NORTH SYDNEY BOYS HIGH SCHOOL

Year 9 Mathematics Task 4 (Yearly)

Wednesday 21 November, 2018

General Instructions

- Working time – 90 minutes.
- All necessary working should be shown.
- Write answers in the space provided.
- Write using **blue or black pen**. Diagrams may be done in pencil.
- Calculators are permitted.
- Attempt all questions.

Class:

- ☐ 9A Mr Pavlovic
- ☐ 9B Mr Zuber
- ☐ 9C Ms Ziaziaris
- ☐ 9D Mr Ireland
- ☐ 9E Mr Berry

Name: _____

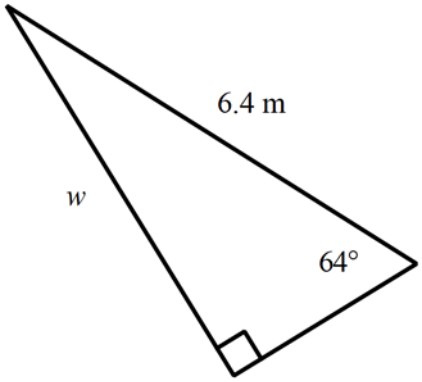
Trig	Alg	Surds & Ind	Eqns	Coord	SA & Vol	Geom	Stats	$$$	WM	Tot	%
1	2	3	4	5	6	7	8	9	10		
/11	/15	/12	/13	/10	/10	/9	/10	/9	/5	/104	

Working out: ☐ Legibility: ☐

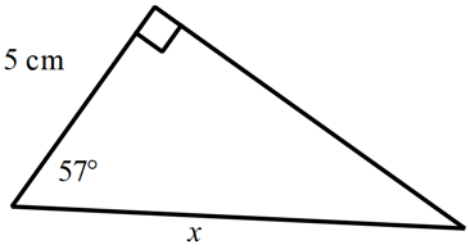
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Question 1: Trigonometry (11 marks) *Marks*

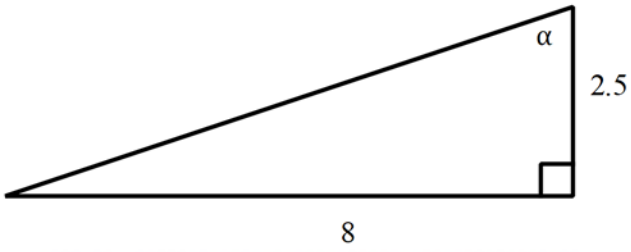
a) Solve for the value of w to 2 significant figures: *2*



b) Solve for the value of x to 2 decimal places: *2*



c) Solve for α to the nearest minute. *2*



d) The sine of an acute angle θ is known to equal $\frac{3}{7}$.

i) What is the **exact** value of $\cos\theta$? *1*

ii) What is the value of $\cos(90° - \theta)$? *1*

e) Two adjacent buildings are 457 m and 454 m tall respectively, separated by a short distance.

A tight-rope walker decides to set up a rope between the top of each building.

Standing at the top of the taller building, the tight-rope walker measures an angle of depression of 2° looking down at the top of the other building.

i) Sketch a diagram of the situation, annotated with the given information. *1*

ii) What length tight-rope is required to join the two buildings? *2*
Round to the nearest metre.



Philippe Petit walks between the Twin Towers August 7, 1974

Question 2: Algebra Techniques (15 marks) *Marks*

a) Expand and simplify $(5x + 3)^2$ *1*

b) Factorise

i) $5a^2 - 20a^4$ *2*

ii) $2xy - 6xz - y^2 + 3yz$ *2*

iii) $p^2 - pq - 6q^2$ *2*

iv) $6x^2 + 11x - 10$ *2*

c) Simplify $\dfrac{a^2 - a}{a} \div \dfrac{3a - 3}{3a - 2}$ *3*

d) Simplify $\dfrac{x}{a^2 + ab} - \dfrac{x}{a^2 - ab}$ *3*

Question 3: Surds and Indices (12 marks) *Marks*

a) Fully simplify $\dfrac{\sqrt{10}+\sqrt{40}}{\sqrt{2}}$. *2*

b) Rationalise the denominator $\dfrac{3-\sqrt{7}}{2+\sqrt{7}}$. *2*

c) Express in index form: $\dfrac{1}{9\sqrt[3]{x^2}}$ *1*

d) Fully simplify $\left(\dfrac{x^{16}}{100y^8z^{-6}}\right)^{-\frac{3}{2}}$ *2*

e) Expand and fully simplify $\left(m^3-2m^{\frac{1}{2}}\right)^2$ *2*

f) A Helium atom is comprised of 2 protons, 2 neutrons and 2 electrons. *3*

The following table provides accurate data for these particles:

Mass of a neutron	1.674929×10^{-27} kg
Mass of a proton	1.672623×10^{-27} kg
Mass of an electron	9.109390×10^{-31} kg

i) What is the mass of 6.0221×10^{23} Helium atoms to 5 significant figures?

ii) Accurate measurement shows this amount of Helium has a mass of 4.0026×10^{-3} kg. How much mass is "missing"?

