Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2017 YEAR 9 YEARLY EXAMINATION

Mathematics

General Instructions

- Working time – 1.5 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Attempt all questions
- Marks may be deducted for incomplete or illegible working

Please tick or highlight)

- ○ Mr Hwang
- ○ Mr Ireland
- ○ Mr Jomaa
- ○ Ms Ren
- ○ Ms Ziaziaris

Student Name: _____

	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	___ /10	___ /17	___ /17	___ /10	___ /14	___ /9	___ /11	___ /11	___ /8	___ /6	___ /113	___ /100

QUESTION 1

(a) Solve for x

$$\frac{2x}{3} = \frac{5x-1}{4}$$ 2

(b) Solve for y

$$\frac{-6-5y}{3} \leq -12$$ 3

(c) Change the subject of the formula to y. 2

$$x = \frac{y}{y+2}$$

(d) Solve the simultaneous equations

$$2x + 5y = 16$$
$$10x - 3y = -4$$ 3

QUESTION 2

(a) Simplify fully $(\sqrt{5}-6)^2$ 1

(b) Simplify fully $5\sqrt{45} - 2\sqrt{32}$ 2

(c) Rationalise $\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ 2

(d) Find one quarter of $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$ 3

(e) Simplify fully $\left(8x^6\right)^{\frac{1}{3}} x^{-3}$ leaving your answer with positive indices. 2

(f) Solve for x : $9\sqrt{3} = 27^{2x}$ 2

(g) How many significant figures in the following numbers

(i) 0.0017 1

(ii) 1.705 1

(iii) 700 1

(h) Express the value of xy^n in scientific notation

where $x = 5.5\times10^3$, $y = 4\times10^{-2}$ and $n = 3$. 2

QUESTION 3

(a) Expand and simplify $12-3(5-2x)$ 1

(b) Simplify $\dfrac{x^2-2x-8}{x^2-4}$ 2

(c) Factorise a^3+a^2+a+1 2

(d) Factorise $2x^2-11x+12$ 2

(e) Factorise fully $2m^2n^2-18n^4$ 2

(f) Factorise $8y^3-a^3$ 2

(g) Simplify $\dfrac{x^2+x-12}{x+3}\div\dfrac{2x^2-7x+3}{x^2+3x}$ 3

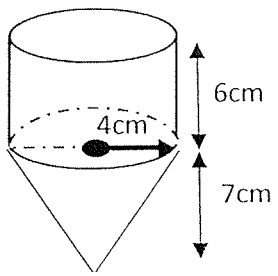
(h) Simplify $\dfrac{3(x+2)}{x^2-1}-\dfrac{4}{x^2-4x-5}$ 3

QUESTION 4

(a) Convert $300\,\text{mm}^2$ to cm^2 . 1

(b) Find the radius of a sphere which has a volume of $400\pi\ \text{cm}^3$.
Give answer as an exact value. 2

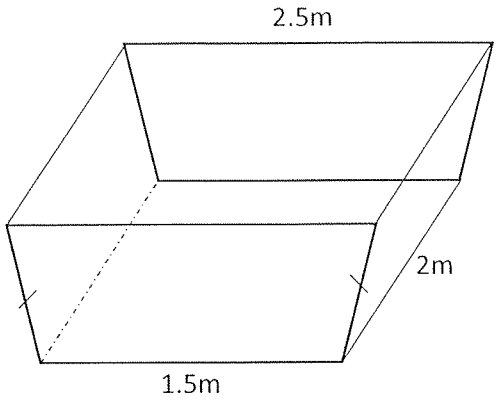
(c)



(i)Find the surface area of the shape above. Give answer to 1 decimal place. 2

(ii) Find the volume of the shape above. Give answer to one decimal place. 2

(d) The capacity of this luxury bath in the shape of a trapezoidal prism below is 10 kL. Find the height of this luxury bath. 3

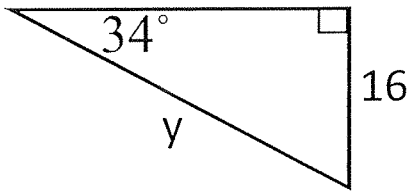


QUESTION 5

(a) If $\tan\theta = 2.1$ and θ is an acute angle, find θ to the nearest minute. 1

