SYDNEY BOYS HIGH SCHOOL

MOORE PARK, SURRY HILLS

Year 9

Yearly Examination 2014

Advanced Mathematics

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- All answers must be given in exact simplified form unless otherwise indicated.
- Clearly indicate your class by placing an X, next to your class

Examiner: *B. Kilmore*

NAME:

Class	Teacher	
9 A	Mr Boros	
9 B	Ms Nesbitt/Ms Likourezos	
9 C	Ms Ward	
9 D	Mr Choy	
9 E	Ms Millar	
9 F	Ms Chen / Mr Elliott	
9 G	Mr Gainford	

Sections	Mark
A	**/15**
B	**/15**
C	**/15**
D	**/15**
E	**/15**
F	**/15**
G	**/15**
Total	**/105**

Section A (15 Marks)

1. Calculate -2^4 **1**

2. Solve $3-x>-2$ **1**

3. Write 0.010031 correct to 3 significant figures. **1**

4. Write 2618000 in scientific notation. **1**

5. Simplify $\frac{\sqrt{45}}{15}$ **1**

6. Write with a radical sign and evaluate: $\left(81^{\frac{1}{4}}\right)^3$ **1**

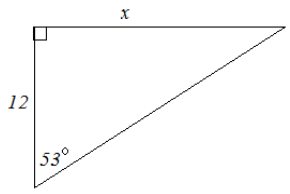
7. Factorise **4**

   a) $24ry^3-42r^3y^2$

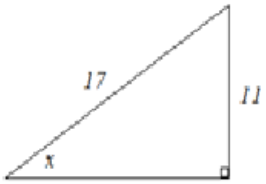
   b) $x^2-11x+24$

   c) $5x^2-20$

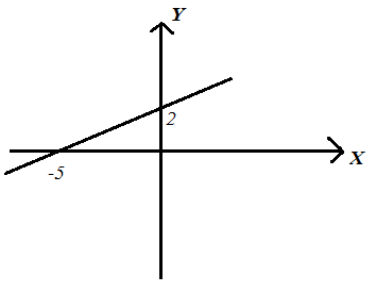
8. Find x correct to one decimal place **1**



9. Find x correct to the nearest minute **1**



10. Find the gradient of this line: **1**

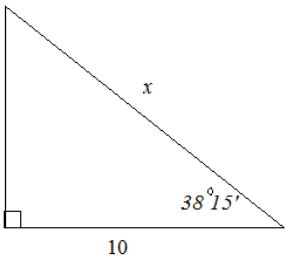


11. If the midpoint of AB is the point $(-1,3)$, and B is $(-5,7)$, find the coordinates of A.

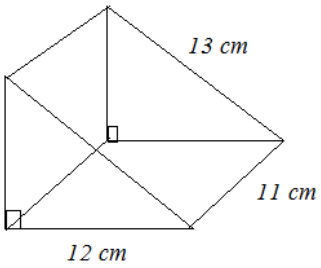
2

Section B (15 Marks)

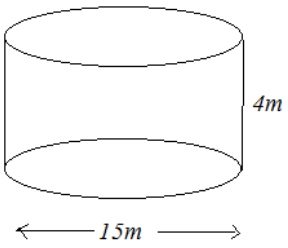
1. Find the value of x correct to one decimal place: **1**



2. Find the volume of this solid **2**



3. Find the surface area of this solid in terms of π **2**



4. Simplify: $\frac{3mn}{4} - \frac{2mn}{6}$ **2**

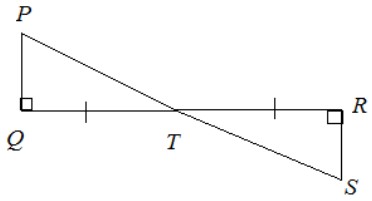
5. Simplify: **4**

a) $3\sqrt{12}+\sqrt{27}$

b) $(3x^3y^{-2})^4$ and write with positive indices.

6. Convert 6400 m^2 to cm^2 **1**

7. Given that QT = TR and $\angle PQT = \angle SRT = 90°$ **3**



Prove that

a) $\Delta PQT \equiv \Delta SRT$ AND

b) PT = ST

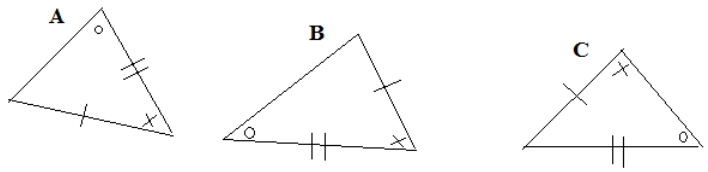
Section C (15 Marks)

1. Rationalise the denominator **3**

a) $\frac{7}{3\sqrt{5}}$

b) $\frac{\sqrt{2}}{3+\sqrt{2}}$

2. Which triangles are congruent in this group of three? State the congruence test you used. **1**



3. Find the length of AB if A = $(-2,5)$ and B = $(4,-2)$ **1**

4. Simplify: $\frac{x}{3}-\frac{x-3}{2}-\frac{5-2x}{5}$ **3**

5. Find the equation of the line with a gradient of 3 passing through the point $(2,-4)$. Express your answer in general form. **2**