Ask your physics teacher where the missing mass went….

Question 4: Equations (13 marks) Marks

a) Solve $\frac{3b-5}{2} - \frac{1-4b}{3} = -1$ *3*

b) Solve $t^2 - 2t - 35 = 0$ *2*

c) Solve $3y^2 + 10y - 5 = 0$ leaving your answer in simplified exact form. *3*

d) Make b the subject, given $a = \frac{3b-1}{b+2}$ *2*

e) Solve the simultaneous equations $\begin{cases} 2x - 3y = 0 \\ 5x + 2y = 1 \end{cases}$ *3*

Question 5: Coordinate Geometry (10 marks) *Marks*

a) An interval AB is defined by the points $A(2,-5),\ B(-3,1)$

i) What is the length of the interval AB? *1*

ii) Find the midpoint of AB. *1*

iii) Find the gradient of AB *1*

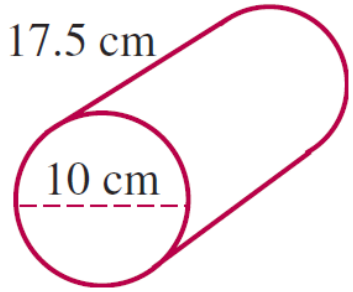
iv) Find the equation of the line perpendicular to AB, passing through the point $C(4,-2)$. Express your answer in general form. *3*

b) Sketch the graph of $3x - 2y + 8 = 0$. *2*

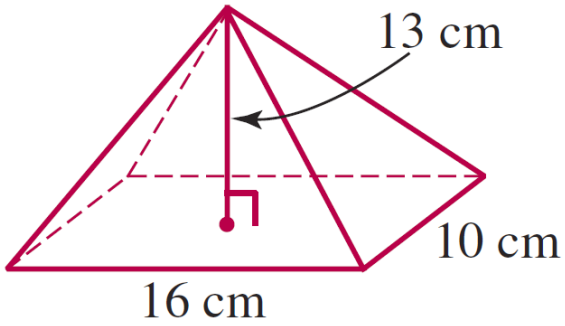
c) Sketch the region defined by the equations $y > x + 5$ *2*

Question 6: Surface Area and Volume (10 marks) *Marks*

a) Find the surface area of this **closed cylinder** in exact form: *3*



b) Find the volume and surface area of this pyramid (exact values) :

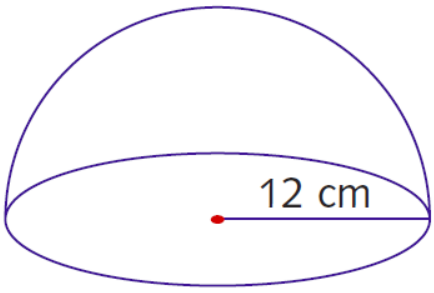


i) What is the volume? *1*

ii) What is the surface area? *3*

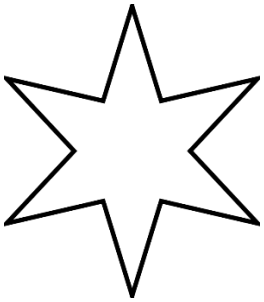
c) A 12 cm radius hemisphere is used as a mold to make a 3 mm thick chocolate dome. *3*
Only the spherical part of the mold is covered.

What is the volume of the resulting chocolate dome to the nearest whole number?

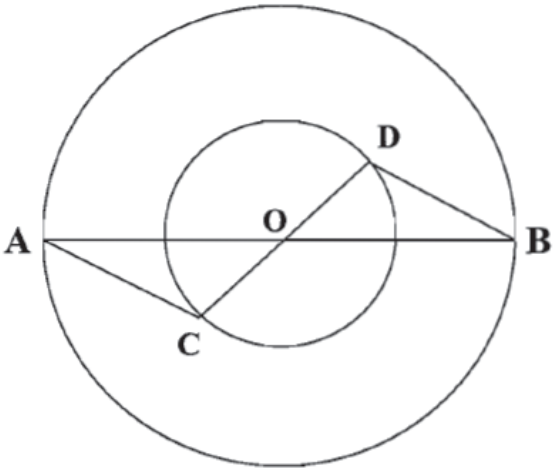


Question 7: Geometry (9 marks) *Marks*

a) What is the sum of the interior angles of this shape? *2*



b) In the diagram below, O is the centre of the two circles.



