NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2020 YEAR 9 YEARLY EXAMINATION

Student Name:_____

Class: _____ **Teacher**_____

General Instructions

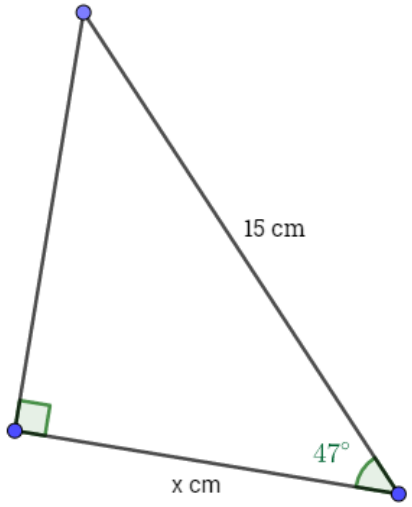
- Working time – **90 minutes**
- Answer questions in the spaces provided.
- Write using blue or black pen.
- **Board approved** calculators are allowed.
- All necessary working should be shown in every question.
- Start each new **question** on a **new page.**
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Section	1 Trig	2 Alg	3 Geo	4 Surds/ Indices	5 Data	6 Eqns	7 Quad	8 SA & V	9 Linear Rel~ns	10 $$$	Total	Total
Mark	/10	/16	/8	/15	/13	/13	/9	/12	/12	/12	/120	/100

Question 1 – Trigonometry (10 Marks)

(a) Calculate the size of *x* to 2 significant figures.



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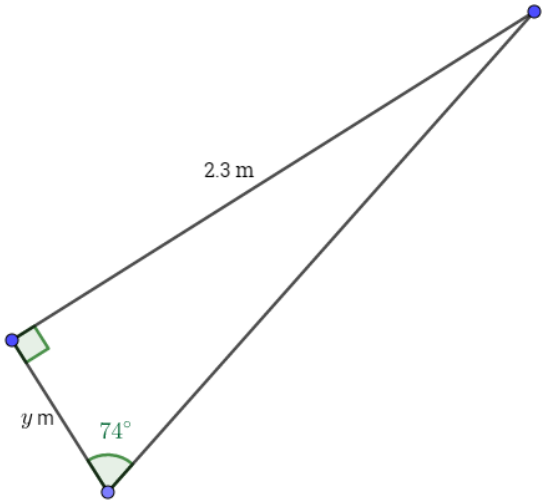
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(b) Calculate the size of *y* to 2 decimal places.



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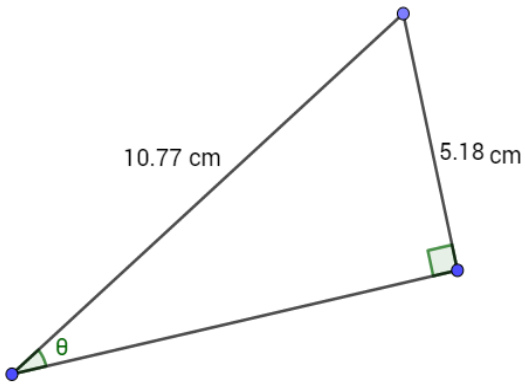
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(c) Solve for θ to the nearest minute.



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(d) Solve for *x*.

$\sin(3x - 10°) = \cos(x + 20°)$

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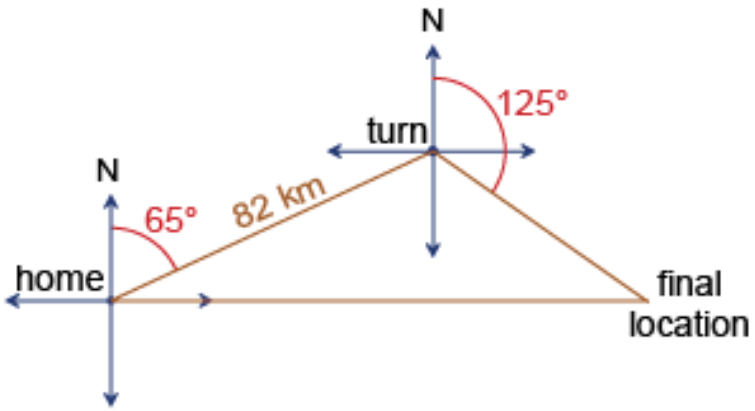
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(e) A cyclist rode 82 km from home one a bearing of 065 before changing direction and cycling on a bearing of 125 until he was due East of his home.



Calculate the distance of the cyclist from his home.

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Question 2 – Algebraic Techniques (16 Marks)

(a) Expand and simplify $(2x - 1)(5x + 2)$

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(b) Factorise fully:

(i) $18x^2 - 50y^2$

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(ii) $3pq + 6pr + 2rq + 4r^2$

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…… **2**

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(iii) $5x^2 + 7x + 2$

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(iv) $a^2 - b^2 + 4bc - 4c^2$

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…… **2**

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(c) Fully simplify:

(i) $\dfrac{x^2+x-6}{x^2+6x+9} \times \dfrac{x^2-9}{x-2}$

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(ii) $\dfrac{a}{x^2+2x+1} + \dfrac{a}{x^2-1}$

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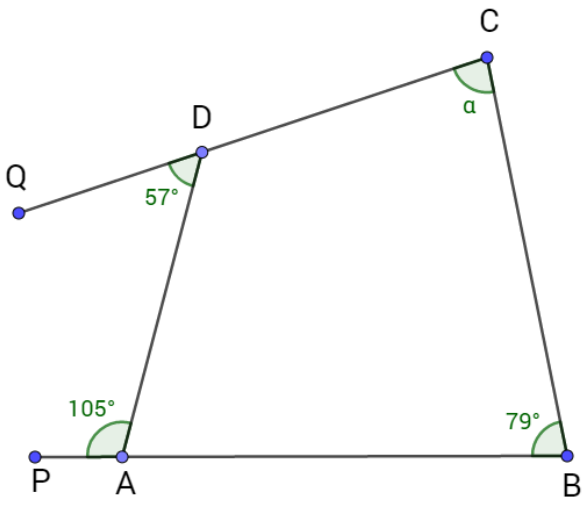
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Question 3 – Properties of Geometric Figures (8 Marks)

(a) Calculate α , giving reasons.