6. A student sits for three subjects. The probability of him passing English is 0.8, the probability of him passing Science is 0.6 and the probability of him passing maths is 0.7. Find

a) the probability of passing all three subjects **1**

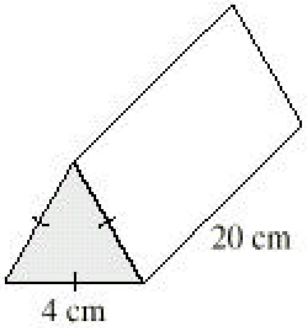
b) the probability of passing two subjects **2**

7. Solve $\frac{1}{1-x}-1=6$ **2**

Section D (15 Marks)

1. Find the equation of the line passing through the point $(4,-1)$ which is perpendicular to the line $2y+7=4x$ **2**

2. A chocolate bar is packed in a box that has the shape of a triangular prism, as shown.



Find the surface area of the box. **2**

3. Solve: **4**

a) $\frac{3n-2}{8}+\frac{5-2n}{2}=-2$

b) Solve: $5(2-3x) \geq 2x-23$

4. An observer in a tower 65 metres high sees a bushfire due west at an angle of depression of 4^o. A second person, due east of the tower, spots the observer in the tower at an angle of elevation of 8^o. How far is this second person from the bushfire (to the nearest metre)?

3

5. Factorise the following expressions: **4**

a) $a^2-b^2-(a-b)^2$

b) $6pq-4p-9q+6$

Section E (15 Marks)

1. Write with a positive index: **3**

a) $\left(\frac{2}{x}\right)^{-2}$

b) $(3x^2y)^4 \times \left(\frac{3x}{2y^{-2}}\right)^{-3}$

2. Factorise the following expressions **7**

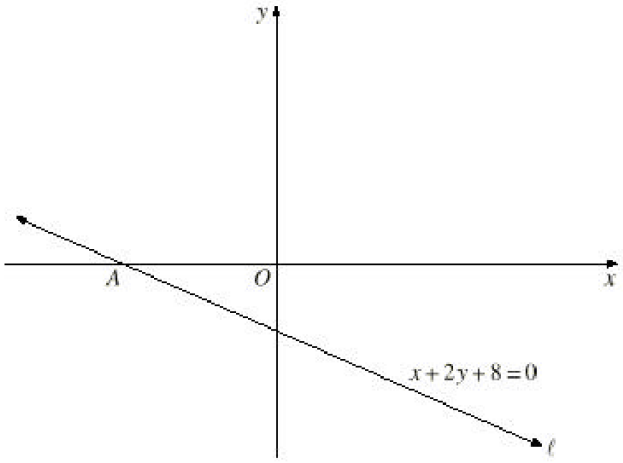
a) $98a - 72ab^2$

b) $10x^2 - 19x + 6$

c) $(3a-b)^2 - (a+b)^2$

3. The line *l* is shown in the diagram. It has equation $x + 2y + 8 = 0$ and cuts the X-axis at A.

The line *k* has equation $y = -\frac{1}{2}x + 6$, and is not shown on the diagram.



(a) Find the coordinates of A. **1**

(b) Explain why *k* is parallel to *l*. **1**

(c) Draw the graph of line *k* on the diagram, indicating where it cuts the axes. **1**

(d) Write down a pair of inequalities which define the region between the lines *k* and *l*. **2**

Section F (15 Marks)

1. What is the size of each angle in a regular octagon? **2**

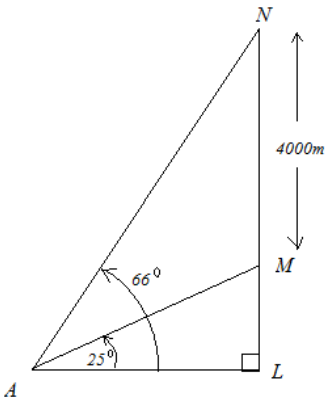
2. If $x = \sqrt{\frac{x+y}{y}}$, make y the subject of the formula. **2**

3. Make a the subject of the formula: **2**

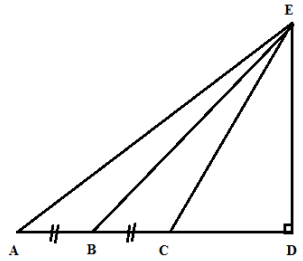
$$\frac{1}{a} = \frac{a}{m} - \frac{a}{n}$$

4. An aeroplane flies due north for 200km and then turns due west and flies for a further distance of 325km. Determine the bearing, to the nearest minute, of the aeroplane from its starting point. **2**

5. A rocket launched vertically from L is observed from A. Soon after launch, when at position M its angle of elevation is 25°. After it climbs 4000m from this position to N its angle of elevation is 66°. Find how far the observer is from the launching pad. **3**