i) Prove that $\Delta AOC \cong \Delta\, ODB$. *3*

ii) Hence prove that $AC \parallel DB$. *2*

iii) Hence or otherwise prove that ADBC is a parallelogram. *2*

Question 8: Statistics (10 marks) *Marks*

a) Given the following set of data,

3	8
4	7 8
5	4
6	2 6
7	1 1 6 7 9 9
8	3 3 6

i) Find the 5 number summary. *2*

ii) Hence draw a box-and-whisker plot to represent the data. *2*

b) Use the following table to determine:

Class	Class Centre	Frequency	$f \times cc$	cf
24 - 26		4		
27 - 29		6		
30 - 32		8		
33 - 35		7		
36 - 38		5		
		$n = 30$	**A**	**B**

You do not need to copy this table into your workbook.

i) What are values for the totals **A** and **B**? *2*

iii) Estimate the mean. *1*

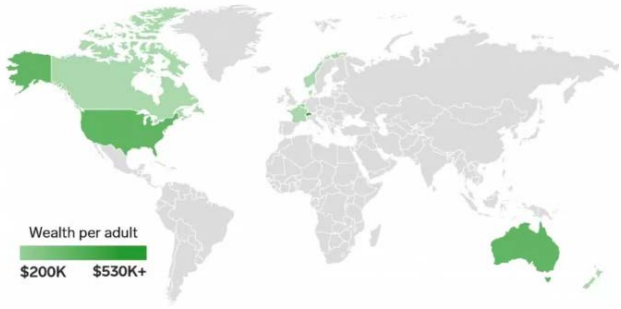
iv) State the median class. *1*

c) Credit Suisse's 2018 Global Wealth Report produced the two charts shown on the next page.

i) Why does the USA appear on one chart, but not on the other? *1*

ii) China was part of the report, so why doesn't it appear in the charts? *1*

Top 10 richest countries based on wealth per adult



RANK	COUNTRY	WEALTH PER ADULT
1	Switzerland	$530,244
2	Australia	$411,060
3	United States	$403,974
4	Belgium	$313,045
5	Norway	$291,103
6	New Zealand	$289,798
7	Canada	$288,263
8	Denmark	$286,712
9	Singapore	$283,118
10	France	$280,580

Top 10 richest countries based on median wealth per adult



RANK	COUNTRY	MEDIAN WEALTH PER ADULT
1	Australia	$191,453
2	Switzerland	$183,339
3	Belgium	$163,429
4	Netherlands	$114,935
5	France	$106,827
6	Canada	$106,342
7	Japan	$103,861
8	New Zealand	$98,613
9	United Kingdom	$97,169
10	Singapore	$91,656

Source: Credit Suisse

Insider Inc.

Shayanne Gal/Business Insider

Question 9: Financial Mathematics (9 marks) *Marks*

a) A $10,000 trust fund earns 6% per annum, compounded monthly. *2*
How much is the trust fund worth at the end of 21 years?

b) Dr Lee, earned $120,000 in the tax year 2017-2018. Each fortnight, she paid $1,000 in PAYG (Pay As You Go) tax payments.

She spent $15,000 for medical indemnity insurance and $3,000 for her professional association fees, which are both tax deductible expenses.

Resident tax rates 2017–18

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $87,000	$3,572 plus 32.5c for each $1 over $37,000
$87,001 – $180,000	$19,822 plus 37c for each $1 over $87,000
$180,001 and over	$54,232 plus 45c for each $1 over $180,000

i) What is Dr Lee's taxable income? *1*

ii) What the amount of tax on this income? *1*

iii) Does Dr Lee owe money, or will she receive a refund? How much? *1*

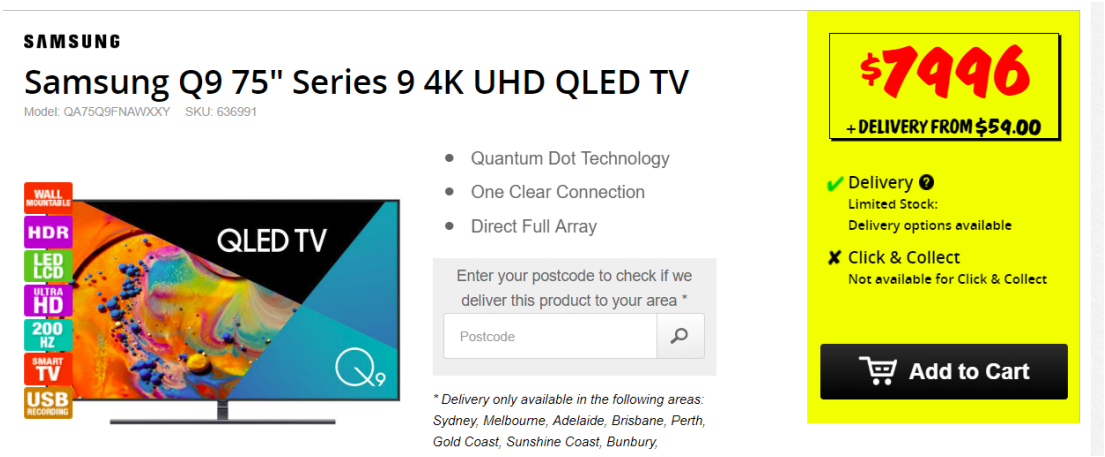
b) The Australian Taxation Office provides a different tax table for foreign residents: *1*