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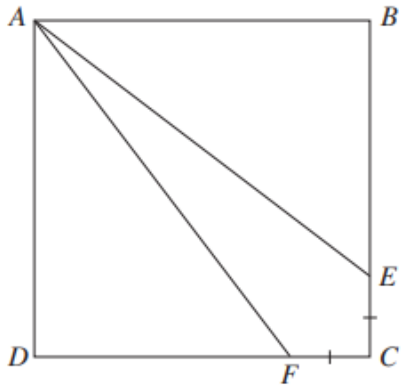
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(b) In the diagram below *ABCD* is a square with side lengths 14 cm. Point *E* is chosen on *BC* and the point *F* is chosen on *CD* such that $EC = FC = 4$ cm.



(i) Prove that ΔADF is congruent to ΔABE.

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(ii) Find the area of *AECF*.

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<u>Question 4 – Surds and Indices & Number in STEM (15 Marks)</u>

(a) Fully simplify each of the following:

(i) $\sqrt{200} + \sqrt{50} + \sqrt{8} + \sqrt{2}$

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(ii) $(\sqrt{3}-\sqrt{2})(\sqrt{8}+\sqrt{3})$

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(b) Rationalise the denominators in each of the following:

(i) $\dfrac{2}{\sqrt{5}}$

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(ii) $\dfrac{1}{\sqrt{2}+\sqrt{3}}$

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(c) Write the following without fractional or negative indices:

(i) x^{-2}

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(ii) $y^{\frac{7}{3}}$

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(iii) $\left(\frac{a^{-2}b^{6}}{27ab^{3}}\right)^{\frac{2}{3}}$

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(d) Express in index form.

$$\frac{1}{5\sqrt[4]{z^{3}}}$$

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2

(e) Avogadro's constant gives us that there are approximately 6.022×10^{23} atoms in a mole of any substance.
If 1 mole of carbon weighs 12.01 grams, calculate the mass of 1 atom of carbon (answer in scientific notation).

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Question 5 – Single Variable Data Analysis (13 Marks)

(a) The stem and leaf plot below shows 11 scores.

Stem	Leaf
3	1 4 7
4	2 5
5	1 6 6 9
6	3 8

(i) Represent this information with a box and whisker diagram. **4**

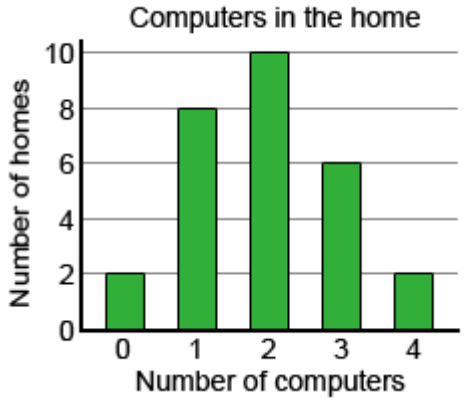
(ii) Calculate the interquartile range.

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(b)



Determine the mean number of computers in a home from the data in this graph.
Give your answer correct to 1 decimal place.

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(c) A group of students sat for an exam (out of 40 marks). The results are shown in the table below.

Class	Class centre	Frequency	Cumulative Frequency
20-24	22	5	
25-29	27	8	
30-34	32	7	
35-39	37	3	

(i) What was the modal class?

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(ii) Complete the cumulative frequency column in the above table. **1**

(iii) In the space below draw a cumulative frequency histogram and polygon (ogive).

2

(iv) Use your diagram in part (iii) to estimate the median of these scores.

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(d) 4.2 7.0 8.2 8.9 9.7 10.6 12.4

When a new score was included in this set the median changed to 9.2. What was the new score?

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Question 6 – Linear Equations, Inequations and Simultaneous Equations (13 Marks)

(a) Solve the following for *x*:

(i) $2(3x-5)-5(x+2)=2x-28$

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(ii) $\frac{2x-1}{3}+\frac{x+1}{4}=\frac{3x}{5}$

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(b) Make *x* the subject of the following.

$ax+2b=c-\frac{x}{2}$

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(c) Solve the inequation and plot your solution on a number line.

$2-3x>8$

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(d) Solve this pair of simultaneous equations.

$3x + 2y = 20$

$4x - 5y = -27$

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Question 7 – Quadratic Equations (9 Marks)

Solve each of the following for *x*:

(a) $x^2 + 5x = 0$ *(solve by factorising)*

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(b) $x^2 + 12x + 5 = 0$ *(solve by completing the square)*

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(c) $3x^2 - 8x - 6 = 0$ *(solve using the quadratic formula)*

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(d) $(x + 2)^4 = 7(x + 2)^2 + 18$

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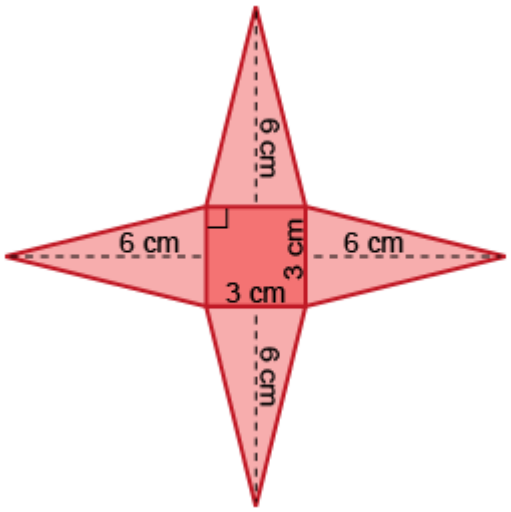
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Question 8 – Surface Area and Volume (12 Marks)

(a) The figure below is the net of a square pyramid.



Calculate:

(i) The surface area.

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(ii) ……

The perpendicular height.

