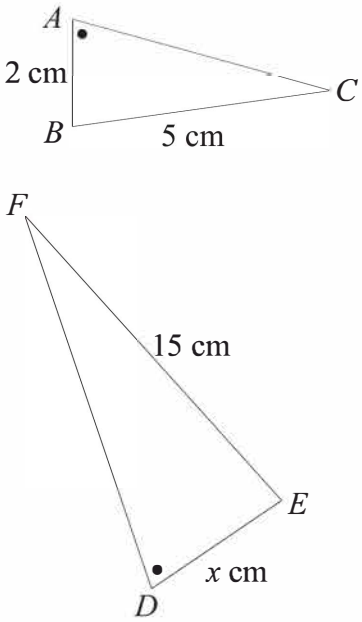
$$5x + y = 23$$
$$3x + y = 13$$

4 Find the surface area of the rectangular prism below. **2**

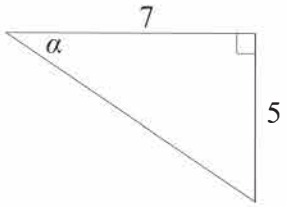


Section 3 (continued)

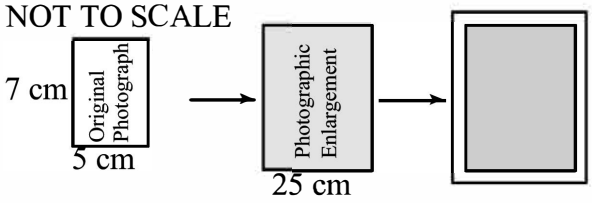
5 In the diagram below, $AB = 2$ cm, $BC = 5$ cm and $EF = 15$ cm. **2**
Given that $\Delta ABC \,|||\, \Delta DEF$, find the value of x.



6 Find the size of α in the triangle below. Answer to the nearest degree. **2**



7 A photograph is enlarged and then mounted on cardboard so that there is a 5 cm border around the enlargement.



What are the dimensions of the cardboard?
(A) 30 cm × 40 cm (B) 30 cm × 54 cm (C) 35 cm × 45 cm (D) 35 cm × 59 cm

8 Solve $x + 3(x - 2) = 6(x + 4)$. **2**

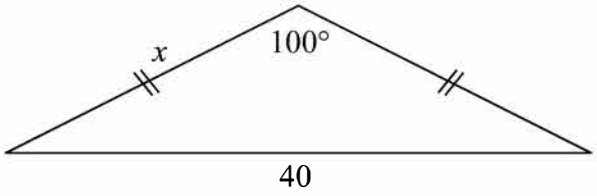
Section 4 (15 Marks)

1 For the straight line $2y = 4x - 9$ find:

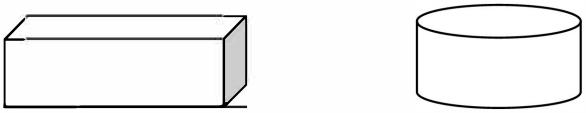
(i) the gradient; **1**

(ii) the y-intercept. **1**

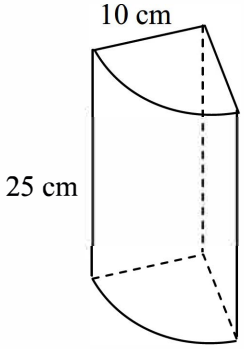
2 Find the size of x in the triangle below, to the nearest integer. **2**



3 A gold ingot in the shape of a rectangular prism 4 cm × 3 cm × 2 cm is melted down and recast as a circular disc 0.5 cm thick. **2**
Calculate the diameter of the disc to the nearest millimeter.

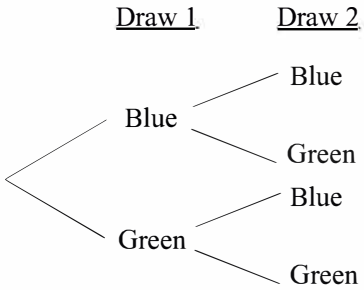


4 The diagram shown is a quarter of a cylinder. The radius is 10 cm and the height is 25 cm. Calculate the total surface area of the solid in terms of π. **2**



5 A bag contains 2 blue marbles and 3 green marbles. Two marbles are drawn from the bag, one after the other, without replacement.

(i) Write the probability of each outcome on the branches below. **2**



(ii) Find the probability that one blue and one green marble are drawn from the bag. **2**

6 Simplify $\frac{x^2-2x}{x^2-7x+10} \div \frac{x^2+6x}{x^2-25}$ **3**

Section 5 (15 Marks)

1 Consider the formula $b = \dfrac{2+\sqrt{a}}{2-\sqrt{a}}$.

Make a the subject of the formula. **3**

2 If $a(c + d) + b(c + d) = 72$ and $c + d = 6$, what is the value of $a + b + c + d$? **3**

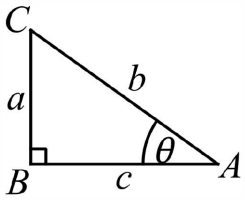
3 Factorise fully $12x^2 + 28x - 5$ **1**

4 Find the equation of the line through the point $(-3, 6)$ which is perpendicular to the line $3x - 2y = 5$. **2**

5 If a, b and c are three different positive integers, and $(ab)^2 = abc$. **1**
Which of the following is a possible value for c?

(A) 1 (B) 5 (C) 9 (D) 12

6 Using the right triangle ABC below, show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$ **3**



7 The hair and eye colour of a class of 24 students is recorded in the table below.

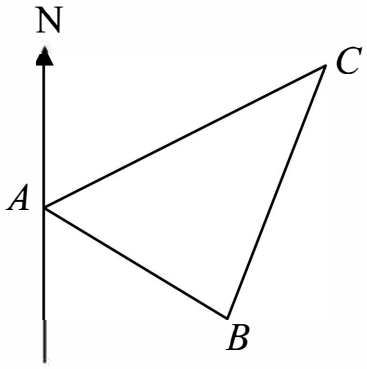
Hair	Eye Colour: Brown	Blue	Green
Dark	13	0	0
Fair	4	5	2

(i) What is the probability that a student chosen at random does not have brown eyes **1**

(ii) What is the probability a student chosen at random has blue eyes given they have fair hair? **1**

Section 6 (15 Marks)

1 Two geologists, Gnaw and Snave, on a large flat mining claim drive 20 km from point A on a bearing of 150° T to a point B.
Snave then drives 40 km on a bearing of 060° T to point C.