Foreign resident tax rates 2017–18

Taxable income	Tax on this income
0 – $87,000	32.5c for each $1
$87,001 – $180,000	$28,275 plus 37c for each $1 over $87,000
$180,001 and over	$62,685 plus 45c for each $1 over $180,000

Describe one way in which foreign residents are taxed differently to residents.

c) Jason wants to purchase this television priced at $7,996.



Payment terms are:

- 5% deposit,
- Simple interest at 12% per annum,
- 40 month contract.

i) How much interest is charged? *2*

ii) What is the monthly repayment? *1*

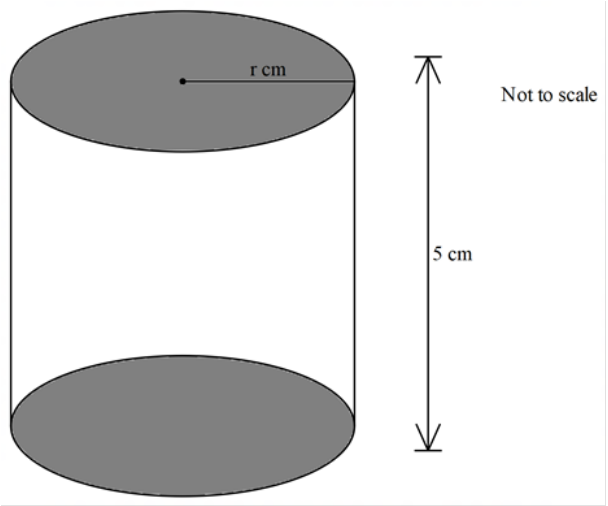
Question 10: Challenge Questions (5 marks) Marks

a) Factorise

i) $a^4 + a^2b^2 + b^4$ 1

ii) $a^4 + 4b^4$ 1

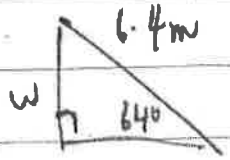
b) The closed cylinder shown below has a height of 5 cm and a surface area of $100\ \text{cm}^2$ *3*



Calculate the volume of the cylinder to the nearest cm^3.

End of the examination

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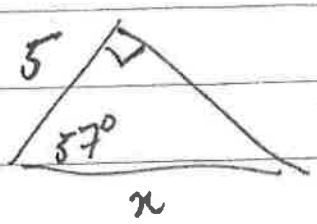
Q1. a) 

$\sin 64° = \frac{w}{6.4}$

$w = 6.4 \sin 64°$

$= 5.7522...$

$= 5.8$ (2 sig figs)

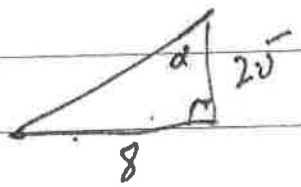
b) 

$\cos 57° = \frac{5}{x}$

$x = \frac{5}{\cos 57}$

$= 9.18039...$

$= 9.18$ (2 dp)

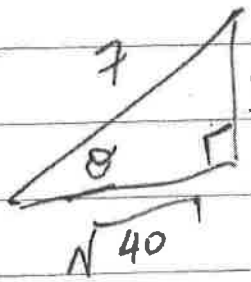
c) 

$\tan \alpha = \frac{8}{2.5}$

$\alpha = 72°38'45$

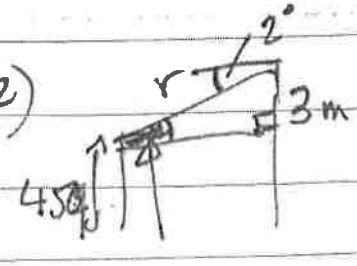
$= 72° 39°$ (nearest degree)

d) $\sin \theta = 3/7$



$\therefore \cos \theta = \frac{\sqrt{40}}{7} = \frac{2\sqrt{10}}{7}$

$\cos(90° - \theta) = \sin \theta = 3/7$

e) 

$\sin 2° = \frac{3}{r}$

$r = \frac{3}{\sin 2°}$

$= 85.96 = 86$ m (nearest metre)

Start here.