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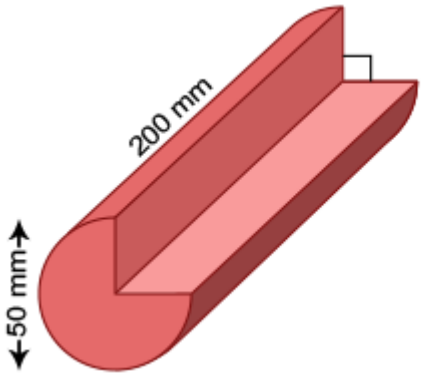
(ii) The volume.

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(b)



For the object above, calculate:

(i) The volume.

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(ii) The surface area.

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(c) The surface area of a right cone is $800\ \text{cm}^2$. Its slant height is three times the radius. Determine the radius correct to the nearest centimetre.

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Question 9 - Linear Relationships (12 Marks)

(a) An interval *AB* is defined by the points $A(-1, 4),\ \ B(5, -2)$.

(i) What is the exact length of AB?

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(ii) What is the midpoint of AB?

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(iii) What is the gradient of AB?

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(iv) By finding the equation of *AB*, or otherwise, find the *x* and *y* intercepts of *AB.*

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(b) Find the equation of the line through $(1, 2)$ which is perpendicular to the line $2x - y + 4 = 0$

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(c) Sketch the region on the number plane where the following inequations are both true

$$x > 2$$
$$x - y \leq 3$$

3

Question 10 – Earning and Investing Money (12 Marks)

(a) The tables below shows the salaries for teachers in public schools in NSW and the 2020-21 rates of income tax.

Band/Level of Accreditation	Salary from the first pay period to commence on or after 1.1.2020 $
Increase	2.5%
Band 1(Graduate)	70,652
Band 2(Proficient)	85,214
Band 2.1	92,492
Band 2.2	96,138
Band 2.3	105,376
Band 3 (Highly Accomplished/ Lead)	112,163

Resident tax rates 2020–21

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $45,000	19 cents for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,001 and over	$51,667 plus 45 cents for each $1 over $180,000

Mr Smythe is a teacher on Band 2.3.

(i) Use the tables above to calculate the amount of income tax that Mr Smythe must pay *(you may disregard the medicare levy for this question).*

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(ii) Calculate the amount of tax Mr Smythe pays as a percentage of his gross salary.

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(iii) $1096 is taken from Mr Smythe's paycheck for PAYG taxes each fortnight. Will Mr Smyth receive a refund or a bill at the end of the tax year and how much is the refund/bill for?

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(iv) Mr Smythe is thinking of applying for a promotion to Band 3 (Highly Accomplished/Lead). If he does this what percentage of his gross salary would he now be paying in tax?

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(v) Is it worth it for Mr Smythe to take the promotion? Explain your answer.

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(b) Mrs Jones invests $1000 into an account that pays 3% p.a interest compounded monthly for 5 years.

(i) How much money will she have at the end of the five years?

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(ii) Mr Smythe invests $1000 into an account which pays simple interest. At the end of five years he has the exact same amount of money as Mrs Jones. What was the rate of interest of the account?

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End of Examination

NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2020 YEAR 9 YEARLY EXAMINATION

Student Name: Solutions

Class: _____ **Teacher** _____

General Instructions

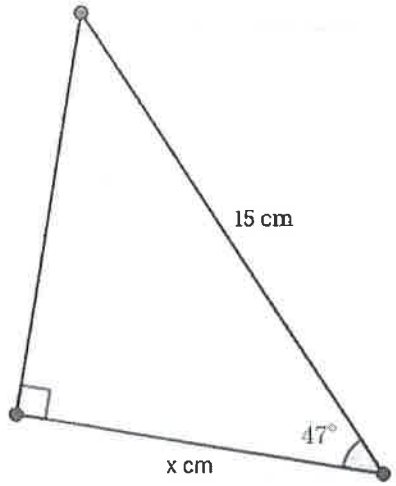
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Question 1 – Trigonometry (10 Marks)

(a) Calculate the size of x to 2 significant figures.



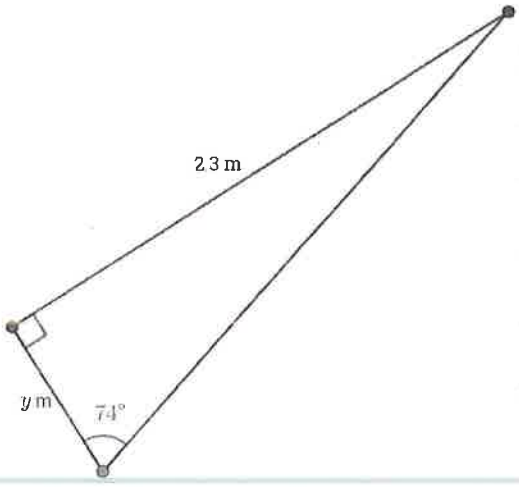
$\cos 47^\circ = x/15$ ✓

$x = 15 \cos 47^\circ$

$= 10.2299754$ ✓

$= 10$ cm (2 s.f) 2

(b) Calculate the size of y to 2 decimal places.



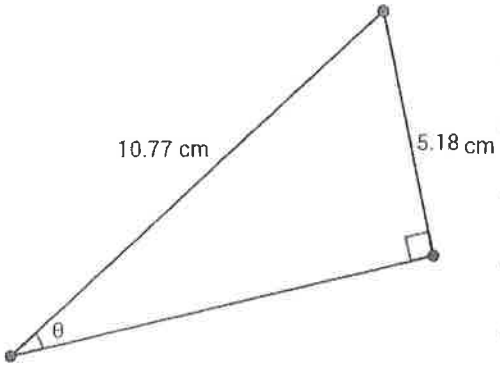
$\tan 74^\circ = 2.3/y$ ✓

$y = 2.3/\tan 74^\circ$

$= 0.659514...$ ✓ 2

$= 0.66$ m (2 d.p)

(c) Solve for θ to the nearest minute.



$\sin\theta = 5.18/10.77$ ✓

$\theta = \sin^{-1}(5.18/10.77)$

$= 28.7484888...$ ✓

$= 28°45'$ 2

(nearest minute)

(d) Solve for x.