(b) Find θ given $\sin 2\theta = \cos 40^\circ$ 1

(c) Find the value of y to one decimal place 2



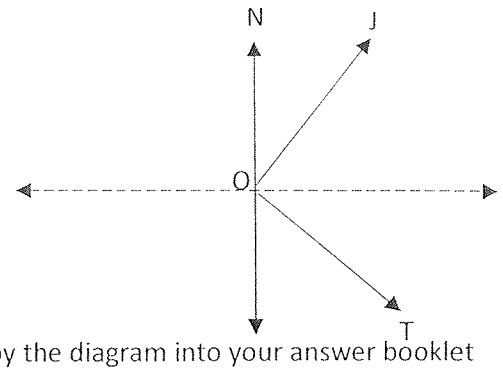
(d) Find the exact value of

(i) $\sin 135^\circ$ 2

(ii) $\tan 660^\circ$ 2

(e) If $\sin\theta = \frac{5}{12}$ and $\cos\theta < 0$ find the exact value of $\tan\theta$. 2

(f) Terry cycles on a bearing of 125° for 43 km while John cycles on a bearing of 035°.



(i) Copy the diagram into your answer booklet

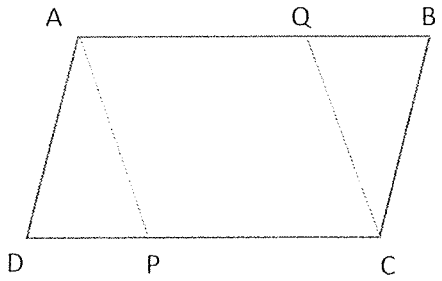
(ii)Label the diagram with the information given. Find the size of $\angle TOJ$ 2

(ii) How far must John travel so that he is directly north of Terry. 2

QUESTION 6

(a) How many sides has a regular polygon whose exterior angle is 24°? 1

(b) $ABCD$ is a parallelogram and $DP = QB$.

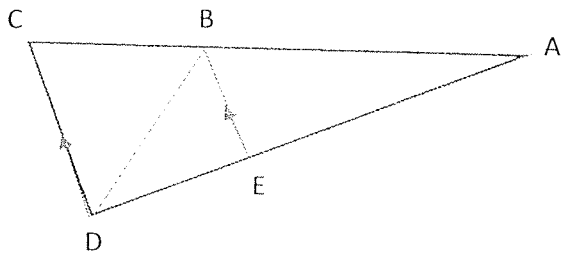


Prove that (i) $\Delta APD \equiv \Delta BQC$ 3

(ii) $AP = QC$ 1

(iii) $AP \| QC$ 2

(c) Given $\angle DBE = \angle ABE$, prove ΔBCD is isosceles. 2



QUESTION 7

(a) Tom is a junior shop assistant and earns $14.35 per hour for the first 30 hours in a week. He receives time-and-a-half for the next 5 hours and double- time after that. Find his pay for a 42 hour week. 2

(b) (i) Find the compound interest earned when $22 000 is invested for 5 years at 6%p.a , compounded monthly. 2

(ii) Find the equivalent simple interest rate p.a. Give answer to 2 decimal places. 1

(c) Sid earns $84365 as a dancer and $462 from his share portfolio. He has allowable tax deductions of $13884 and throughout the year pays a total of $15 988 in PAYG tax instalments.

Find (i) his total income 1

(ii) his taxable income 1

(iii) the tax payable on his taxable income, using the table below 2

(iv) Sid must pay a medicare levy of 2% of the taxable income. Find the amount of this levy. 1

(v) Including the medicare levy that needs to be paid as well as the tax find the tax refund or tax that is still required to be paid. 1

Taxable income	Tax on this income
0 - $18 200	Nil
$18 201 - $37 000	19c for each $1 over $18 200
$37 001 - $87000	$3 572 plus 32.5c for each $1 over $37 000
$87 001 - $180 000	$19 822 plus 37c for each $1 over $87 000
$180 000 and over	$54 232 plus 45c for each $1 over $180 000

QUESTION 8

(a) The times (in seconds) for a 200m swim were expressed in the form of a stem and leaf plot below.