2. a) $(5x+3)^2 = 25x^2 + 30x + 9$ ✓

b) i) $5a^2 - 20a^4$

$= 5a^2(1 - 4a^2)$ ✓

$= 5a^2(1-2a)(1+2a)$ ✓

ii) $2xy - 6xz - y^2 + 3yz$

$= 2x(y-3z) - y(y-3z)$ ✓

$= (y-3z)(2x-y)$ ✓

iii) $p^2 - pq - 6q^2$

$= (p+2q)(p-3q)$ ✓2

iv) $6x^2 + 11x - 10$

$= \frac{(6x+15)(6x-4)}{6}$ ✓2

$= (2x+5)(3x-2)$

c) $\frac{a^2-a}{a} \div \frac{3a-3}{3a-2}$

$= \frac{a(a-1)}{a} \times \frac{3a-2}{3(a-1)}$

$= \frac{3a-2}{3}$

d) $\frac{x}{a^2+ab} - \frac{x}{a^2-ab}$

$= \frac{x}{a(a+b)} - \frac{x}{a(a-b)}$

$= \frac{x(a-b) - x(a+b)}{x(a+b)(a-b)}$

$= \frac{((a-b)-(a+b))x}{x(a+b)(a-b)}$

$= \frac{(a-b-a-b)x}{a(a+b)(a-b)}$

$= \frac{-2bx}{a(a+b)(a-b)}$

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3.

a) $\frac{\sqrt{10}+\sqrt{40}}{\sqrt{2}} = \frac{\sqrt{10}+2\sqrt{10}}{\sqrt{2}}$

$= \frac{3\sqrt{10}}{\sqrt{2}}$

$= 3\sqrt{\frac{10}{2}}$

$= 3\sqrt{5}$

b) $\frac{3-\sqrt{7}}{2+\sqrt{7}} = \frac{(3-\sqrt{7})(2-\sqrt{7})}{4-7}$

$= \frac{6-3\sqrt{7}-2\sqrt{7}+7}{-3}$

$= \frac{13-5\sqrt{7}}{-3}$

$= \frac{5\sqrt{7}-13}{3}$

c) $\frac{1}{9\sqrt[3]{x^2}} = \frac{1}{9}x^{-2/3}$

or $(9^{-1})(x^{-2/3})$

d) $\left(\frac{x^{16}z^6}{100y^8}\right)^{-3/2} = \left(\frac{100y^8}{x^{16}z^6}\right)^{3/2} = \frac{1000y^{12}}{x^{24}z^9}$

e) $(m^3 - 2m^{1/2})^2$

$= m^6 - 2(m^3)(2m^{1/2}) + 4m$

$= m^6 - 4m^{7/2} + 4m$ (or $m^6 - 4m^3\sqrt{m} + 4m$)

f) mass $= \left(\left(1.674929 \times 10^{-27}\right) + \left(1.672623 \times 10^{-27}\right) + 0.0009710 \times 10^{-27}\right) \times 2$

$= 3.3484629 \times 10^{-27}$

$\times 2$

$= 6.696 \times 10^{-27}$

$\times 6.0221 \times 10^{23}$

$= 0.00403295$ kg

$= 4.0330 \times 10^{-3}$ kg (5 sig fig)

ii) expected 4.0026×10^{-3}

missing mass $= 0.0304$ gm.

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4 a) $\frac{3b-5}{2} - \frac{1-4b}{3} = -1$