6. In the diagram shown AB = BC

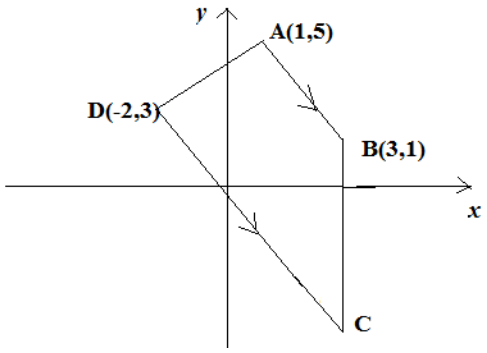


a) Prove $BD = \frac{1}{2}(AD + CD)$ **2**

b) Hence show $\tan \angle DEB = \frac{1}{2}(\tan \angle AED + \tan \angle CED)$ **2**

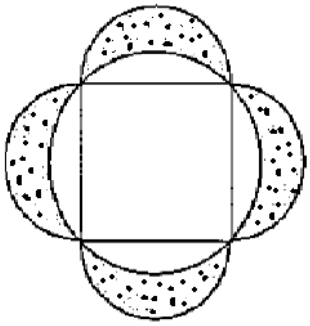
Section G (15 Marks)

1. ABCD is a trapezium. AB is parallel to DC. BC is parallel to the y-axis.

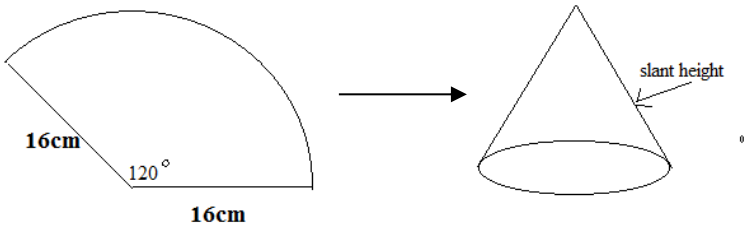


Find the co-ordinates of the point C. **3**

2. A square of side a is inscribed in a circle, and semi-circles are constructed on its sides as shown. Find the total area of the four lunes (the shaded area). **3**



3. A cone is to be formed by joining the radii of the sector shown. **4**



In the cone that is formed, find as an exact answer,

a) the slant height

b) the radius

c) the perpendicular height of the cone.

4. (a) By grouping the numbers from 1 to 100 into pairs that give the same sum , show that the sum of the integers $1 + 2 + 3 + 4 + ... + 100$ is 5050. **2**

(b) Hence, or otherwise, find the sum of all the fractions between 0 and 1 that have denominators between 2 and 100 inclusive:

$$\frac{1}{2}+\frac{1}{3}+\frac{2}{3}+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+...+\frac{98}{100}+\frac{99}{100}$$

3

End of Test

Extra Working Page

Extra Working Page

Extra Working Page

Year 9 Yearly 2014 - Solutions

Section A (15 Marks)