NOT TO SCALE

(i) By putting the above information on to the diagram above, show that $\angle ABC = 90°$. **1**

(ii) Find the bearing of Snave, at point C, from point A, to the nearest degree. **2**

2 Solve the following pair of simultaneous equations. **2**

$$4x + 3y = 11$$
$$x + y = 2$$

3 If x lies in the range $-2 \leq x < 3$, within what range does $2 - x$ lie? **1**

4 Simplify $\sqrt[4]{p \times \sqrt[3]{p^{-1}}}$, giving your answer in surd form. **2**

5 Factorise completely $(3x - 1)(2x + 5) - (x - 3)(2x + 5)$ **2**

6 A line passing through the point of intersection of $2x + 3y - 1 = 0$ and $4x - 9y + 3 = 0$ may be written in the form $2x + 3y - 1 + \lambda(4x - 9y + 3) = 0$ for some constant λ. For what value of λ does this line pass through the point (2, 1)? **2**

7 (i) Expand $(a \times 10^5 + b)(c \times 10^5 + d)$ **1**

(ii) By using part (i) and your calculator write down ALL the digits of **2**

$$7\ 166\ 432\ 102 \times 4\ 366\ 295\ 023$$

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Section 7 (15 Marks)

1 The shaded region below is to be cut to complete the net of a solid. **2**

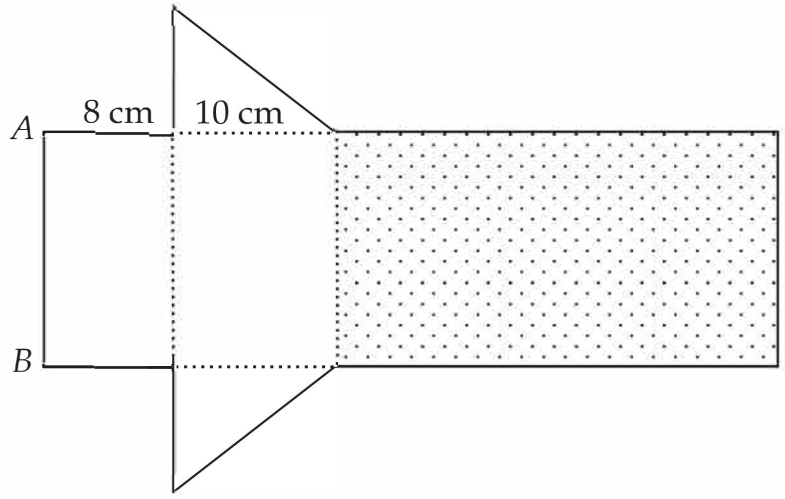
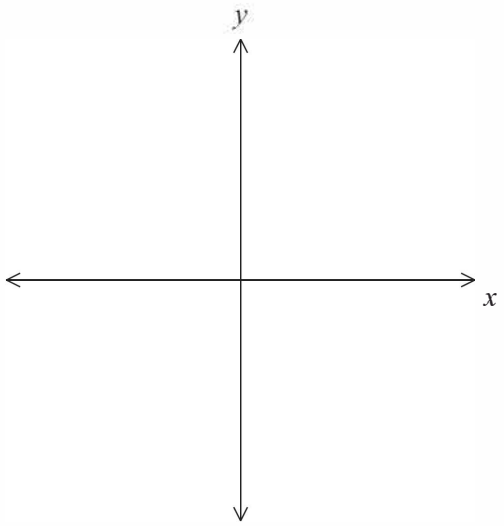


DIAGRAM DRAWN TO SCALE

Mark accurately where the shaded section must be cut and then calculate its volume

2 (i) Sketch $y = 2^x$ and $y = 3 - x$ on the same set of axes carefully showing all intercepts with the axes. **2**



(ii) Hence write down the solution of $2^x = 3 - x$. **1**

3 The prime factorisation of 45 is $3^2 \times 5^1$.
In general, the prime factorisation of an integer $n \geq 2$ is of the form

$$p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times ... \times p_k^{a_k},$$

where $p_1, p_2, \ldots, p_k$ are different prime numbers and $a_1, a_2, \ldots, a_k$ are positive integers.
Given an input of an integer $n \geq 2$, the *Barbeau Process* outputs the number equal to

$$n\left(\frac{a_1}{p_1} + \frac{a_2}{p_2} + ... + \frac{a_k}{p_k}\right).$$

For example, given an input of 45, the Barbeau Process outputs $45\left(\frac{2}{3} + \frac{1}{5}\right) = 39$.

(i) Given an input of 126, what number does the Barbeau Process output? **2**

(ii) Simplify $p^2q\left(\frac{2}{p} + \frac{1}{q}\right)$, leaving your answer factorized. **1**

(iii) Using part (ii), determine all pairs (p, q) of different prime numbers such that the Barbeau Process with input p^2q outputs 135. **3**

4. A train which is 15 metres long travels at a constant speed of 18 km/h. **4**
Eldned and Gnowk are running parallel to the train tracks. Eldned is running in the same direction as the train, while Gnowk is running towards the train. They are both running at a constant speed. The train takes 6 seconds to pass Eldned. It meets Gnowk 32 seconds later, taking 2 seconds to pass him.
How long after the train passes Gnowk will the runners meet?

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Spare Questions

5 In a Super Smash Bros tournament, $\frac{1}{2}$ of the contestants play as Fox, $\frac{1}{3}$ of the contestants play as Falco and $\frac{1}{6}$ of the contestants play as Peach.
Given that there were 40 more people who played either Fox or Falco than who played Peach, how many contestants attended the tournament?

A BINGO card has twenty-five different integers arranged into five rows and five columns labeled B, I, N, G, and O such that:

- The middle integer is always 0.
- Integers in column B are between 1 and 15 inclusive.
- Integers in column I are between 16 and 30 inclusive.
- Integers in column N are between 31 and 45 inclusive (other than the middle integer being 0).
- Integers in column G are between 46 and 60 inclusive.
- Integers in column O are between 61 and 75 inclusive.

Here is an example of a BINGO card.

B	I	N	G	O
5	24	36	48	61
2	29	31	53	64
11	18	0	60	68
15	20	44	51	69
3	26	42	47	70



(a) What is the smallest possible sum of the numbers in a row on a BINGO card?



(b) Carrie's BINGO card has a row and a diagonal each with the same sum. What is the smallest possible such sum? Show that there is a BINGO card with this sum and explain why there is no BINGO card with a smaller such sum.



(c) In the BINGO card shown, numbers in a diagonal and in the 3^{rd} row are missing. Determine with justification the number of ways to complete this BINGO card so that the sum of the numbers in this diagonal is equal to 177 and the sum of the numbers in the 3^{rd} row is also equal to 177.