$3(3b-5) - 2(1-4b) = -6$

$9b - 15 - 2 + 8b = -6$

$17b - 17 = -6$

$17b = +11$

$b = \frac{11}{17}$

b) $t^2 - 2t - 35 = 0$

$(t-7)(t+5) = 0$

$t = -5$ or $t = 7$

c) $3y^2 + 10y - 5 = 0$

$\Delta = b^2 - 4ac$

$= 100 - 4(3)(-5)$

$= 160$

$y = \frac{-10 \pm \sqrt{160}}{2(3)}$

$y = \frac{-10 \pm 4\sqrt{10}}{2(3)}$

$y = \frac{-5 \pm 2\sqrt{10}}{3}$

d) $a = \frac{3b-1}{b+2}$

$a(b+2) = 3b-1$

$ab+2a = 3b-1$

$2a+1 = 3b-ab$

$2a+1 = b(3-a)$

$b = \frac{2a+1}{3-a}$

e) $\begin{cases} 2x-3y=0 & (A) \\ 5x+2y=1 & (B) \end{cases}$

A×5 : $10x+15y=0$

B×(-2): $-10x-4y=-2$

Add: $19y=-2$

$y=\frac{2}{19}$

~~subst into (B) $5x+2(\frac{2}{19})$~~

subst into (A) $2x-3(\frac{2}{19})=0$

$2x=\frac{6}{19}$

$x=\frac{3}{19}$

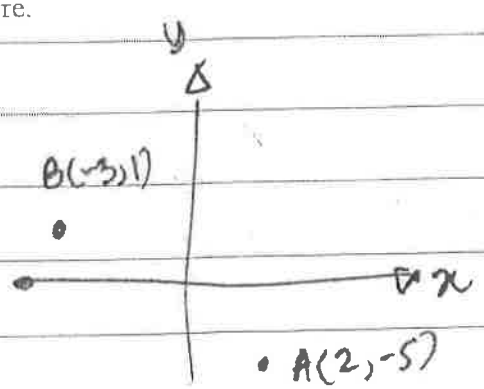
$\boxed{x=\frac{3}{19}, y=\frac{2}{19}}$

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a)