Stem	Leaf
5	3
6	1 2 5 8 9
7	0 2 3 6 7 9
8	4 4 6

Find

(i) the median time 1

(ii) the mode 1

(iii) the interquartile range 2

(iv) draw a box and whisker plot to scale 3

(b)

Class	Frequency	Cumulative Frequency
41 - 50	5	
51 - 60	0	
61 - 70	12	
71 - 80	10	
81 - 90	9	
91 – 100	4	

(i) Copy and complete the table in your workbook. 1

(ii) Find the median class 1

(iii) Find the mean 2

QUESTION 9

(a) The midpoint of interval AB is given by $M(-3,4)$. If A has coordinates $(6,-2)$, find the coordinates of B. 2

(b) Find the gradient of the interval joining the points $(-5,6)$ and $(3,-4)$. 1

(c) Find the equation of a line which passes through the point $(-6,2)$ and is perpendicular to the line with equation $3x-4y+1=0$. Give answer in general form. 3

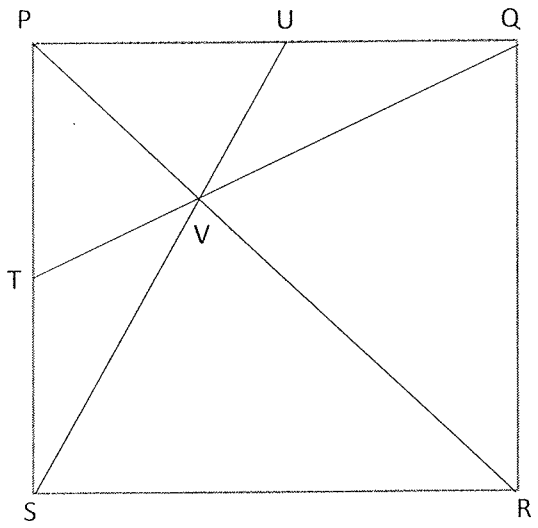
(d) Sketch the region given by $y \geq 1-3x$ 2

QUESTION 10

(a) Factorise fully $12a - 30b + 4a^2 - 25b^2$ 2

(b) A committee is to have 3 more women than men and the number on the committee must be at least 7 but no more than 15. By using an equation, give the range of the possible number of women on the committee if the committee must contain men and women? 2

(c) PQRS is a square. T and U are midpoints of the sides PS and PQ respectively. TQ and SU intersect at V. What fraction of the area of the square is the area of quadrilateral QVSR. 2



YR9 YEARLY 2017 SOLN's

Q1.

a) $\frac{2x}{3} = \frac{5x-1}{4}$

$8x = 15x - 3$

$-7x = -3$

$x = \frac{3}{7}$

b) $\frac{-6-5y}{3} \leq -12$

$-6-5y \leq -36$

$-5y \leq -30$

$y \geq 6$

c) $x = \frac{y}{y+2}$

$xy + 2x = y$

$xy - y = -2x$

$y(x-1) = -2x$

$y = \frac{-2x}{x-1}$

d) $2x + 5y = 16$ — ①

$10x - 3y = 4$ — ②

① × 5 $10x + 25y = 80$ — ③

③ − ② $28y = 84$

$y = 3$

Sub $y = 3$ into ①

$2x + 15 = 16$

$2x = 1$

$x = \frac{1}{2}$ $\left(\frac{1}{2}, 3\right)$

Q2. a) $(\sqrt{5}-6)^2$

$= 5 - 12\sqrt{5} + 36$

$= 41 - 12\sqrt{5}$.

b) $5\sqrt{9}.\sqrt{5} - 2\sqrt{16}.\sqrt{2}$

$= 15\sqrt{5} - 8\sqrt{2}$.

c) $\frac{(2\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \frac{6 - 2\sqrt{6} + \sqrt{6} - 2}{3 - 2}$

$= 4 - \sqrt{6}$.

d) $\frac{1}{4} \times \left(\frac{4}{9}\right)^{-\frac{1}{2}}$

$= \frac{1}{4} \times \left(\frac{9}{4}\right)^{\frac{1}{2}}$

$= \frac{1}{4} \times \frac{\sqrt{9}}{\sqrt{4}}$

$= \frac{3}{8}$

e) $(8x^6)^{\frac{1}{3}} x^{-3}$

$= 2x^2 x^{-3}$

$= 2x^{-1}$

$= \frac{2}{x}$

f) $9\sqrt{3} = 27^{2x}$

$3^2 \times 3^{\frac{1}{2}} = (3^3)^{2x}$

$3^{5/2} = 3^{6x}$

$6x = 5/2$ $\therefore x = 5/12$

g) (i) 2
(ii) 4
(iii) 1, 2, or 3.