B	I	N	G	O
	23	35	47	65
5		31	52	63
		0		
11	20	40		69
9	18	38	48	

Among grandfather's papers a bill was found:

72 turkeys $—67.9—

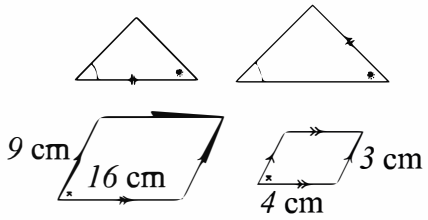
The first and last digit of the number that obviously represented the total price of those fowls are replaced here by blanks, for they have faded and are now illegible.

What are the two faded digits and what was the price of one turkey?

A town has 2017 houses. Of these 2017 houses, 1820 have a dog, 1651 have a cat, and 1182 have a turtle. If x is the largest possible number of houses that have a dog, a cat, and a turtle, and y is the smallest possible number of houses that have a dog, a cat, and a turtle, then $x - y$ is

. Express $(0^3 - 350)(1^3 - 349)(2^3 - 348)...(350^3 - 0)$ concisely as possible.

Consider the diagrams and statements below.

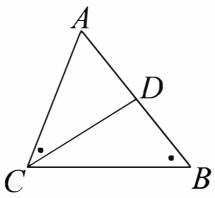


I: The two triangles above are similar.

II: The two parallelograms above are similar.

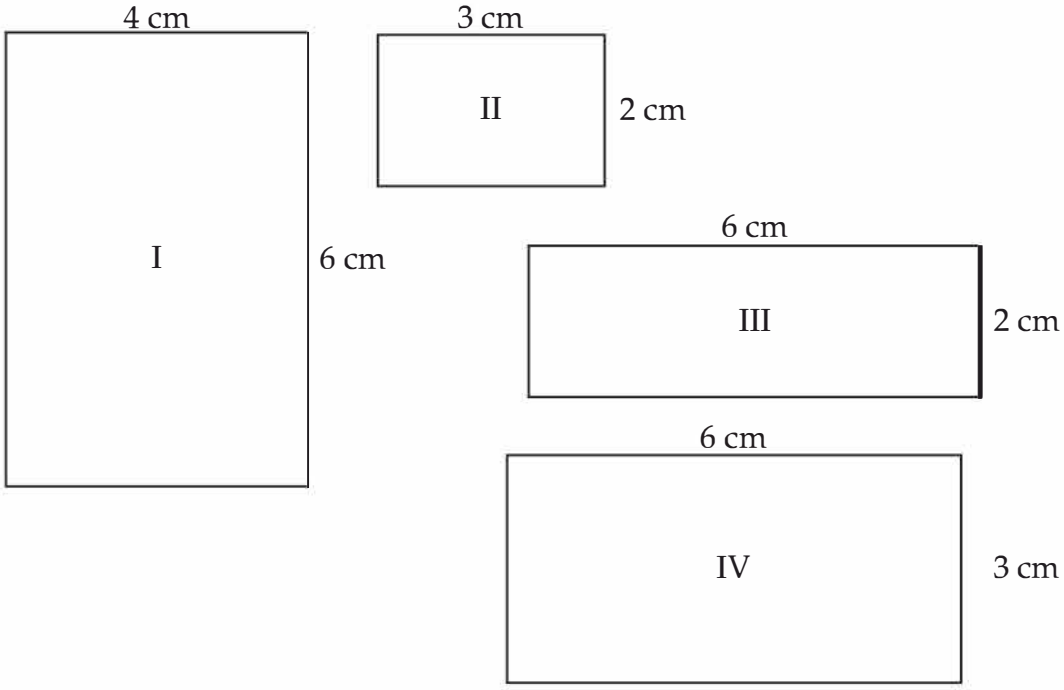
Which statement is true?

(A) I only is true
(B) II only is true
(C) Both I and II are true
(D) Neither I nor II is true



D is a point on AB in the triangle ABC. Angle ACD = Angle DBC.
Name a pair of similar triangles.

Which two rectangles are similar?



(A) I and II (B) II and III (C) III and IV (D) I and IV

The shaded section of this diagram is to be cut to complete the net of a solid.

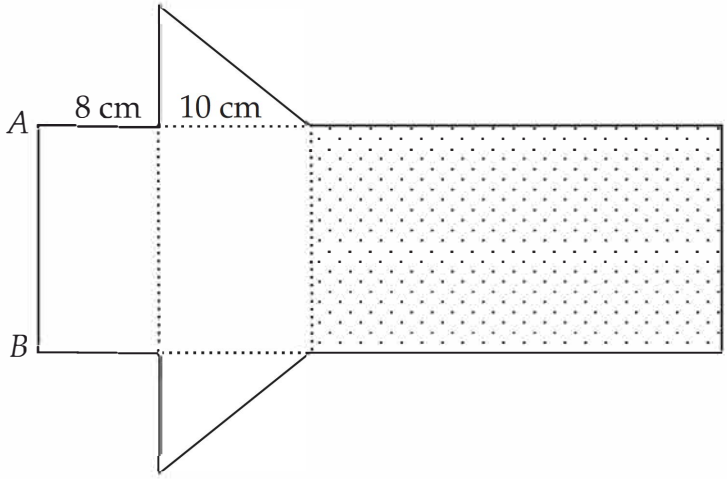


DIAGRAM
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(a) Mark accurately where the shaded section must be cut.

(b) Name the solid that can be formed.

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(c) By measurement and calculation, find the length of side *AB*.

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(d) When assembled, the net folds into a drink container.
David has 12 litres of juice.

How many of these containers can he fill? (1 litre = 1000 cm^3)

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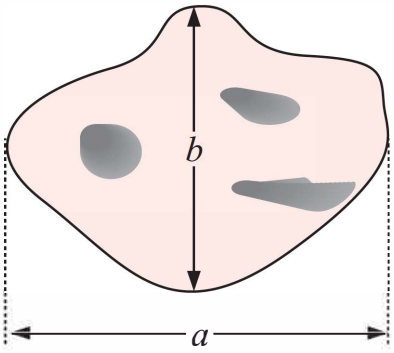
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The diagram given shows a microscopic organism *enlarged* using the scale 1 : 0.001. Find the actual length of the dimensions marked:

a width, a

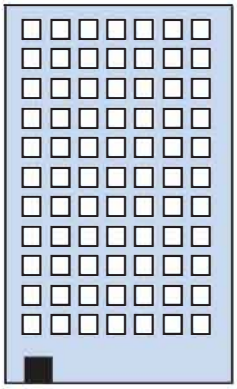
b height, b.