$\sin(3x - 10°) = \cos(x + 20°)$

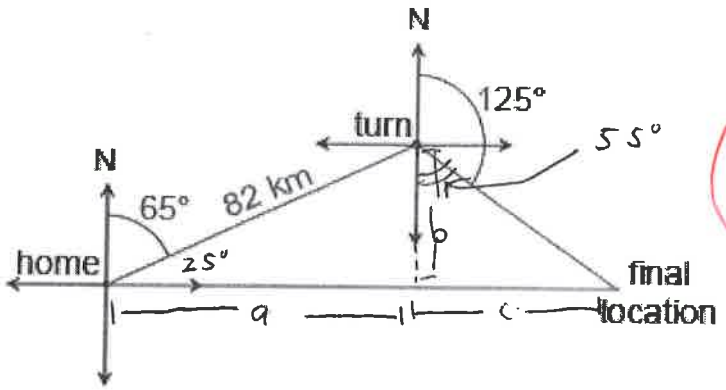
$(3x - 10°) + (x + 20°) = 90°$

$4x + 10° = 90°$

$4x = 80°$ $\therefore x = 20°$ ✓

1

(e) A cyclist rode 82 km from home one a bearing of 065 before changing direction and cycling on a bearing of 125 until he was due East of his home.



✓ finding necessary angles

Calculate the distance of the cyclist from his home.

let a, b & c be as shown

$\cos 25° = \frac{a}{82}$

$\therefore a = 82\cos 25°$ ✓

$\sin 25° = \frac{b}{82}$

$\therefore b = 82\sin 25°$

$\tan 55° = \frac{c}{b}$

$\therefore c = b\tan 55°$

$= 82\sin 25°\tan 55°$

$\therefore a + c = 82\cos 25° + 82\sin 25°\tan 55°$

$= 123.80927556\ldots$ km ✓

3

✓ 10

Question 2 – Algebraic Techniques (16 Marks)

(a) Expand and simplify $(2x-1)(5x+2)$

$10x^2 + 4x - 5x - 2$ ✓

$= 10x^2 - x - 2$ ✓ **2**

(b) Factorise fully:

(i) $18x^2 - 50y^2$

$2(9x^2 - 25y^2)$ ✓

$= 2(3x - 5y)(3x + 5y)$ ✓ **2**

(ii) $3pq + 6pr + 2rq + 4r^2$

$3p(q + 2r) + 2r(q + 2r)$ ✓

$= (q + 2r)(3p + 2r)$ ✓ **2**

(iii) $5x^2 + 7x + 2$

$5x^2 + 5x + 2x + 2$

$= 5x(x + 1) + 2(x + 1)$

$= (x + 1)(5x + 2)$ ✓✓ **2**

(iv) $a^2 - b^2 + 4bc - 4c^2$

$a^2 - (b^2 - 4bc + 4c^2)$ ✓

$= a^2 - (b - 2c)^2$

$= (a - (b - 2c))(a + (b - 2c))$

$= (a - b + 2c)(a + b - 2c)$ ✓ **2**

(c) Fully simplify:

(i) $\frac{x^2+x-6}{x^2+6x+9} \times \frac{x^2-9}{x-2}$

✓ factorising
✓ cancelling
✓ answer

$\frac{(x+2)(x+3)}{(x+3)^2} \times \frac{(x-3)(x+3)}{(x+2)}$

$= x-3$ 3

(ii) $\frac{a}{x^2+2x+1} + \frac{a}{x^2-1}$

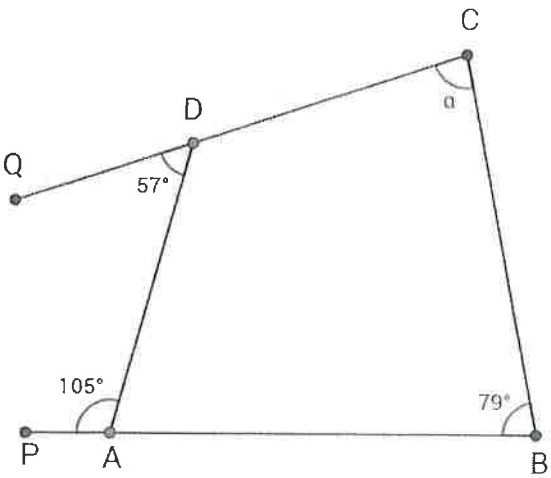
$\frac{a}{(x+1)^2} + \frac{a}{(x-1)(x+1)}$ ✓ $= \frac{ax-a+ax+a}{(x-1)(x+1)^2}$

$= \frac{a(x-1)+a(x+1)}{(x-1)(x+1)^2}$ ✓ $= \frac{2ax}{(x-1)(x+1)^2}$ ✓ 3

16

Question 3 – Properties of Geometric Figures (8 Marks)

(a) Calculate α, giving reasons.



$\angle ADC = 180° - 57°$ (angles on straight line are supplementary)

$= 123°$

$\angle DAB = 180° - 105°$ (as above)

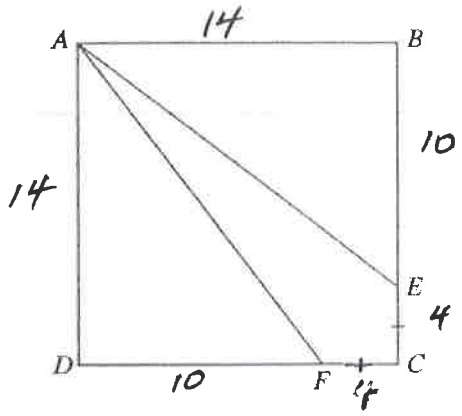
$= 75°$ 2

$\alpha = 360° - 123° - 75° - 79°$

$= 83°$ (angle sum of a quadrilateral)

✓ answer
✓ reasons

(b) In the diagram below *ABCD* is a square with side lengths 14 cm. Point *E* is chosen on *BC* and the point *F* is chosen on *CD* such that $EC = FC = 4$ cm.



(i) Prove that ΔADF is congruent to ΔABE.