h) $(5.5 \times 10^3)(4 \times 10^{-2})^3$

$= 0.352$
$= 3.52 \times 10^{-1}$

Q3. a) $12 - 3(5 - 2x)$
$12 - 15 + 6x$
$-3 + 6x$

b) $\frac{x^2 - 2x - 8}{x^2 - 4}$
$= \frac{(x-4)(x+2)}{(x-2)(x+2)}$
$= \frac{x-4}{x-2}$

c) $a^3 + a^2 + a + 1$
$= a^2(a+1) + 1(a+1)$
$= (a^2+1)(a+1)$

d) $2x^2 + 11x + 12$
$2x \times -3$
$x \times -4$
$(2x-3)(x-4)$

e) $2m^2n^2 - 18n^4$
$2n^2(m^2 - 9n^2)$
$2n^2(m-3n)(m+3n)$

f) $8y^3 - a^3$
$= (2y-a)(4y^2 + 2ya + a^2)$

g)

g) $\frac{x^2+x-12}{x+3} \div \frac{2x^2-7x+3}{x^2+3x}$

$\frac{(x+4)(x-3)}{x+3} \times \frac{x(x+3)}{(2x-1)(x-3)}$

$2x \times -1$
$x \times -3$

$= \frac{x(x+4)}{2x-1}$

h) $\frac{3(x+2)}{x^2-1} - \frac{4}{x^2-4x-5}$

$= \frac{3(x+2)}{(x-1)(x+1)} - \frac{4}{(x-5)(x+1)}$

$= \frac{3(x+2)(x-5) - 4(x-1)}{(x-1)(x+1)(x-5)}$

$= \frac{3(x^2-3x-10) - 4x+4}{(x-1)(x+1)(x-5)}$

$= \frac{3x^2-9x-30-4x+4}{(x-1)(x+1)(x-5)}$

$= \frac{3x^2-13x-26}{(x-1)(x+1)(x-5)}$

Q4. a) $300mm^2 = 3cm^2$

b) $V = \frac{4}{3}\pi r^3$.

$400\pi = \frac{4}{3} \times \pi \times r^3$.

$\frac{1200\pi}{4\pi} = r^3$

$300 = r^3$

$r = \sqrt[3]{300}$ cm.

c) (i) $SA = \pi r^2 + 2\pi rh + \pi rs$.

$s = \sqrt{4^2+7^2} = \sqrt{65}$

$\therefore SA = \pi \times (16) + 2\pi(4)(6) + \pi(4)\sqrt{65}$

$= 16\pi + 48\pi + 4\sqrt{65}\pi$

$= 302.4\ cm^2$

(ii) $V = \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \pi(16)(6) + \frac{1}{3} \times \pi(16)(7)$

$= 418.9\ cm^3$

d) $10kL = 10000L$.

$= 10m^3$

$V = A_b \times h$.

$10 = \frac{1}{2}h(2.5+1.5) \times 2$.

$10 = 4h$

$h = 2.5$ m.

Q5.

a) $\tan\theta = 2.1$

$\theta = 64^\circ 32'$

b) $\sin 2\theta = \cos 40^\circ$

$2\theta = 50^\circ$

$\theta = 25$

c)

$\sin 34^\circ = \frac{16}{y}$

$y = \frac{16}{\sin 34^\circ}$

$y = 28.6$

d) (i) $\sin 135^\circ = \sin 45^\circ$

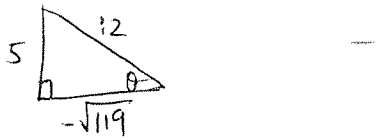
$= \frac{1}{\sqrt{2}}$

(ii) $\tan 660^\circ = \tan 300^\circ$

$= -\tan 60^\circ$

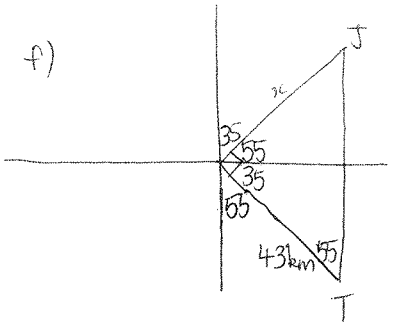
$= -\sqrt{3}$.

e)



$\tan\theta = \frac{-5}{\sqrt{119}} = \frac{-5\sqrt{119}}{119}$.

f)



(i) $\angle TOJ = 90^\circ$.