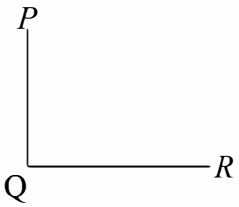
A scale diagram of a building is shown with scale 1 : 1000.

a If the height is 5 cm and width is 3 cm on the drawing, find the actual height and width of the building in metres.

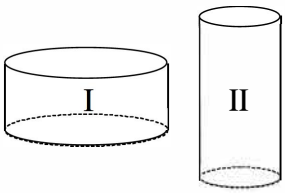
b If the height of the windows on the drawing are 2.5 mm, how high are the actual windows?

c If the actual height of the entrance door is 3.2 m, what is its height on the scale drawing?



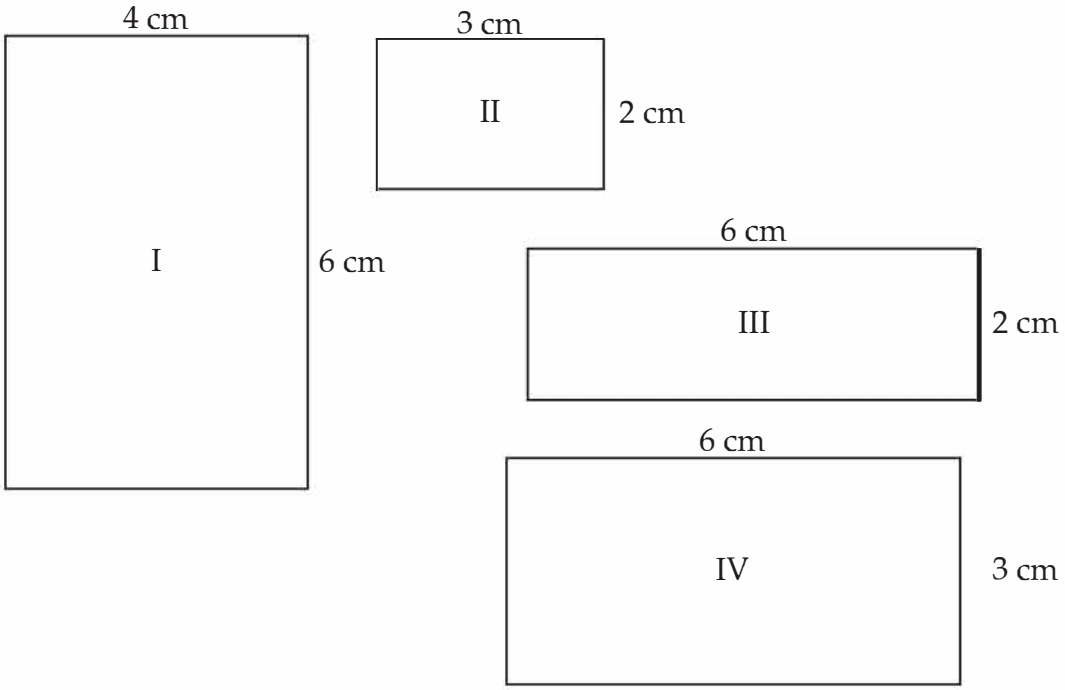


PQ and *QR* are two sides of a four-sided paddock, drawn to a scale of *1 : 10 000*. By measurement and calculation, find the length of the side *QR* in metres.



The diameter of cylinder I is twice that of cylinder II.
The height of cylinder II is twice that of cylinder I.
The volume of cylinder I is V_1.
The volume of cylinder II is V_2.
$V_1 : V_2 = ?$

(A) *1 : 2* (B) *1 : 1* (C) *2 : 1* (D) *4 : 1*


4 cm
I
6 cm
3 cm
II
2 cm
6 cm
III
2 cm
6 cm
IV
3 cm

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2017

Year 9 Yearly

Advanced Mathematics

Suggested Solutions

Sections	Marker
1	**JM**
2	**RE**
3	**AF**
4	**EC**
5	**MK**
6	**ES**
7	**AYW**

Section 1

1. $12.0957 = 12.1$ (3 sig. figs.) **C**

2. m (gradient of AB) $= \frac{-4-6}{4-(-1)}$

$= \frac{-10}{5}$

$= -2$ **A**

3. $0.\dot{4}\dot{9}$ **C**

4. $3x^2 - 10x - 8$ P: -24, S: -10 $\Rightarrow -12, 2$

$= \frac{(3x-12)(3x+2)}{3}$

$= \frac{3(x-4)(3x+2)}{3}$

$= (x-4)(3x+2)$ **D**

5. $2x - 3y + 3 = 0 \Rightarrow 3y = 2x + 3$

$y = \frac{2}{3}x + 1$

$\therefore$ gradient of perpendicular line $= \frac{-1}{2/3}$

$= -\frac{3}{2}$

$\therefore y = -\frac{3}{2}x + b$ (where b is a constant)

$2y = -3x + 2b$

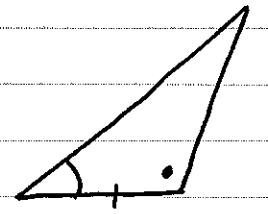
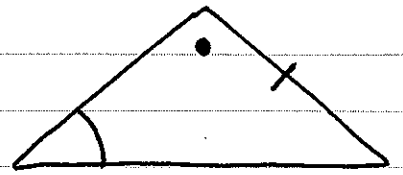
$3x + 2y - 2b = 0$ **C**

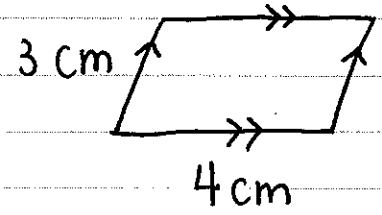
6. $20 500 - $1500 = $19 000 (taxable income)

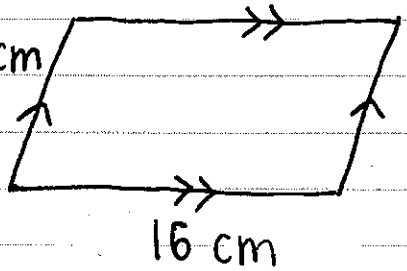
$\therefore$ 1800 + 0.29 x (19 000 - 12 600)
= 1800 + 6 400 x 0.29 **A**

7. $\sin\theta = \cos(90-\theta)$
$\therefore \sin 40 = \cos(90-40)$
$\sin 40 = \cos 50$ **B**

8. $\frac{a^6 + a^3}{a^3} = \frac{a^3(a^3+1)}{a^3}$
$= a^3 + 1$ **C**

9. 