In ΔADF & ΔABE

$\angle ADF = \angle ABE$ (90° - vertex of square) ✓

$AD = AB$ (sides of square equal) ✓

$DF = BF$ ($DC - FC = BC - EC$ as $DC = BC$ (square) & $FC = EC$ (given)) ✓

$\therefore \Delta ADF \equiv \Delta ABE$ (SAS) ✓

4

(ii) Find the area of *AECF*.

$AECF = ABCD - ADF - ABE$ ✓

$= (14 \times 14) - \left(\frac{1}{2} \times 14 \times 10\right) - \left(\frac{1}{2} \times 14 \times 10\right)$

$= 196 - 70 - 70$

$= 56\ cm^2$ ✓

2

8

Question 4 – Surds and Indices & Number in STEM (15 Marks)

(a) Fully simplify each of the following:

(i) $\sqrt{200} + \sqrt{50} + \sqrt{8} + \sqrt{2}$

$10\sqrt{2} + 5\sqrt{2} + 2\sqrt{2} + \sqrt{2}$ ✓

$= 18\sqrt{2}$ ✓

2

(ii) $(\sqrt{3}-\sqrt{2})(\sqrt{8}+\sqrt{3})$

$\sqrt{24} + \sqrt{9} - \sqrt{16} - \sqrt{6}$ ✓

$= 2\sqrt{6} + 3 - 4 - \sqrt{6}$

$= \sqrt{6} - 1$ ✓ 2

(b) Rationalise the denominators in each of the following:

(i) $\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{5}}{5}$ ✓ 1

(ii) $\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ ✓

$= \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{2}-\sqrt{3}}{-1}$

$= \sqrt{3}-\sqrt{2}$ ✓ 2

(c) Write the following without fractional or negative indices:

(i) x^{-2}

$\frac{1}{x^2}$ ✓ 1

(ii) $y^{\frac{7}{3}}$

$\sqrt[3]{y^7}$ ✓ 1

(iii) $\left(\frac{a^{-2}b^{6}}{27ab^{3}}\right)^{\frac{2}{3}}$

$= \left(\frac{b^6}{27a^3b^3}\right)^{\frac{2}{3}} = \left(\frac{b^3}{27a^3}\right)^{2/3}$ ✓ $= \left(\frac{b}{3a}\right)^2$ $= \frac{b^2}{9a^2}$ ✓ 2

(d) Express in index form.

$\frac{1}{5\sqrt[4]{z^3}}$ (note: ← looks like $\frac{1}{5^4 \cdot \sqrt{z^3}}$)

✓ negative index
✓ fractional index

$\frac{1}{5}(z)^{-3/4}$ or $5^{-1}z^{-3/4}$ 2

(e) Avogadro's constant gives us that there are approximately 6.022×10^{23} atoms in a mole of any substance.
If 1 mole of carbon weighs 12.01 grams, calculate the mass of 1 atom of carbon (answer in scientific notation).

$12.01 \div 6.022 \times 10^{23}$

$= 1.994 \times 10^{-23}$ (4 s.f.)

✓ answer
✓ scientific notation

2

15

Question 5 – Single Variable Data Analysis (13 Marks)

(a) The stem and leaf plot below shows 11 scores.

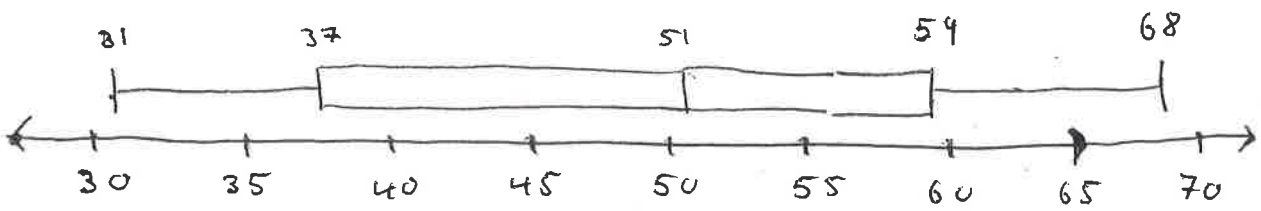
Stem	Leaf
3	(1) 4 (7)
4	2 5
5	(1) 6 6 (9)
6	3 (8)

5 point summary

31, 37, 51, 59, 68

✓ 5 pt summary
✓ number line
✓✓ box plot

(i) Represent this information with a box and whisker diagram. 4

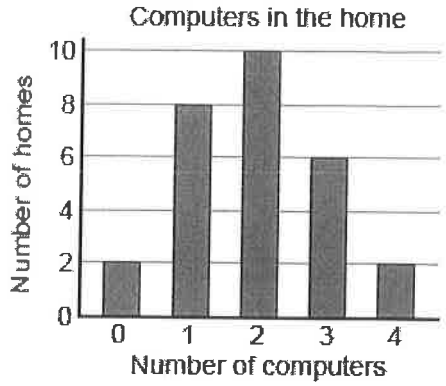


(ii) Calculate the interquartile range.

59 - 37 = 22 ✓

1

(b)



Determine the mean number of computers in a home from the data in this graph.
Give your answer correct to 1 decimal place.

$\frac{(0\times2)+(1\times8)+(2\times10)+(3\times6)+(4\times2)}{2+8+10+6+2}$ ✓

$= \frac{54}{28} = 1\frac{13}{14}$ or 1.9 (1 d.p.) ✓

2

(c) A group of students sat for an exam (out of 40 marks). The results are shown in the table below.

Class	Class centre	Frequency	Cumulative Frequency
20-24	22	5	5
25-29	27	8	13
30-34	32	7	20
35-39	37	3	23

✓

(i) What was the modal class?