(ii) $\tan 55^\circ = \frac{x}{43}$

$x = 43\tan 55^\circ$

$x = 61.4$ km.

John travels 61.4 km (1 dp).

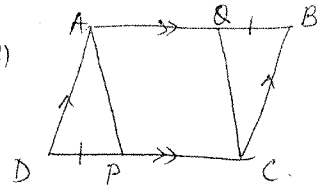
Q6.

a) No. of ext. angles $= \frac{360}{24}$

$= 15$.

$\therefore$ No. of sides $= 15$.

b) (i)



(1) In Δ's ADP & CBQ

1. DP = BQ given
2. BC = DA opposite sides of parallelogram are equal
3. $\angle ADP = \angle CBQ$ opposite angles of parallelogram are equal

$\therefore \Delta ADP \equiv \Delta CBQ$ (SAS).

(ii) AP = QC (corresponding sides of congruent triangles are equal)

(iii) AQ = AB − BQ

PC = DC − DP

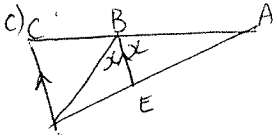
$\therefore$ AQ = PC (since AB = DC opposite sides of a parallelog. and BQ = DP given)

$\therefore$ AQCP is a parallelogram (2 pairs of opposite sides equal).

$\therefore$ AP $\parallel$ QC

OR any other method.

c)



$\angle BDC = x$ (alternate angles, BE $\parallel$ CD)

$\angle BCD = x$ (corresponding angles, BE $\parallel$ CD)

$\therefore \Delta BCD$ is isosceles (2 equal angles)

Q7. a) $30 \times 14.35 + 5 \times 14.35 \times 1.5 + 7 \times 14.35 \times 2$

$= \$739.03$

b)(i) $A_n = P(1+R)^n$

$= 22000(1.005)^{60}$

$= \$29674.70$.

$\therefore$ Interest $= 29674.70 - 22000$

$= \$7674.70$.

(ii) $I = PRT$

$7674.70 = 22000 \times R \times 5$

$R = \frac{7674.70}{22000 \times 5}$

$R = 0.06977$

$R = 6.98\%$ p.a.

c) (i) Total income $= 84365 + 462$.

$= \$84827$

(ii) Taxable income $= 84827 - 13884$

$= \$70943$

(iii) Amount Over $= 70943 - 37000$

$= \$33943$

Tax Payable $= 3572 + 0.325 \times 33943$

$= \$14603.48$

(iv) Medicare Levy $= 0.02 \times 70943$

$= \$1418.86$.

(v) Tax + Medicare $= 1418.86 + 14603.48$

$= \$16022.34$

Tax Bill $= 16022.34 - 15988$

$= \$34.34$

Q8. a) (i) Median = 72.

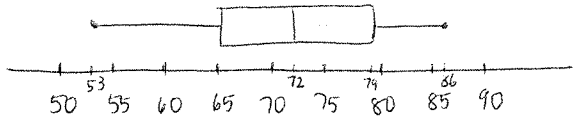
(ii) Mode = 84

(iii) $Q_3 - Q_1$

$= 79 - 65$

$= 14$

(iv)



b)

Class	f	cf	c.c.	fx c.c.
41-50	5	5	45.5	227.5
51-60	0	5	55.5	0
61-70	12	17	65.5	786
71-80	10	27	75.5	755
81-90	9	36	85.5	769.5
91-100	4	40	95.5	382