(equiangular)





(sides aren't the same ratio)
$\therefore$ not similar

A

10. Best buy: (A) $0.0018 /mL
(B) $0.00173 /mL ✓
(C) $0.00185 /mL
(D) $0.00183 /mL

$\therefore$ **B**

11. $\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

12. $\frac{2\sqrt{3}+3\sqrt{2}}{3} \times \frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$

$= \frac{4(3)-9(2)}{3(2\sqrt{3}-3\sqrt{2})}$

$= \frac{-6}{3(2\sqrt{3}-3\sqrt{2})}$

$= \frac{-2}{2\sqrt{3}-3\sqrt{2}}$ **C**

13. $\tan 87° = \frac{y}{7}$

$\therefore y = 7 \times \tan 87°$

$= 133.5679568...$

$= 133.57$ (2.d.p)

14. $\frac{8490 + 8490 \times 0.16 \times 5}{5 \times 12} = \frac{15\,282}{60}$

$= \$254.70$

Section 2 (15 Marks)

1 Simplify $4\sqrt{2} \times 3\sqrt{5}$ **1**

½ mark
- $12\sqrt{2} \times \sqrt{5}$
- $\sqrt{1440}$

$12\sqrt{10}$

2 Express 4^{-3} as a fraction with no indices. **1**

½ mark
- $\frac{1}{4^3}$

$\frac{1}{64}$

3 Write down the equation of the vertical line through the point $(1, -3)$. **1**

$x = 1$

4 Write down the coordinates of the midpoint of the interval joining the points $A(8, 3)$ and $B(2, -1)$. **1**

½ mark
- Correct working w/ wrong answer E.g. (3,2)
- 1 correct answer.

$(5, 1)$

5 Factorise fully

½ mark

(i) $14x^2 - 7x$ **1**

$7x(2x-1)$

(ii) $x^2 - 36$ **1**

- $(x-6)^2$ → ie. 1 solution
- $(x-4)(x+4)$

$(x-6)(x+6)$

(iii) $x^2 + 10x + 9$ **1**

- Correct working with wrong solution E.g. $(x^2+9)(x+1)$

$(x+9)(x+1)$

(iv) $x^2 + 5x - 24$ **1**

$(x+8)(x-3)$

6 Find the distance between the points $E(-2, 7)$ and $F(1, 3)$ **1**

½ mark
- Correct working with wrong answer.

5

½ mark

- $m = \frac{y}{x} - b$
- $m = \frac{b-y}{x}$
- $m = \frac{b+y}{x}$

7 Consider the formula $y = mx + b$

(i) Make m the subject of the formula **1**

$m = \frac{y-b}{x}$

(ii) Hence find the value of m when $y = 20$, $x = 8$ and $b = -12$ **1**

$m = 4$

Full marks given if they used the wrong formula above, correctly.

½ mark

- Correct working with wrong answer.

8 If $\sin 87° = \frac{7}{x}$, find x to two decimal places. **1**

$x \doteq 7.01$

9 Solve the following pair of simultaneous equations. **1**

$y = 2x - 3$

$x = 4$

½ mark

- Missing x value.

$x = 4, y = 5$ OR (4,5)

10 Sneh Pets tossed a coin three times. **1**
What is the probability she got three tails in a row?

$\frac{1}{8}$ OR 12.5% OR 0.125

½ mark

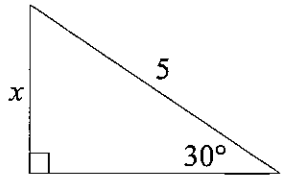
- Gradient of $-1, 3, -\frac{1}{3}$.

11 Write down the gradient of a line perpendicular to $y = \frac{1}{3}x - 2$. **1**

-3

Section 3 (15 Marks)

1 Find the value of x in the triangle below. **2**



$\sin 30^\circ = \frac{x}{5}$

$x = 5\sin 30^\circ$

$x = \frac{5}{2}$.

2 Given $x = 999\,999\,999\,999$, determine the exact value of $\frac{x^2-4}{x-2}$. **2**

Show all your calculations.

$\frac{(x-2)(x+2)}{x-2}$

$= 999\,999\,999\,999 + 2$

$= 1000\,000\,000\,001$

3 Solve the simultaneous equations **2**

$5x + y = 23$ ——— ①

$3x + y = 13$ ——— ②

① − ②

$2x = 10$

$x = 5$

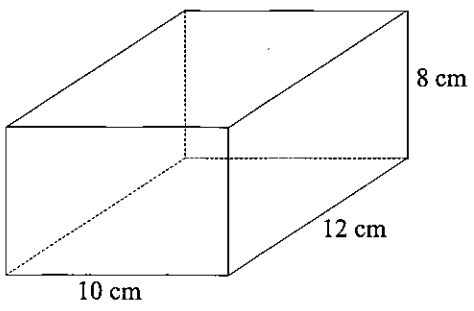
sub into ①

$5(5) + y = 23$

$y = -2$

$x = 5, y = -2$

4 Find the surface area of the rectangular prism below. **2**

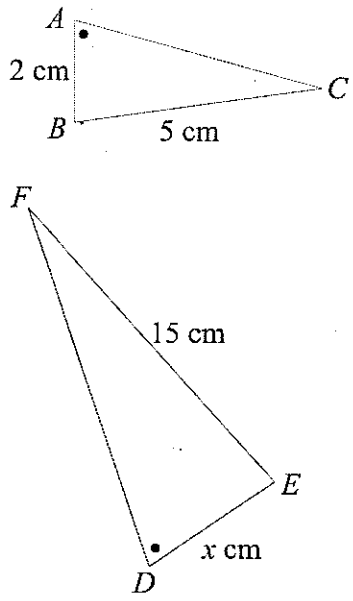


$SA = 2\times(10\times8) + 2\times(10\times12) + 2\times(8\times12)$

$= 592 \text{ cm}^2$.

Section 3 (continued)

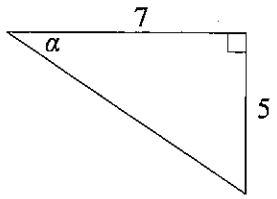
5 In the diagram below, $AB = 2$ cm, $BC = 5$ cm and $EF = 15$ cm. **2**
Given that $\Delta ABC \parallel\!\parallel \Delta DEF$, find the value of x.