1. Calculate -2^4 **1**

-16

2. Solve $3-x>-2$ **1**

$-x > -5$

$x < 5$

$x < 5$

3. Write 0.010031 correct to 3 significant figures. **1**

1.00×10^{-2}

0.0100

4. Write 2618000 in scientific notation. **1**

2.618×10^6

5. Simplify $\frac{\sqrt{45}}{15}$ **1**

$= \frac{3\sqrt{5}}{15}$

$= \frac{\sqrt{5}}{5}$

$\frac{1}{\sqrt{5}}$

6. Write with a radical sign and evaluate: $\left(81^{\frac{1}{4}}\right)^3$ **1**

$\left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

7. Factorise **4**

a) $24ry^3 - 42r^3y^2$

$6ry^2(4y - 7r^2)$

b) $x^2 - 11x + 24$

$x \times -8$

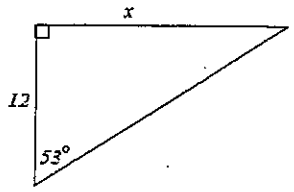
$x \times -3$

$(x-8)(x-3)$

c) $5x^2 - 20$

$5(x-2)(x+2)$

8. Find x correct to one decimal place **1**

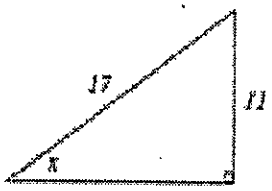


$\frac{x}{12} = \tan 53^\circ$

$x = 12 \tan 53^\circ$

$x = 15.9$

9. Find x correct to the nearest minute **1**

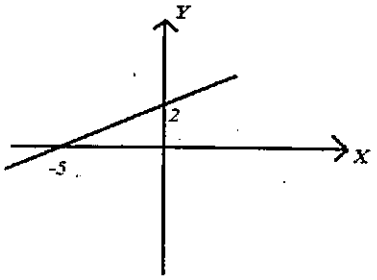


$\sin x = \frac{11}{17}$

$x = \sin^{-1}\frac{11}{17}$

$x = 40^\circ 19'$

10. Find the gradient of this line: **1**



$m = \frac{2}{5}$

11. If the midpoint of AB is the point $(-1,3)$, and B is $(-5,7)$, find the coordinates of A. **2**

$-1 = \frac{x-5}{2}$

$-2 = x - 5$

$x = 3$

$3 = \frac{y+7}{2}$

$6 = y + 7$

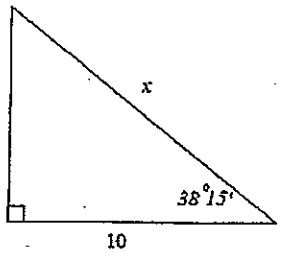
$0 = y + 1$

$y = -1$

$\therefore A(3,-1)$

Section B (15 Marks)

1. Find the value of x correct to one decimal place: 1

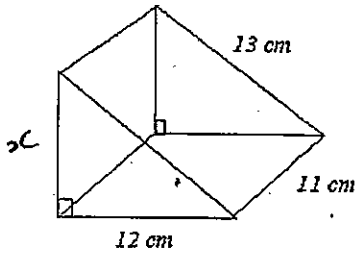


$\cos 38^\circ 15' = \frac{10}{x}$

$x = \frac{10}{\cos 38^\circ 15}$

$= 12.7$

2. Find the volume of this solid 2



Let height $= x$

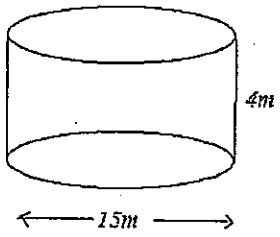
$x^2 + 12^2 = 13^2$

$x = 5$

Volume $= \frac{1}{2}(12 \times 5) \times 11$

$= 330\ cm^3$

3. Find the surface area of this solid in terms of π 2



$SA = 2\pi r^2 + 2\pi r h$

$= 2\pi\left(\frac{15}{2}\right)^2 + 2\pi\left(\frac{15}{2}\right)4$

$= \pi\left(\frac{225}{2} + 60\right)$

$= 172\frac{1}{2}\pi\ m^2$

4. Simplify: $\frac{3mn}{4} - \frac{2mn}{6}$ 2

$= \frac{9mn - 4mn}{12}$

$= \frac{5mn}{12}$

5. Simplify: 4

a) $3\sqrt{12}+\sqrt{27}$

$= 3\sqrt{4\times3} + \sqrt{9\times3}$

$= 6\sqrt{3} + 3\sqrt{3}$

$= 9\sqrt{3}$

b) $(3x^3y^{-2})^4$ and write with positive indices.

$= 3^4 x^{12} y^{-8}$

$= \frac{3^4 x^{12}}{y^8}$

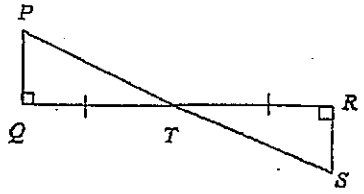
6. Convert 6400 m^2 to cm^2 1

$1\ m = 100\ cm$

$1\ m^2 = 10,000\ cm^2$

$\therefore 6400\ m^2 = 64,000,000\ cm^2$

7. Given that QT = TR and $\angle PQT = \angle SRT = 90°$ 3



Prove that

a) $\Delta PQT \equiv \Delta SRT$ AND

b) PT = ST

a) $P\hat{Q}T = S\hat{R}T = 90°$ GIVEN

$QT = RT$ GIVEN

$P\hat{T}Q = R\hat{T}S$ VERT. OPP ANGLES

$\therefore \Delta PQT \equiv \Delta SRT$ (ASA)

b) $PT = ST$ COR. SIDES OF CONGRUENT TRIANGLES

Section C (15 Marks)

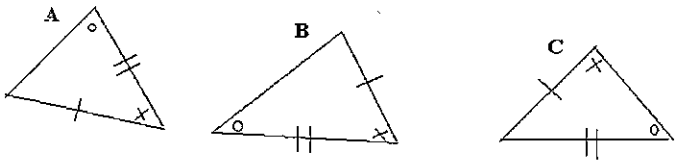
1. Rationalise the denominator **3**

a) $\frac{7}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{15}$

b) $\frac{\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3\sqrt{2}-2}{9-3\sqrt{2}+3\sqrt{2}-2}$

$= \frac{3\sqrt{2}-2}{7}$

2. Which triangles are congruent in this group of three? State the congruence test you used. **1**



$\Delta A \equiv \Delta B$ (SAS or AAS)

3. Find the length of AB if A = (–2,5) and B = (4,–2) **1**

$d_{AB} = \sqrt{(4-(-2))^2+((-2)-5)^2}$

$= \sqrt{36+49}$

$= \sqrt{85}$

4. Simplify: $\frac{x}{3}-\frac{x-3}{2}-\frac{5-2x}{5}$ **3**

$= \frac{10x-15(x-3)-6(5-2x)}{30}$

$= \frac{10x-15x+45-30+12x}{30}$

$= \frac{7x+15}{30}$

5. Find the equation of the line with a gradient of 3 passing through the point $(2,-4)$. Express your answer in general form. **2**