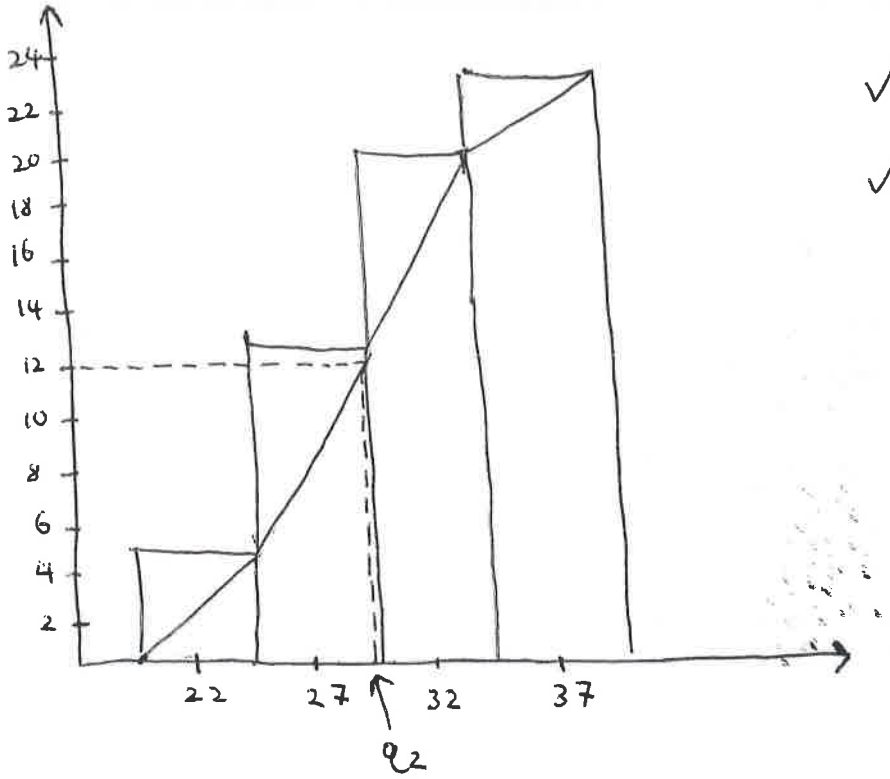
25 - 29 ✓

1

(ii) Complete the cumulative frequency column in the above table.

1

(iii) In the space below draw a cumulative frequency histogram and polygon (ogive).



✓ histogram
✓ polygon

2

(iv) Use your diagram in part (iii) to estimate the median of these scores.

29 or 30 ✓

1

(d) 4.2 7.0 8.2 8.9 9.7 10.6 12.4

When a new score was included in this set the median changed to 9.2. What was the new score?

8 scores ⇒ median between 4th & 5th

$\frac{8.9 + x}{2} = 9.2$

$8.9 + x = 18.4$

$x = 9.5$ ✓

1

13

Question 6 – Linear Equations, Inequations and Simultaneous Equations (13 Marks)

(a) Solve the following for x:

(i) $2(3x-5)-5(x+2)=2x-28$

$6x-10-5x-10 = 2x-28$ ✓

$x-20 = 2x-28$ ✓

$x = 8$ ✓ 3

(ii) $\frac{2x-1}{3}+\frac{x+1}{4}=\frac{3x}{5}$ $\times 60$ ✓

$20(2x-1)+15(x+1) = 12\times 3x$

$40x-20+15x+15 = 36x$ ✓

$55x-5 = 36x$ 3

$19x = 5$

$x = 5/19$ ✓

(b) Make x the subject of the following.

$ax+2b=c-\frac{x}{2}$ $\times 2$

$2ax+4b = 2c-x$

$2ax+x = 2c-4b$ ✓

$x(2a+1) = 2c-4b$ 2

$\therefore x = \frac{2c-4b}{2a+1}$

or

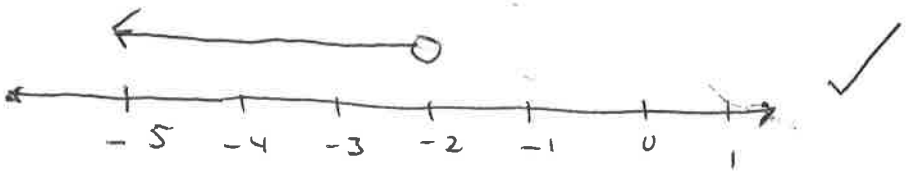
$x = \frac{2(c-2b)}{2a+1}$ ✓

(c) Solve the inequation and plot your solution on a number line.

$2-3x>8$

$-3x > 6$

$x < -2$ ✓ 2



✓

(d) Solve this pair of simultaneous equations.

✓ method
✓ x
✓ y

(1) $3x + 2y = 20$ × 4

(2) $4x - 5y = -27$ × 3

$12x + 8y = 80$ ... (3)

$12x - 15y = -81$ ... (4)

(3) − (4)

$23y = 161$

$y = 7$

sub into (1)

$3x + 2(7) = 20$

$3x + 14 = 20$

$3x = 6$

$x = 2$

∴ $x = 2$

$y = 7$

3

13

Question 7 – Quadratic Equations (9 Marks)

Solve each of the following for x:

(a) $x^2 + 5x = 0$ *(solve by factorising)*

$x(x + 5) = 0$

∴ $x = 0$ or $x = -5$

✓ ✓

2

(b) $x^2 + 12x + 5 = 0$ *(solve by completing the square)*

$x^2 + 12x = -5$

$x^2 + 12x + 36 = -5 + 36$ ✓

$(x + 6)^2 = 31$

$x + 6 = \pm\sqrt{31}$

$x = -6 \pm \sqrt{31}$ ✓

2

(c) $3x^2 - 8x - 6 = 0$ *(solve using the quadratic formula)*

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{8 \pm \sqrt{(-8)^2 - 4 \times 3(-6)}}{2 \times 3}$ ✓