$\frac{x}{2} = \frac{15}{5}$ (corresponding sides in same ratio)

$x = 6$ cm

6 Find the size of α in the triangle below. Answer to the nearest degree. **2**



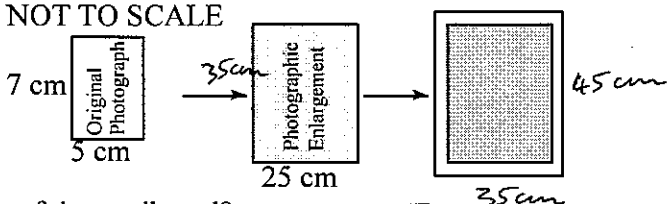
$\tan\alpha = \frac{5}{7}$

$\alpha = \tan^{-1}\left(\frac{5}{7}\right)$

$\alpha = 36^\circ$

7 A photograph is enlarged and then mounted on cardboard so that there is a 5 cm border around the enlargement.

NOT TO SCALE



What are the dimensions of the cardboard?

(A) 30 cm × 40 cm (B) 30 cm × 54 cm (C) 35 cm × 45 cm (D) 35 cm × 59 cm

8 Solve $x + 3(x - 2) = 6(x + 4)$. **2**

$x + 3x - 6 = 6x + 24$

$4x - 6 = 6x + 24$

$-30 = 2x$

$x = -15$

Section 4 (15 Marks)

1 For the straight line $2y = 4x - 9$ find:

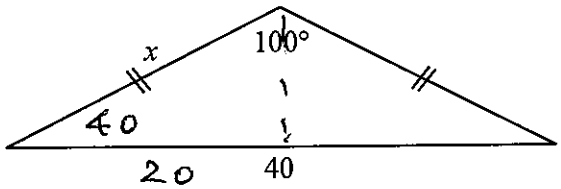
(i) the gradient; **1**

2

(ii) the y-intercept. **1**

$-9/2$

2 Find the size of x in the triangle below, to the nearest integer. **2**



$\cos 40 = 20/x$

$x = \frac{20}{\cos 40} = 26$

3 A gold ingot in the shape of a rectangular prism 4 cm × 3 cm × 2 cm is melted down and recast as a circular disc 0.5 cm thick. **2**
Calculate the diameter of the disc to the nearest millimeter.



(i) $\pi r^2 \times \frac{1}{2} = 24$

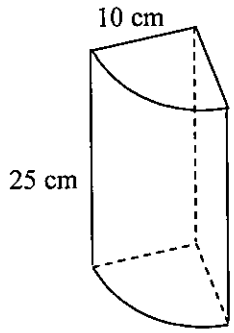
$\pi r^2 = 48$

$r^2 = \frac{48}{\pi}$

$d = 2\sqrt{\frac{48}{\pi}} = 8\sqrt{\frac{3}{\pi}}$

$= 78$ mm

4 The diagram shown is a quarter of a cylinder. The radius is 10 cm and the height is 25 cm. Calculate the total surface area of the solid in terms of π. **2**



2 bases $2 \times \frac{1}{4} \times \pi \times 100$

$= 50\pi$

$\frac{1}{4} \times C = 5\pi$

Curve SA $= 25 \times 5\pi = 125\pi$

2 rectangle $= 500$

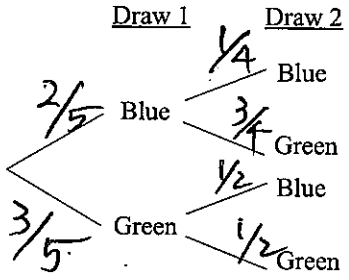
$\therefore S = 125\pi + 50\pi + 500$

$= 175\pi + 500$ cm²

Section 4 (continued)

5 A bag contains 2 blue marbles and 3 green marbles. Two marbles are drawn from the bag, one after the other, without replacement.

(i) Write the probability of each outcome on the branches below. **2**



(ii) Find the probability that one blue and one green marble are drawn from the bag. **2**

$$P(BG) + P(GB)$$
$$= \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{1}{2}$$
$$= \frac{3}{5}$$

6 Simplify $\dfrac{x^2-2x}{x^2-7x+10} \div \dfrac{x^2+6x}{x^2-25}$ **3**

$$\frac{x(x-2)}{(x-5)(x-2)} \times \frac{(x-5)(x+5)}{x(x+6)}$$
$$= \frac{x+5}{x+6}$$

Section 5 (15 Marks)

1 Consider the formula $b = \dfrac{2+\sqrt{a}}{2-\sqrt{a}}$. **3**

Make a the subject of the formula.

$b = \dfrac{2+\sqrt{a}}{2-\sqrt{a}}$

$(2-\sqrt{a})b = 2+\sqrt{a}$ ✓ ① mark for rearranging / expanding

$2b - b\sqrt{a} = 2+\sqrt{a}$

$2b-2 = \sqrt{a}+b\sqrt{a}$

$2(b-1) = \sqrt{a}(b+1)$ ✓ ① mark for factorising

$\sqrt{a} = \dfrac{2(b-1)}{b+1}$

$\therefore a = \dfrac{4(b-1)^2}{(b+1)^2}$ ✓ ① mark for squaring correctly

2 If $a(c+d) + b(c+d) = 72$ and $c+d = 6$, what is the value of $a+b+c+d$? **3**

$a(6)+b(6) = 72$ ✓ ① mark

$6a+6b = 72$

$a+b = 12$ ✓ ① mark

$a+b+c+d = 12+6$

$= 18$ ✓ ① mark

3 Factorise fully $12x^2 + 28x - 5$ **1**

$(2x+5)(6x-1)$ ✓ ① mark

4 Find the equation of the line through the point (–3, 6) which is perpendicular to the line $3x - 2y = 5$. **2**

$3x-2y = 5$

$2y = 3x-5$

$y = \dfrac{3}{2}x - \dfrac{5}{2}$

$m_1 = \dfrac{3}{2}$

$m_2 = -\dfrac{2}{3}$ ✓ ① mark

$\Rightarrow y-6 = -\dfrac{2}{3}(x+3)$ ① mark for correct point gradient formula

$y-6 = -\dfrac{2}{3}x - 2$

$y = -\dfrac{2}{3}x + 4$ ✓

OR

$0 = 2x+3y-12$

① mark to notice change in gradient for perpendicular

5 If a, b and c are three different positive integers, and $(ab)^2 = abc$. **1**
Which of the following is a possible value for c?

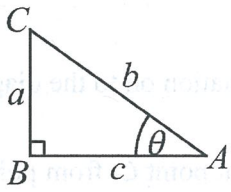
(A) 1 (B) 5 (C) 9 (D) 12

$(ab)^2 = abc$
$a^2b^2 = abc$
$ab = c$

∴ Answer must be D since it is the only answer for products of different integers.