$y - y_1 = m(x - x_1)$ $\therefore 3x - y - 10 = 0$

$y - (-4) = 3(x - 2)$

$y + 4 = 3x - 6$

6. A student sits for three subjects. The probability of him passing English is 0.8, the probability of him passing Science is 0.6 and the probability of him passing maths is 0.7. Find

a) the probability of passing all three subjects **1**

$P(\text{passing all three}) = 0.8 \times 0.6 \times 0.7$

$= 0.336 \quad (42/125)$

b) the probability of passing two subjects **2**

$P(\text{passing two}) = 0.8 \times 0.6 \times 0.3$

$+ 0.8 \times 0.4 \times 0.7$

$+ 0.2 \times 0.6 \times 0.7$

$= 0.452 \quad (113/250)$

7. Solve $\frac{1}{1-x} - 1 = 6$ $x \neq 1$ **2**

$\frac{1}{1-x} = 7$

$1 = 7(1 - x)$

$1 = 7 - 7x$

$-6 = -7x$

$x = \frac{6}{7}$

Section D (15 Marks)

1. Find the equation of the line passing through the point $(4,-1)$ which is perpendicular to the line $2y+7=4x$ 2

$m = \frac{-1}{2}$

$y+1 = -\frac{1}{2}(x-4)$

$2y+2 = -x+4$

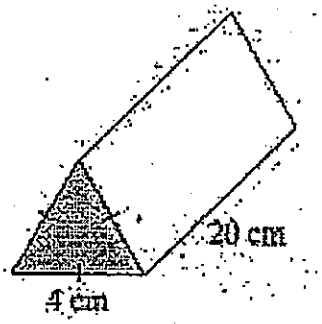
$\boxed{x+2y-2=0}$

or

$y = -\frac{1}{2}x + \frac{2}{2}$

$y = -\frac{1}{2}x + 1$

2. A chocolate bar is packed in a box that has the shape of a triangular prism, as shown.



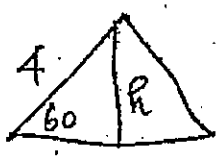
Find the surface area of the box. 2

$\frac{1}{2} \times 16 \times \frac{\sqrt{3}}{2} \times 2$

$+ 20 \times 4 \times 3$

$= 8\sqrt{3} + 240$

$\boxed{A = 8(\sqrt{3}+30) \text{ or } 4(\sqrt{12}+60)}$ 253.856



$\frac{h}{4} = \frac{\sqrt{3}}{2}$

$h = 2\sqrt{3}$

3. Solve: 4

a) $\frac{3n-2}{8} + \frac{5-2n}{2} = -2$

$3n-2+4(5-2n) = -16$

$-5n+18 = -16$

$-5n = -34$

$n = \frac{34}{5}$

6.8

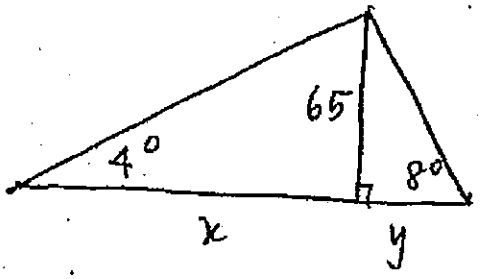
b) Solve: $5(2-3x) \geq 2x-23$

$10-15x \geq 2x-23$

$33 \geq 17x$

$x \leq \frac{33}{17}$

4. An observer in a tower 65 metres high sees a bushfire due west at an angle of depression of 4°. A second person, due east of the tower, spots the observer in the tower at an angle of elevation of 8°. How far is this second person from the bushfire (to the nearest metre)? 3



$\frac{65}{y} = \tan 8°$

$y = \frac{65}{\tan 8°} = 462.499$

$x = \frac{65}{\tan 4°} = 929.543$

$\therefore x+y = 1392$ m.

5. Factorise the following expressions: 4

a) $a^2-b^2-(a-b)^2$

$(a+b)(a-b) - (a-b)^2$

$= (a-b)(a+b-a+b)$

$= 2b(a-b)$

b) $6pq-4p-9q+6$

$2p(3q-2) - 3(3q-2)$

$= (2p-3)(3q-2)$.