$= \dfrac{8 \pm \sqrt{136}}{6}$

$= \dfrac{8 \pm 2\sqrt{34}}{6}$

$= \dfrac{4 \pm \sqrt{34}}{3}$ ✓

2

(d) $(x + 2)^4 = 7(x + 2)^2 + 18$

let $u = (x+2)^2$ ✓

$\therefore u^2 = 7u + 18$

$u^2 - 7u - 18 = 0$

$(u - 9)(u + 2) = 0$

$\therefore u = 9$ or $u = -2$ ✓

$(x+2)^2 = 9$ or $(x+2)^2 = -2$

$x + 2 = \pm 3$ no solutions

$x = -2 \pm 3$

$\therefore x = -2 - 3$ or $x = -2 + 3$

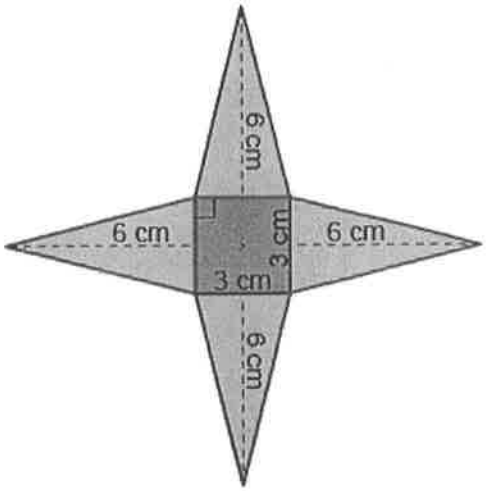
$x = -5$ or $x = 1$ ✓

3

9

Question 8 – Surface Area and Volume (12 Marks)

(a) The figure below is the net of a square pyramid.



Calculate:

(i) The surface area.

$SA = (3 \times 3) + 4 \times \left(\frac{1}{2} \times 3 \times 6\right)$ ✓

$= 9 + 36$

$= 45 \text{ cm}^2$ ✓

2

(ii) The perpendicular height.

$h^2 + 1.5^2 = 6^2$ ✓

$h^2 + 2.25 = 36$

$h^2 = 33.75$

$h = 5.809475\ldots$ cm ✓

$h^2 = \frac{135}{4} \therefore h = \frac{\sqrt{135}}{2}$ cm $\pm$ 5.8094...

2

(ii) The volume.

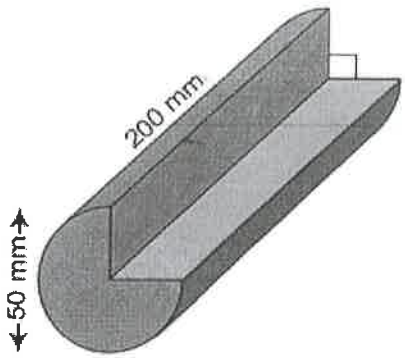
$V = \frac{1}{3} Ah$

$= \frac{1}{3} \times 3 \times 3 \times 5.809475\ldots$

$= 17.42842506\ldots \text{ cm}^3$ ✓

1

(b)



For the object above, calculate:

(i) The volume.

$V = Ah$ ✓

$= \frac{3}{4} \times \pi \times 25^2 \times 200$

$= 294\,524.3113 \text{ mm}^3$ ✓

(or $93750\pi \text{ mm}^3$)

2

(ii) The surface area.

$SA = 3/4 \times \pi \times 25^2 \times 2$ (sectors) ✓ $= 2945.24$

$+ \ 200 \times 25 \times 2$ (flat rectangles) ✓ $+ 10\,000$

$+ \ 3/4 \times \pi \times 50 \times 200$ (curved rectangle) $+ 23561.94$

$= 36507.18801 \text{ mm}^2$ ✓

3

12

(c) The surface area of a right cone is 800 cm^2. Its slant height is three times the radius. Determine the radius correct to the nearest centimetre.

$SA = \pi r s + \pi r^2$ ✓

$800 = \pi \times r \times 3r + \pi r^2$

$800 = 4r^2\pi$

$r^2 = \frac{800}{4\pi}$

$= \sqrt{\frac{200}{\pi}}$

$= 7.9788\ldots$

$= 8cm$ (nearest cm) ✓

2

Question 9 - Linear Relationships (12 Marks)

(a) An interval *AB* is defined by the points $A(-1, 4)$, $B(5, -2)$.

(i) What is the exact length of AB?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-5)^2 + (4-(-2))^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$ ✓

$= 6\sqrt{2}$ units

1

(ii) What is the midpoint of AB?

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-1+5}{2}, \frac{4+(-2)}{2}\right)$

$= \left(\frac{4}{2}, \frac{2}{2}\right)$

$= (2, 1)$ ✓

1

(iii) What is the gradient of AB?

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-4}{5-(-1)}$

$= \frac{-6}{6}$

$= -1$ ✓

1

(iv) By finding the equation of AB, or otherwise, find the x and y intercepts of AB.

$y = mx + b$

$m = -1 \quad (x, y) = (-1, 4)$

$\therefore 4 = (-1)(-1) + b$

$4 = 1 + b$

$b = 3$

$y = -x + 3$

when $y = 0$

$0 = -x + 3$

$x = 3$

$\therefore$ x-int is $(3, 0)$ ✓

y-int is $(0, 3)$ ✓

✓ method

3

(b) Find the equation of the line through $(1, 2)$ which is perpendicular to the line $2x - y + 4 = 0$

$2x - y + 4 = 0$

$\therefore y = 2x + 4$ ✓

$\therefore m_1 = 2 \quad \& \quad m_2 = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 1)$ ✓