1 mark

6 Using the right triangle ABC below, show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$ **3**



$a^2 + c^2 = b^2$ (Pythagoras' Theorem) ✓ 1 mark for noticing Pythagoras

$\sin\theta = \frac{a}{b} \Rightarrow a = b\sin\theta$

$\cos\theta = \frac{c}{b} \Rightarrow c = b\cos\theta$

$(b\sin\theta)^2 + (b\cos\theta)^2 = b^2$

$\Rightarrow b^2(\sin\theta)^2 + b^2(\cos\theta)^2 = b^2$

$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$

1 mark for both $\sin\theta$ and $\cos\theta$

1 mark for showing.

7 The hair and eye colour of a class of 24 students is recorded in the table below.

Hair \ Eye Colour	Brown	Blue	Green	
Dark	13	0	0	13
Fair	4	5	2	11
	17	5	2	24

(i) What is the probability that a student chosen at random does not have brown eyes **1**

$\frac{7}{24}$ ✓ 1 mark

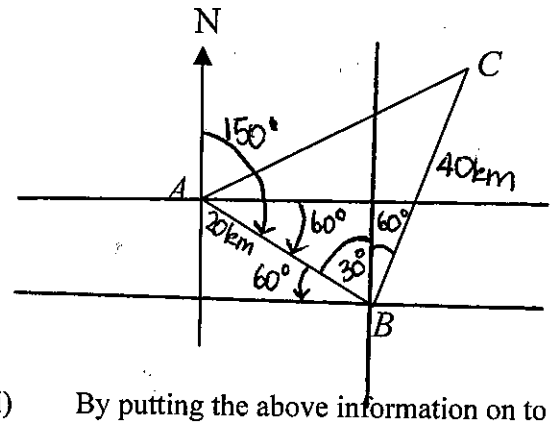
(ii) What is the probability a student chosen at random has blue eyes given they have fair hair? **1**

$\frac{5}{11}$ ✓ 1 mark

$\frac{1}{2}$ mark only if they noticed to change denominator due to fair hair.

Section 6 (15 Marks)

1 Two geologists, Gnaw and Snave, on a large flat mining claim drive 20 km from point A on a bearing of 150° T to a point B.
Snave then drives 40 km on a bearing of 060° T to point C.

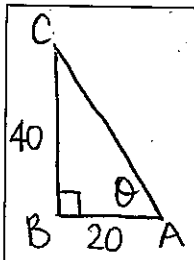


NOT TO SCALE

* No half marks.

(i) By putting the above information on to the diagram above, show that $\angle ABC = 90°$. **1**

(ii) Find the bearing of Snave, at point C, from point A, to the nearest degree. **2**



$\text{Tan}\,\theta = \frac{40}{20}$

$\theta = 63°26' \doteqdot 63°$ ← * 1 mark for getting to here

$150 - 63° = 87°$

∴ Bearing from A to C
= 87° (to nearest degree)

2 Solve the following pair of simultaneous equations. **2**

$4x + 3y = 11$ ①
$x + y = 2$ ②

② → $y = 2 - x$ → ①

$= 4x + 3(2-x) = 11$
$4x + 6 - 3x = 11$
$x = 5$

→ ② → $5 + y = 2$
$y = -3$

$(x = 5, y = -3)$

* 1 mark for 1 correct pronumeral.
* 1½ marks for correct method or subbing in incorrect solution to first part.

Section 6 (continued)

3 If x lies in the range $-2 \leq x < 3$, within what range does $2 - x$ lie? **1**

* $2-(-2) = 4$
* $2-(3) = -1$

$\Rightarrow \quad -1 < 2-x \leq 4$

* Half mark for one correct boundary or incorrect + − signs.

4 Simplify $\sqrt[4]{p \times \sqrt[3]{p^{-1}}}$, giving your answer in surd form. **2**

$\sqrt[4]{p \times p^{-1/3}} = \sqrt[4]{p^{2/3}}$

$= p^{\frac{2/3}{4}}$

$= p^{1/6}$

$= \sqrt[6]{p}$

$\frac{2}{3} \div 4$

$= \frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$

$= \frac{1}{6}$

* lost half a mark for leaving as $p^{1/6}$

5 Factorise completely $(3x-1)(2x+5) - (x-3)(2x+5)$ **2**

$(3x-1)(2x+5) - (x-3)(2x+5)$

* → half mark for correct expansion

$= 6x^2 + 15x - 2x - 5 - (2x^2 + 5x - 6x - 15)$

$= 6x^2 + 13x - 5 - 2x^2 + x + 15$

$= 4x^2 + 14x + 10$

$= 2(2x^2 + 7x + 5)$

$2x \quad 5$

$x \quad 1$

$= 2(2x+5)(x+1)$

* lost half mark for { $(4x+10)(x+1)$ / $(2x+5)(2x+2)$

6 A line passing through the point of intersection of $2x + 3y - 1 = 0$ and $4x - 9y + 3 = 0$ may be written in the form $2x + 3y - 1 + \lambda(4x - 9y + 3) = 0$ for some constant λ. For what value of λ does this line pass through the point (2, 1)? **2**

$2(2) + 3(1) - 1 + \lambda[4(2) - 9(1) + 3] = 0$

$4 + 3 - 1 + \lambda(8 - 9 + 3) = 0.$

$6 + 2\lambda = 0$

$2\lambda = -6$

$\lambda = -3$

* lost half mark for $\lambda = 3$

7 (i) Expand $(a \times 10^5 + b)(c \times 10^5 + d)$ **1**

$ac \times 10^{10} + da \times 10^5 + bc \times 10^5 + bd$

(ii) By using part (i) and your calculator write down ALL the digits of **2**

7 166 432 102 × 4 366 295 023

$(71664 \times 10^5 + 32102) \times (43662 \times 10^5 + 95023)$

$= 3128993568 \times 10^{10} + 6809728272 \times 10^5$

$+ 1401637524 \times 10^5 + 30504283 46$

$= 3128993568 \times 10^{10} + 8211365796 \times 10^5$

$+ 30504283 46.$

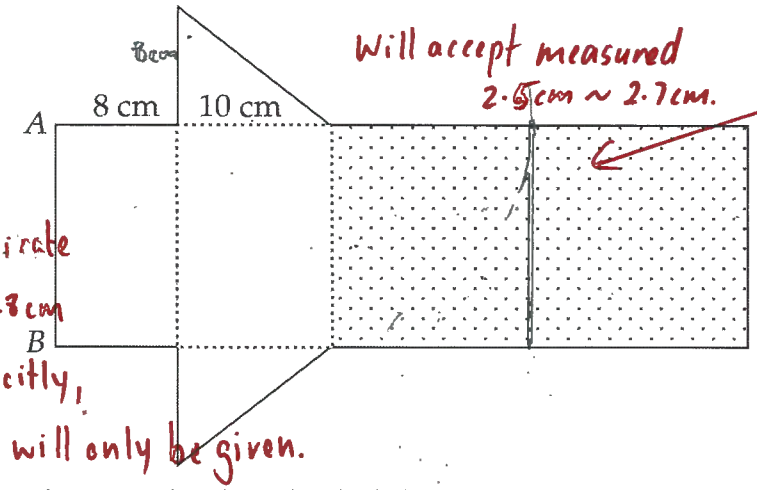
$= + \begin{array}{r} 312899356800000000000 \\ 821136579600000 \\ 30504283 46 \\ \hline 3129075681963002834 6 \end{array}$

* 1 mark for writing out
3129075682000000000000.