SECTION E

1a $\left(\frac{2}{x}\right)^{-2} = \left(\frac{x}{2}\right)^{2}$

$= \frac{x^2}{4}$

(NO PART MARKS)

1b) Consider

$\left(\frac{3x}{2y^{-2}}\right)^{-3} = \left(\frac{3xy^2}{2}\right)^{-3}$

$= \left(\frac{2}{3xy^2}\right)^{3}$

$= \frac{8}{27x^3y^6}$

Then $(3x^2y)^4 \times \left(\frac{3x}{2y^{-2}}\right)^{-3}$

$= 81x^8y^4 \times \frac{8}{27x^3y^6}$

$= \frac{24x^5}{y^2}$ (NO PART MARKS)

2a) $98a - 72ab^2$

$= 2a(49 - 36b^2)$

$= 2a(7-6b)(7+6b)$

b) $10x^2 - 19x + 6 = (5x-2)(2x-3)$ (NO PART MARKS)

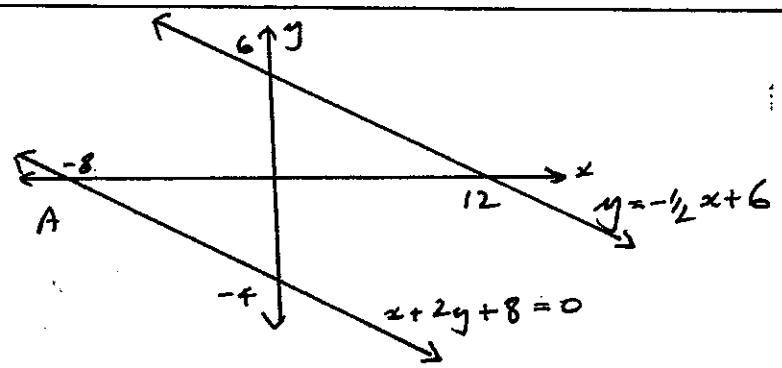
c) $(3a-b)^2 - (a+b)^2$

$= (3a-b+a+b)(3a-b-a-b)$

$= 4a(2a-2b)$

$= 8a(a-b)$

3



a) A(-8, 0)

b) Both l & k have gradients of $-\frac{1}{2}$

c) As per diagram

d) (x, y):

$y < -\frac{1}{2}x + 6 \cap x + 2y + 8 > 0$

Section F (15 Marks)

1. What is the size of each angle in a regular octagon? **2**

$$\frac{(n-2)\times 180^\circ}{n}$$

$$= \frac{(8-2)\times 180^\circ}{8} = 135^\circ$$

2. If $x=\sqrt{\frac{x+y}{y}}$, make y the subject of the formula. **2**

$$x^2 = \frac{x+y}{y}$$

$$x^2 y = x+y$$

$$x^2 y - y = x$$

$$y(x^2-1) = x$$

$$\therefore y = \frac{x}{x^2-1}$$

3. Make a the subject of the formula: **2**

$$\frac{1}{a}=\frac{a}{m}-\frac{a}{n}$$

$$\frac{1}{a} = \frac{an-am}{mn}$$

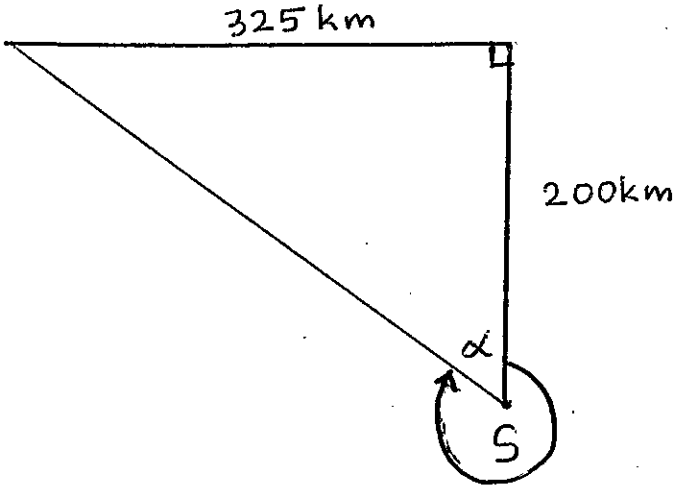
$$1 = \frac{a^2n-a^2m}{mn}$$

$$mn = a^2(n-m)$$

$$\frac{mn}{n-m} = a^2$$

$$\therefore a = \pm\sqrt{\frac{mn}{n-m}}$$

4. An aeroplane flies due north for 200km and then turns due west and flies for a further distance of 325km. Determine the bearing, to the nearest minute, of the aeroplane from its starting point. 2



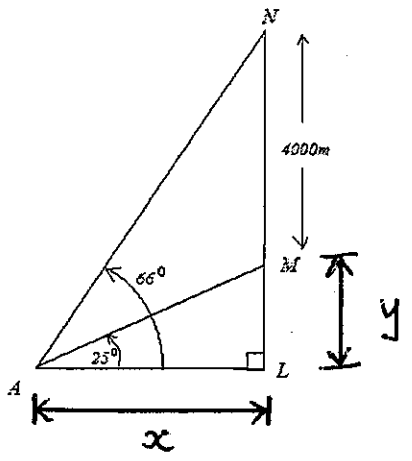
$\tan\alpha = \frac{325}{200}$

$\alpha = 58°24'$

$360° - \alpha = 301°36'$

∴ bearing of aeroplane from starting point is $301°36'$

5. A rocket launched vertically from L is observed from A. Soon after launch, when at position M its angle of elevation is 25°. After it climbs 4000m from this position to N its angle of elevation is 66°. Find how far the observer is from the launching pad. 3