i) $d^2 = (2-(-3))^2 + (1+(-5))^2$

$= 5^2 + 6^2$

$= 25 + 36$

$= 61$

$\therefore d = \sqrt{61}$ u

ii) Midpoint $\left(\frac{-3+2}{2}, \frac{1+(-5)}{2}\right)$

$= \left(-\frac{1}{2}, -2\right)$

iii) gradient $= \frac{-5-1}{2-(-3)}$

$= -\frac{6}{5}$

(iv) Gradient $\perp$ $= +5/6$ ✓

Line passing through C(4,-2) $\perp$ AB is

$y+2 = 5/6(x-4)$ ✓

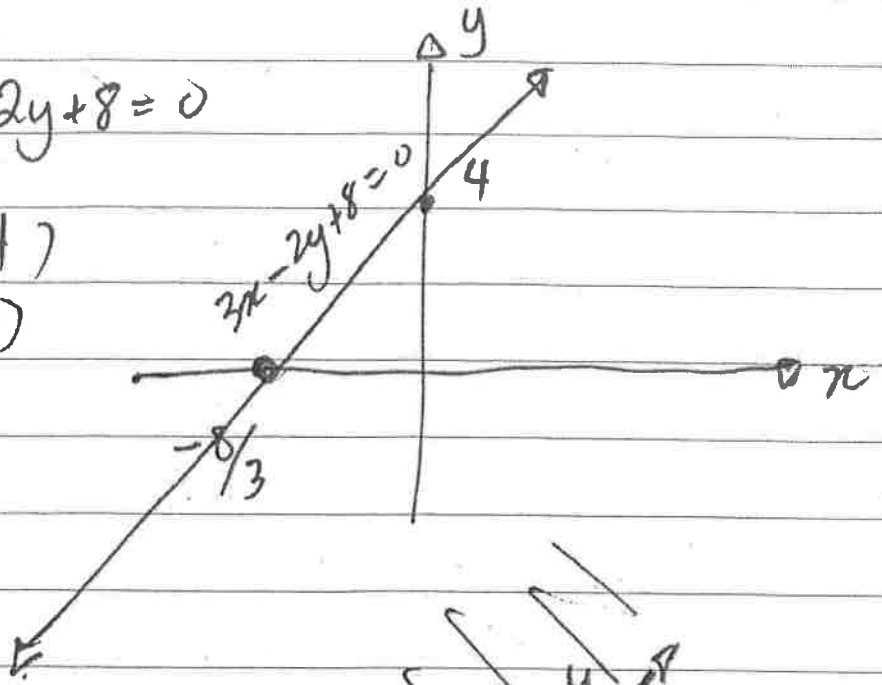
$6y + 12 = 5x - 20$

$\boxed{5x - 6y - 32 = 0}$ general form ✓

b) $3x - 2y + 8 = 0$

$(0, 4)$

$(-\frac{8}{3}, 0)$

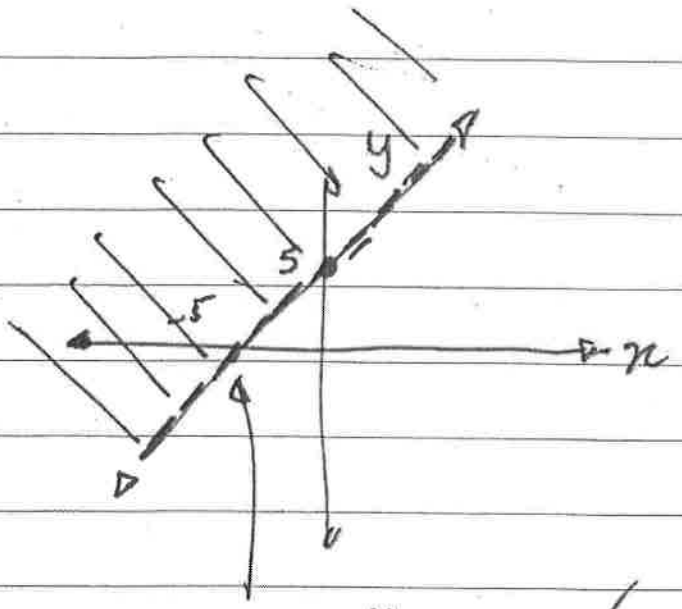


c) $y > x + 5$

Test $(0, 0)$

$0 ? 0 + 5$

false



dashed line ✓

correct region ✓

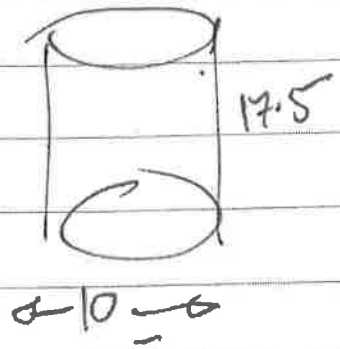
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Q6.

a)



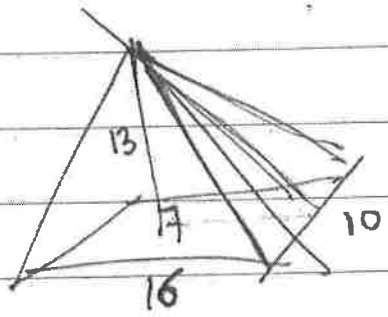
$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi r(r+h)$

$= 2\pi(5)(5+17.5)$

$= 225\pi \text{ cm}^2$

b)

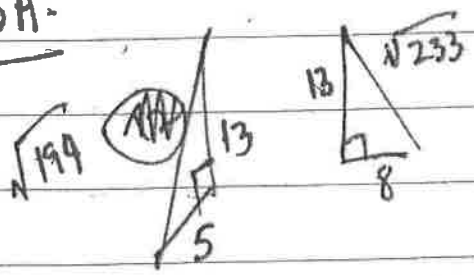


$V = \frac{1}{3}b \times h$

$= \frac{1}{3}(16)(10)(13) \text{ cm}^3$

$= 693.33 = \frac{2080}{3} \text{ cm}^3$

SA:



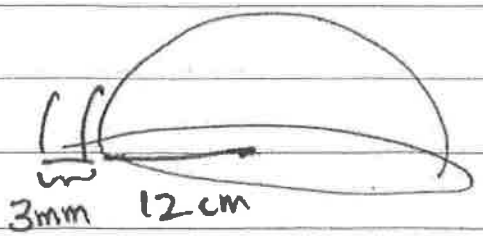
$SA = b_{base} + 2\triangle + 2\triangle$

$= (16 \times 10) + 2\left(\frac{1}{2}\right)(16)(\sqrt{194}) + 2\left(\frac{1}{2}\right)(\sqrt{233})(10)$

$= 535.497$

$= 535.5 \text{ cm}^2$

c)



$V \approx \frac{1}{2}\left(\frac{4}{3}\pi\right)(12.3^3 - 12^3)$

≈ 278.27

$\approx 278\ cm^3$

or $SA = 4\pi(12)^2 \times \frac{1}{2}$

$V \approx 3\ mm \times 4\pi(12)^2 \times \frac{1}{2}$

$= 0.3 \times 4\pi(144) \times \frac{1}{2}$

$= 271.4$

$= 271\ cm^3$

error is: 2.5%

2 marks for this approach.

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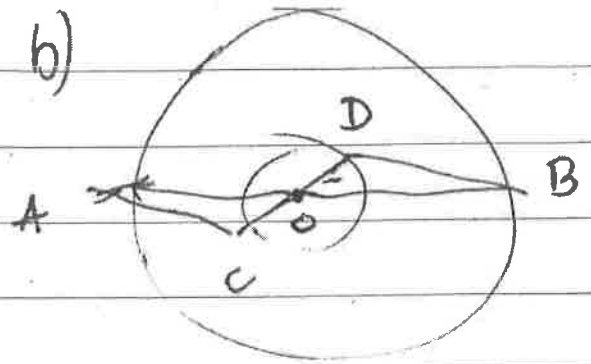
Start here.