$y - 2 = -\frac{1}{2}(x - 1)$

$2y - 4 = -(x - 1)$

$2y - 4 = -x + 1$

$\therefore x + 2y - 5 = 0$ ✓

or

$y = -\frac{1}{2}x + \frac{5}{2}$

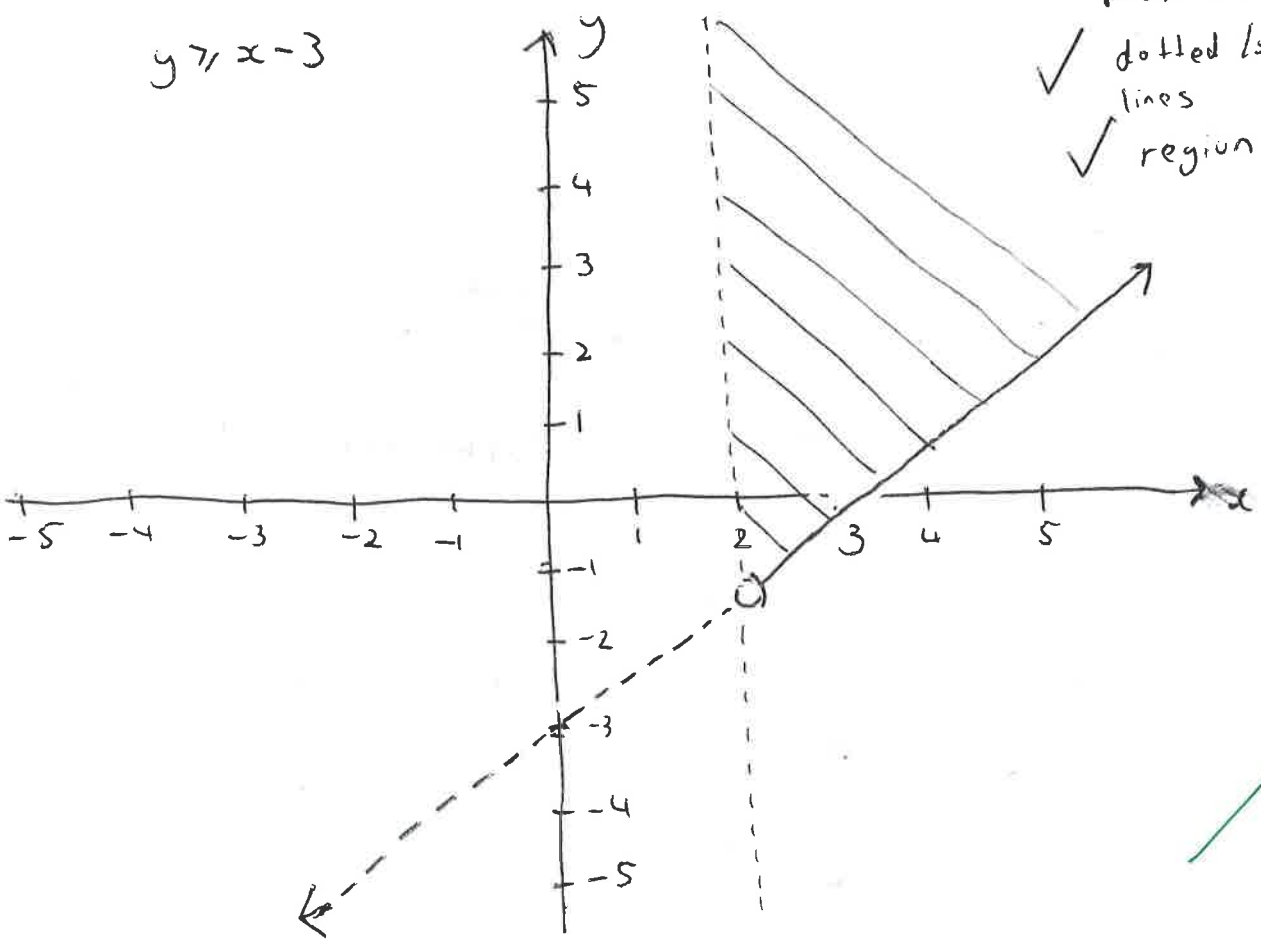
3

(c) Sketch the region on the number plane where the following inequations are both true

$x > 2$

$x - y \leq 3$

$y \geq x - 3$



✓ lines correct positions

✓ dotted / solid lines

✓ region.

3

12

Question 10 – Earning and Investing Money (12 Marks)

(a) The tables below shows the salaries for teachers in public schools in NSW and the 2020-21 rates of income tax.

Band/Level of Accreditation	Salary from the first pay period to commence on or after 1.1.2020 $
Increase	2.5%
Band 1(Graduate)	70,652
Band 2(Proficient)	85,214
Band 2.1	92,492
Band 2.2	96,138
Band 2.3	105,376
Band 3 (Highly Accomplished/ Lead)	112,163

Resident tax rates 2020–21

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $45,000	19 cents for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,001 and over	$51,667 plus 45 cents for each $1 over $180,000

Mr Smythe is a teacher on Band 2.3.

(i) Use the tables above to calculate the amount of income tax that Mr Smythe must pay *(you may disregard the medicare levy for this question).*

60 376

$5092 + 0.325(105\,376 - 45000)$ ✓

$= 19622.20$

$= \$24\,714.20$ ✓

2

(ii) Calculate the amount of tax Mr Smythe pays as a percentage of his gross salary.

$\frac{24714.20}{105376} \times 100\%$

$= 23.45334801...\%$ ✓ 1

(iii) $1096 is taken from Mr Smythe's paycheck for PAYG taxes each fortnight. Will Mr Smyth receive a refund or a bill at the end of the tax year and how much is the refund/bill for?

$\$1096 \times 26 = \$28496$ ✓

$\$28496 - \$24714.20 = \$3781.80$

∴ tax refund of $3781.80 ✓ 2

(iv) Mr Smythe is thinking of applying for a promotion to Band 3 (Highly Accomplished/Lead). If he does this what percentage of his gross salary would he now be paying in tax?

$tax = 5092 + 0.325(112163 - 45000)$

$= \$26919.975$ ✓

$percentage = \frac{26919.975}{112163} \times 100\%$ 2

$= 24.00076228...\%$ ✓

(v) Is it worth it for Mr Smythe to take the promotion? Explain your answer.

Yes as his net income will be higher even though he is paying a higher percentage of his income in tax. ✓ 1

$(1 - 0.325)(112163 - 105376)$

$= \$4581.225$ more net income.

(b) Mrs Jones invests $1000 into an account that pays 3% p.a interest compounded monthly for 5 years.

(i) How much money will she have at the end of the five years?

$r = 0.25\%$ $n = 60$ ✓

$A = P(1+r)^n$

$= 1000(1+0.0025)^{60}$

$= \$1161.616782\ldots$

$= \$1161.62$ (2 d.p.) ✓

2

(ii) Mr Smythe invests $1000 into an account which pays simple interest. At the end of five years he has the exact same amount of money as Mrs Jones. What was the rate of interest of the account?

Interest $= \$1161.616782\ldots - \$1000$

$= \$161.616782\ldots$ ✓

$I = Prt$

$r = \dfrac{I}{Pt}$

$= \dfrac{\$161.616782\ldots}{1000 \times 5} \times 100\%$

$= 3.232335631\ldots\%$ ✓

2

12

End of Examination