$\tan 25° = \frac{y}{x}$

$\therefore y = x\tan 25°$ ——— (1)

$\tan 66° = \frac{4000 + y}{x}$

$x\tan 66° = 4000 + y$

$\therefore y = x\tan 66° - 4000$ ——— (2)

equating (1) and (2),

$x\tan 25° = x\tan 66° - 4000$

$x\tan 25° - x\tan 66° = -4000$

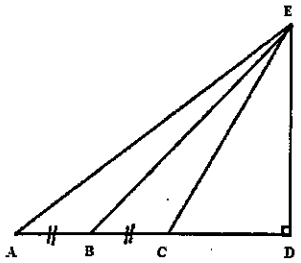
$x(\tan 25° - \tan 66°) = -4000$

$x = \frac{-4000}{\tan 25° - \tan 66°}$

$= 2247.533...$

∴ observer is 2248 m from the launching pad (to the nearest metre)

6. In the diagram shown AB = BC



a) Prove $BD = \frac{1}{2}(AD + CD)$ 2

Let x = AB = BC

y = CD

$$\begin{aligned} RHS &= \tfrac{1}{2}(2x + y + y) \\ &= \tfrac{1}{2}(2x + 2y) \\ &= x + y \\ &= BC + CD \\ &= BD \\ &= LHS \end{aligned}$$

b) Hence show $\tan \angle DEB = \frac{1}{2}(\tan \angle AED + \tan \angle CED)$ 2

Let DE = h

$$\begin{aligned} RHS &= \tfrac{1}{2}(\tan \angle AED + \tan \angle CED) \\ &= \tfrac{1}{2}\left[\frac{2x+y}{h} + \frac{y}{h}\right] \\ &= \tfrac{1}{2}\left[\frac{2x+2y}{h}\right] \\ &= \frac{x+y}{h} \\ &= \tan \angle DEB \\ &= LHS \end{aligned}$$

(4)

(i) Gradient of AD is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{1-2} = \frac{2}{3}$.

Gradient of AB is $m = \frac{1-5}{3-1} = \frac{-4}{2} = -2$.

Told AB || DC so DC has $m = -2$.

Eqn of DC is $(y - y_1) = m(x - x_1)$

using $m = -2$, $D(-2, 3)$

$y - 3 = -2(x + 2)$

$y - 3 = -2x - 4$

$y = -2x - 1$

since C lies on $y = -2x - 1$,

C $(3, y)$ from data

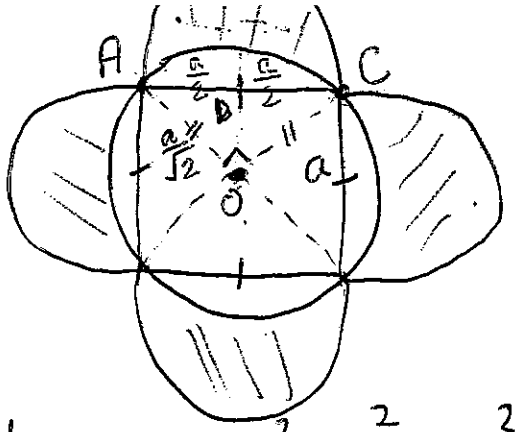
so $y = -2 \times 3 - 1 = -7$

C $(3, -7)$ (3)

check using D $(-2, 3)$ D $(3, -7)$

$m = \frac{-7-3}{3-2} = \frac{-10}{5} = -2$ ✓

(2)



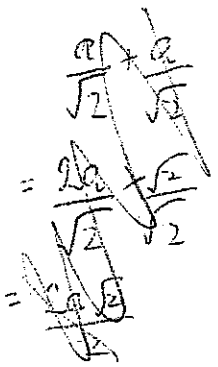
area of 4 lunes (shaded area).

By Pythagoras $x^2 + x^2 = a^2$

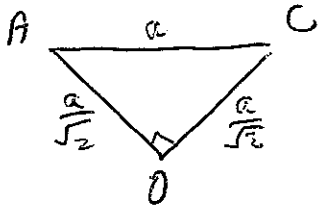
$2x^2 = a^2$

$x^2 = \frac{a^2}{2}$

$x = \frac{a}{\sqrt{2}}$



radius of inner circle $\frac{a}{\sqrt{2}}$



length of side of square is 'a', this is the diameter of the lune.