Q7. a) Shape is a 12-gon

Interior angle sum = $180(12-2)$

$= 1800°$

b)



i) In $\triangle AOC$, $\triangle ODB$,

$\angle AOC = \angle BOD$ (vertically opposite angles are equal)

$AO = BO$ (radii same circle)

$CO = DO$ (radii same circle)

$\therefore \triangle AOC \equiv \triangle BOD$ (SAS)

ii) $\angle OCA = \angle ODB$ (corresponding angles in congruent triangles are equal)

$\therefore AC \parallel DB$ (alternate angles are are equal, therefore lines are parallel)

(iii) $AC = DB$ (corresponding sides in congruent triangles are equal)

$AC \parallel DB$ (proven in ii)

$\therefore$ ADBC is a parallelogram. (opposite sides equal and parallel.)

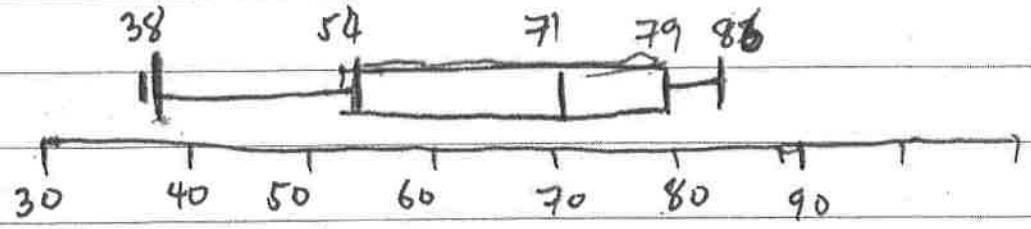
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Q8. a) 15 scores

Median is 8th score = 71

Q1 is 54, Q3 is 79

∴ 5 number summary is: 38-54-71-79-86



b) A = 939 B = 87 (meaningless)

(iv) Mean = 939/30 = 31.30

∴ Median class 30-32

c) i) One chart is average other is median

US is not in top 10 median because/ vast wealth concentrated right at the top.

- Data is heavily skewed (positively skewed)
- A few people have most of the wealth

(ii) China has huge population — so averages are much lower. even though some regions of great wealth.

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Q9. a) $A = \$10,000\left(1 + \frac{5}{100} \times \frac{1}{12}\right)^{21\times12}$

$= \$35,143.71$

b) i) Taxable income

$= \$120,000 - (\$15,000 + \$3,000)$

$= \$102,000$

(ii) Tax bracket is: \$19,822 plus 37¢ over \$87,000

(a) Tax owe $= \$19,822 + 0.37(102,000 - 87,000)$

$= \$25,372$

(iii) PAYG paid $= \$1000 \times 26$ weeks

$= \$26000$

Tax owed $= \$25,375 - \$26,000$

$= -\$628.00$

Dr Lee will get a Refund of \$628.00

b) Foreign residents

✓1 — have no tax free threshold

✓1 — pay at a rate of ~~B7~~ ~~market~~ dollar 32.5¢ / $
when residents pay 0¢ or 19¢

✓1 — they are taxed much higher [need to qualify]
eg: At $87,001
the have paid $28,275
versus $19,822

So high income foreign resident pay $16,947 more in tax.

c) Price: $7,996
Borrow $7996 × 0.95

Interest is $7996 × 0.95 × $\frac{12}{100} \times \left(\frac{40}{12}\right)$ ✓(1)
= $3038

Monthly repayment $\frac{(\$3038 + \$7996 \times 0.95)}{40}$ ✓✓ (2)
= $265.855 = $265.86/month.

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(accept $265.87)

Start here.

Q10 e) i) $a^4 + a^2b^2 + b^4$

$= a^4 + 2a^2b^2 + b^4 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$ ✓

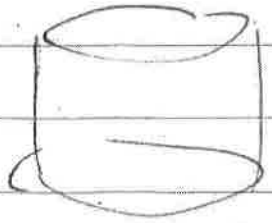
(ii) $a^4 + 4b^4$

$= a^4 + 4a^2b^2 + 4b^4 - 4a^2b^2$

$= (a^2 + 2b^2) - (2ab)^2$

$= (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$ ✓

Q10



$h = 5$ $SA = 100$

$SA = 2\pi r(r+h)$

$100 = 2\pi r(r+5)$

$100 = 2\pi r^2 + 10\pi r$

$2\pi r^2 + 10\pi r - 100 = 0$

$\pi r^2 + 5\pi r - 50 = 0$

$r = \dfrac{-5\pi \pm \sqrt{25\pi^2 + 4\pi(50)}}{2\pi}$

$\simeq \dfrac{-5\pi \pm 29.58}{2\pi}$

$r \simeq 2.208$ (ignore $r<0$)

$\therefore V = \pi r^2 h$

$\simeq \pi(2.208)^2(5)$

$\simeq 76.58$

$\simeq 77\ cm^3$

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