(i) 71-80

(ii) Mean $= \frac{\Sigma f x c.c}{\Sigma f}$

$= \frac{2920}{40}$

$= 73$

Q9. a) $(-3,4) = \left(\frac{6+x_2}{2}, \frac{-2+y_2}{2}\right)$

$\therefore \frac{x_2+6}{2} = -3 \qquad 4 = \frac{-2+y_2}{2}$

$x_2 = -6-6 \qquad y_2 = 8+2$

$x_2 = -12 \qquad y_2 = 10$

$\therefore B(-12,10)$

b) $m = \frac{6+4}{-5-3}$

$= \frac{10}{-8}$

$= \frac{-5}{4}$

c) m of line $= \frac{3}{4}$

$\therefore$ m of perpendic. line $= \frac{-4}{3}$

Eqn: $y - 2 = \frac{-4}{3}(x+6)$

$3y - 6 = -4x - 24$

$4x + 3y + 18 = 0$

d) $y \geq 1 - 3x$

$y = 1 - 3x$

Xint, $y = 0$

$3x = 1$

$x = \frac{1}{3}$

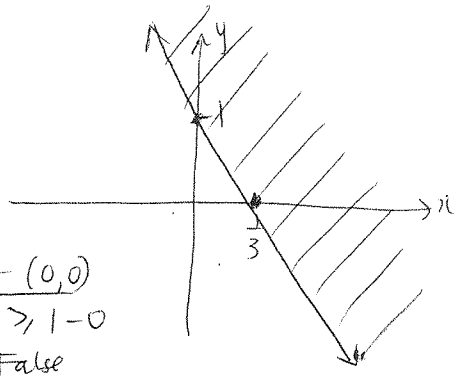
Yint: $x = 0$

$y = 1$

Test (0,0)

$0 \geq 1 - 0$

False



Q10 a) $12a - 30b + 4a^2 - 25b^2$

$= 6(2a-5b) + (2a-5b)(2a+5b)$

$= (2a-5b)(6 + 2a+5b)$

b) Let the number of men be x

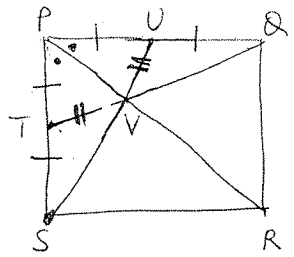
$\therefore$ the number of women will be $x+3$.

$$7 \le x + (x+3) \le 15.$$
$$7 \le 2x+3 \le 15$$
$$4 \le 2x \le 12$$
$$2 \le x \le 6.$$

$\therefore$ No of women will be:

$$5 \le \text{No. of women} \le 9$$

c)



$\Delta PUV \equiv \Delta PTV$ (SAS)

ΔPVT and ΔVTS have equal areas as bases are the same and same perpendicular height from V to PS.

$\therefore$ Sim, ΔPUV and ΔUQV have equal areas.

$\therefore$ Area of ΔQUV = Area of ΔUVP = Area of PVT = Area of VTS.

But Area of ΔPUS is $\frac{1}{4}$ of Area of square (U is midpt)

$\therefore$ Area of $\Delta QVU = \frac{1}{4}$ of square

$\therefore$ Area of quad $QVSR = 1 - \left(\frac{1}{4} + \frac{1}{12}\right)$

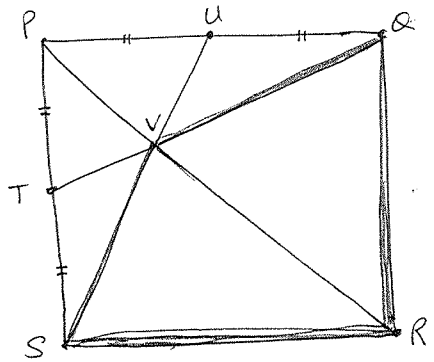
$= 1 - \frac{4}{12}$

$= \frac{8}{12}$

$= \frac{2}{3}$ of square.

OR alternative solution

(c)



Align the square in a number plane so that S is at the origin, and R is at (2,0), P at (0,2).

Then SU has equation $y = 2x$.

$PR \perp SQ \therefore m_{PR} = -1$

$\therefore$ PR has equation $y = -x+2$.

Then V's coordinates are got by solving $y = 2x$ & $y = -x+2$

Simultaneously $\rightarrow$ $2x = -x+2$ $\therefore x = \frac{2}{3}$, $y = \frac{4}{3}$ $\therefore V = \left(\frac{2}{3}, \frac{4}{3}\right)$.

So $\therefore$ area $\Delta VSR = \frac{1}{2} \times 2 \times \frac{4}{3} = \frac{4}{3} u^2$.

likewise, $\Delta VQR = \frac{4}{3} u^2$. $\therefore$ area $QVSR = 2 + \frac{4}{3} = \frac{8}{3} u^2$.

$\therefore$ fraction of square is $\frac{\frac{8}{3}}{2 \times 2} = \frac{2}{3}$. #