So radius of semicircle (or lune) is $\frac{1}{2} \times a = \frac{a}{2}$

area of 1 lune is area of ① semicircle ABCD + area △ ② OAC − area of ③ quarter circle (sector AOC)

① $A = \frac{1}{2} r^2 \pi = \frac{1}{2} \times \pi \times \left(\frac{a}{2}\right)^2 = \frac{\pi}{2} \times \frac{a^2}{4} = \frac{\pi a^2}{8}$

② $A = \frac{1}{2} bh = \frac{1}{2} \times \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{4}$

③ $A = \frac{1}{4} r^2 \pi = \frac{1}{4} \times \left(\frac{a}{\sqrt{2}}\right)^2 \pi = \frac{\pi}{4} \times \frac{a^2}{2} = \frac{\pi a^2}{8}$

area of 4 lunes $= 4 \times \left[\frac{\pi a^2}{8} + \frac{a^2}{4} - \frac{\pi a^2}{8} \right]$

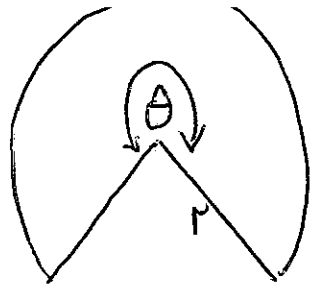
PTO

$$4 \times \left[\frac{\pi a^2}{8} + \frac{2a^2}{8} - \frac{\pi a^2}{8} \right]$$

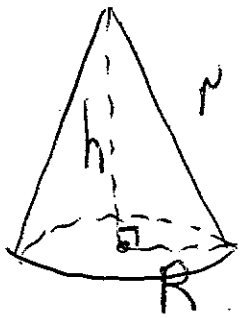
$$= \frac{8a^2}{8} = a^2 \quad u^2 \; //$$

(3)

(3)



⇒



① ②

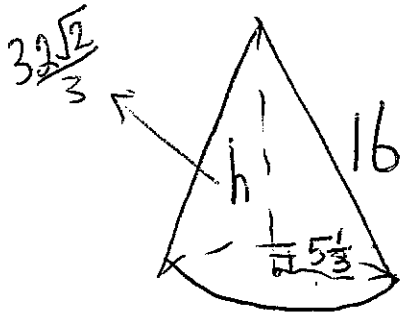
arc length of ① = circumference of base circle ②

$$\frac{\theta}{360} \times 2\pi r = 2\pi R$$

$$R = \frac{2\pi r\theta}{2\pi\, 360} \qquad R = \frac{r\theta}{360^\circ}$$

(b) So in ② $R = 16 \times \frac{120^\circ}{360^\circ}$

$= 16 \times \frac{1}{3} = 5\frac{1}{3}$ cm. (1)



(a) 16 cm (1)

(c) $h^2 + \left(\frac{16}{3}\right)^2 = 16^2$

$h^2 + \frac{256}{9} = 256$

$h^2 = 256 - \frac{256}{9}$

$= \frac{2304 - 256}{9}$

$= \frac{2048}{9}$

$h = \frac{\sqrt{2048}}{3} = \frac{\sqrt{8 \times 256}}{3}$

$$\frac{2\sqrt{2} \times 16}{3} = \frac{32\sqrt{2}}{3} \text{ cm}$$ (2)

$$\fallingdotseq 15.08 \text{ cm}.$$

4 (a) $1+2+3+4+5+\cdots\cdots+96+97+98+99+100$

Pair to give same sum:

$1+100 = 101$
$2+99 = 101$
$3+98 = 101$
$4+97 = 101$
$5+96 = 101$
etc.

How many pairs?

$10+10+10+10+10$

$50 \times 101 = 5050$

(2)

✓ $10+91$
⋮
✓ $20+81$
⋮
✓ $30+71$
⋮
✓ $40+61$
⋮
$50+51$

(b) $\frac{1}{2} \rightarrow \frac{1}{2}$ ✓

$\frac{1}{3} + \frac{2}{3} \rightarrow \frac{3}{3} = 1$ ✓

$\frac{1}{4} + \frac{2}{4} + \frac{3}{4} \rightarrow \frac{6}{4}$ $(\frac{3}{2})$ ✓

$\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} \rightarrow \frac{10}{5}$ (2) ✓

$\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} \rightarrow \frac{15}{6}$ $(2\frac{1}{2})$ ✓

$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7} \rightarrow \frac{21}{7}$ (3)

$\frac{1}{8} + \frac{2}{8} + \cdots \frac{7}{8} = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$ $(3\frac{1}{2})$

$\frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \cdots + \frac{99}{100} \rightarrow \frac{4950}{100}$

二 三 四 五 六 七 八 九 十

Sum $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + 3 + 3\frac{1}{2} + 4 + 4\frac{1}{2}$

Fractions denom.	sum
2	$\frac{1}{2}$
3	1
4	$1\frac{1}{2}$
5	2
6	$2\frac{1}{2}$
~~7~~	3
~~8~~	$3\frac{1}{2}$
9	4
10	$4\frac{1}{2}$
100	$49\frac{1}{2}$

$n = 99$ sums

first sum $\frac{1}{2}$

adding to each sum $\frac{1}{2}$

sum pairs $\frac{99}{2}\left(2 \times \frac{1}{2} + (99-1) \times \frac{1}{2}\right)$

$= \frac{99}{2}(1 + 49)$

$= 2475$ ✓(3)