16mm : 8cm
2mm . 1cm
1mm . 0·5cm

Section 7 (15 Marks)

1 The shaded region below is to be cut to complete the net of a solid. 2



Will accept measured 2·5cm ~ 2·7cm.

0·5 mark for correct line

DIAGRAM DRAWN TO SCALE

Note: If Students indicate that AB is 2·8cm or 3cm explicitly, full marks will only be given.

Mark accurately where the shaded section must be cut and then calculate its volume

Scale: 16mm to 8cm
16 : 80
1 : 5

AB = 2·9cm (measured)

∴ Actual measurement
= 2·9 × 5 = 14·5cm

0·5 mark for using scale

∴ Volume of triangular prism = $\frac{8 \times 10}{2} \times 14.5$

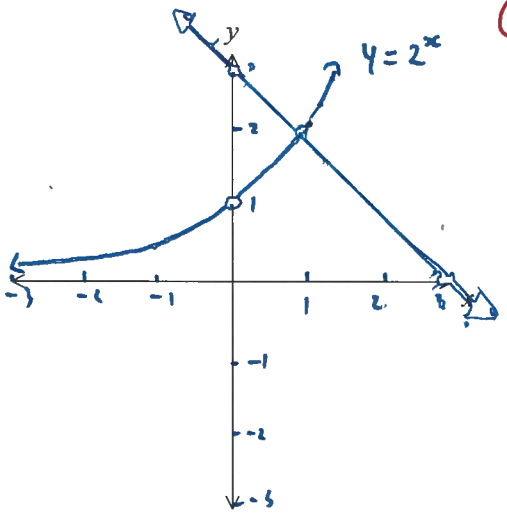
= 580cm³

0·5 mark correct answer

0·5 calculation mark

2 (i) Sketch $y = 2^x$ and $y = 3 - x$ on the same set of axes carefully showing all intercepts with the axes. 2

* Students were penalised for not having a scale on axes.



1 mark for correct graph and intercept of $y=2^x$

1 mark for correct graph and intercepts of $y=3-x$

(ii) Hence write down the solution of $2^x = 3 - x$. 1

x = 1 (1)

Need to use the graph to answer the question.

Note: Student need to realise that finding the solution means finding the x-value only in this question.

5

3 The prime factorisation of 45 is $3^2 \times 5^1$.
In general, the prime factorisation of an integer $n \geq 2$ is of the form

$$p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times \ldots \times p_k^{a_k},$$

where $p_1, p_2, \ldots, p_k$ are different prime numbers and $a_1, a_2, \ldots, a_k$ are positive integers.
Given an input of an integer $n \geq 2$, the *Barbeau Process* outputs the number equal to

$$n\left(\frac{a_1}{p_1}+\frac{a_2}{p_2}+\ldots+\frac{a_k}{p_k}\right).$$

For example, given an input of 45, the Barbeau Process outputs $45\left(\frac{2}{3}+\frac{1}{5}\right)=39$.

(i) Given an input of 126, what number does the Barbeau Process output? **2**

$126 = 2 \times 3^2 \times 7$

$\therefore$ Barbeau Process $= 126 \times \left(\frac{1}{2}+\frac{2}{3}+\frac{1}{7}\right)$ (1) mark

$= 165$ (1) mark

(ii) Simplify $p^2q\left(\frac{2}{p}+\frac{1}{q}\right)$, leaving your answer factorized. **1**

$p^2q\left(\frac{2}{p}+\frac{1}{q}\right)$

$= p^2q\left(\frac{2q+p}{pq}\right)$

$= p(2q+p)$ (1)

(iii) Using part (ii), determine all pairs (p, q) of different prime numbers such that the Barbeau Process with input p^2q outputs 135. **3**

$135 = 5 \times 27$

$= 5 \times 3^3$ (0.5) mark

$\therefore$ Only 2 possible primes.

If $p = 5$

$5 \times (2q+5) = 135$

$2q + 5 = 27$

$2q = 22$

$q = 11$ (1) mark for working

If $q = 5$

$p(10+p) = 135$

If $p = 3$

$3 \times (2q+3) = 135$

$6q + 9 = 135$

$q = 21$

But $q = 21$ is not prime

$\therefore$ Only $(5, 11)$ (1) correct answer

(0.5) mark to test or state why 3 won't work.

6

4. A train which is 15 metres long travels at a constant speed of 18 km/h. Eldned and Gnowk are running parallel to the train tracks. Eldned is running in the same direction as the train, while Gnowk is running towards the train. They are both running at a constant speed. The train takes 6 seconds to pass Eldned. It meets Gnowk 32 seconds later, taking 2 seconds to pass him. How long after the train passes Gnowk will the runners meet? **4**

Train Speed: $\frac{18\text{km}}{1\text{h}} = \frac{18000\text{m}}{3600\text{s}} = 5\text{m/s}$ 0.5 mark

Eldned Speed = V_E

For the whole train passing Eldned

Distance = Speed x time

$15\text{m} = (5 - V_E) \times 6$

$2.5 = 5 - V_E$

$V_E = 2.5\text{ m/s}$ 1 mark with working

Gnowk Speed = V_G

For the whole train passing Gnowk

Distance = Speed x time

$15\text{m} = (5 + V_G) \times 2$

$7.5 = 5 + V_G$

$V_G = 2.5\text{m/s}$ 1 mark with working.

∴ Eldned and Gnowk are travelling same speed.

In 34 seconds the whole train passes Eldned and Gnowk

train distance = $34\text{s} \times 5\text{m/s}$
$= 170\text{m}$

Eldned distance = $34 \times 2.5\text{m/s}$
$= 85\text{m}$

∴ Since Eldned and Gnowk are travelling towards each other, they should travel same distance to meet each other.

i.e. $\frac{85}{2} = 42.5\text{m}$.

∴ Time taken to meet = $\frac{\text{Distance}}{\text{Speed}} = \frac{42.5}{2.5}$ 1 mark

$= 17\text{seconds}$ 0.5 mark

End of paper

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