NORTH SYDNEY BOYS HIGH SCHOOL

2016 YEAR 9 COMMON TEST 4

STAGE 5.3 MATHEMATICS

Student Name:_____

Class: _____ **Teacher**_____

Number of pages used : _____(Fill at the end of exam)

General Instructions

- Working time – **90 minutes**
- Write on **both sides of the booklet pages**.
- Write using blue or black pen.
- **Board approved** calculators are allowed.
- All necessary working should be shown in every question.
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Section	A	B	C	D	E	F	G	H	I	J	Total	Total %
Mark	___/14	___/11	___/6	___/12	___/12	___/20	___/15	___/8	___/10	___/12	___/120	___/100

Section A (14 marks) **Commence a NEW page** **Mark**

1. Evaluate $\frac{9 \cdot 1^2}{\sqrt{1-0 \cdot 9^2}}$ correct to 2 significant figures. 2

2. Simplify fully:

   (a) $\sqrt{54}$ 1

   (b) $\left(\sqrt{6}-3\right)^2$ 2

3. Express $\frac{2}{4-\sqrt{5}}$ with a rational denominator. 2

4. Write $\frac{1}{\sqrt{x}}$ in index form. 1

5. Express $0 \cdot 05307$

   (a) Correct to 4 decimal places. 1

   (b) Correct to 3 significant figures. 1

   (c) In scientific notation. 1

6. Simplify $2x(2x^2)^2$ 1

7. Simplify $\frac{2^n \times 4^{n+1}}{8^{n-2}}$ expressing your answer with positive indices. 2

Section B (11 marks) **Commence a NEW page** **Mark**

1. Use the 2015-16 personal tax rate table to answer this question (Note: it does not include the 2% Medicare levy)

Taxable income	Tax on this income
0 - $18,200	Nil
$18,201 - $37,000	19c for each $1 over $18,200
$37,001 - $80,000	$3,572 plus 32•5c for each $1 over $37,000
$80,001 - $180,000	$17,547 plus 37c for each $1 over $80, 000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

Darnell received a salary of $ 75,400 plus investment income of $ 8,200, while his allowable expenses amounted to $ 4,900.

(a) What is Darnell's taxable income? 1

(b) Using the table, calculate the income tax that Darnell owes. 2

(c) How much does Darnell owe for the Medicare levy? (rate = 2%) 1

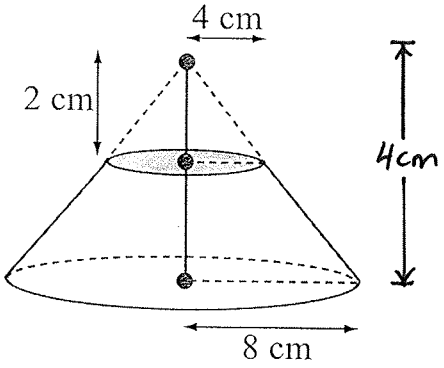
(d) If Darnell's employer deducts $710 a fortnight for PAYG tax, what is Darnell's tax refund/bill ? 2

2. $28000 is invested for 5 years at 3·6% per annum, compounded monthly.
What does this sum grow to, and what total interest does it earn? 3

3. After a 24 % discount was given, Jada paid $ 60 for a dinner set.
What was the original price? 2

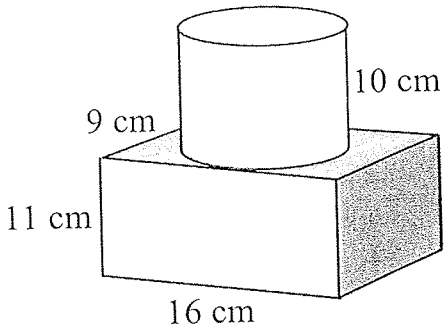
Section C (6 marks) **Commence a NEW page** **Mark**

1. Calculate the volume of this composite solid (it's a solid cone that has had the top sliced off). 3



Not to scale

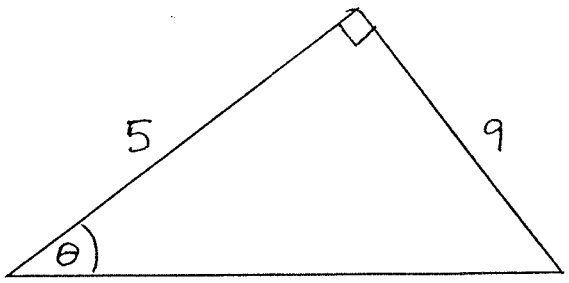
2. A solid cylinder was placed on top of a rectangular prism, as shown.
Calculate the surface area of the composite solid (including the base). 3



Not to scale

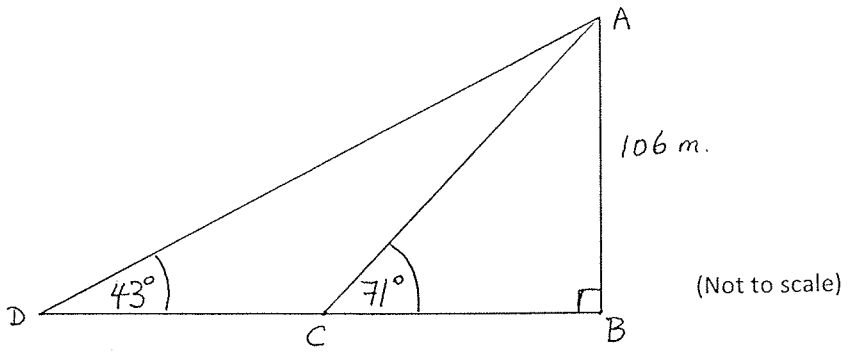
Section D (12 marks) **Commence a NEW page** **Mark**

1. Calculate the value of θ correct to the nearest minute: 2



(Not to scale)

2. Calculate the length of *CD*, correct to the nearest metre: 3



3. Use about one-third of a page to sketch $y = 4\cos 2\theta$ for $0° \le \theta \le 360°$ 3

4. In which quadrant does θ lie if $\tan\theta < 0$ and $\cos\theta > 0$? 1

5. Give the exact values of

   (a) $\sin 60°$ 1

   (b) $\cos 225°$ 2

Section E (12 marks) **Commence a NEW page** **Mark**

1. If the straight line $ax - 3y + 4 = 0$ passes through $(2,1)$, find the value of a. 2

2. (a) Find the equation of the straight line that passes through $(3,7)$ and which is parallel to the line $y = 5x - 2$. 2

   (b) What angle θ does the line $y = 5x - 2$ make with the positive direction of the x-axis ? 2

3. Show (without drawing a picture) that the points $P(0,1)$, $Q(-4,7)$, and $R(10,-14)$ are collinear. 3

4. Given the points $A(1,-2)$ and $B(-3,4)$, find in general form the equation of the line through the origin that is perpendicular to the interval AB. 3

Section F (20 marks) **Commence a NEW page** **Mark**

1. Solve these equations:

   (a) $\frac{5}{x-2}=\frac{2}{x}$ 2

   (b) $2(4x-1)=5x+6$ 2

   (c) $\frac{2x+1}{3}=\frac{x}{2}-1$ 2

   (d) $x^2=10x$ 2

   (e) $x^2=3x+1$ (give exact answer) 2

   (f) $(x-3)^2=2$ (give exact answer) 2

2. Solve simultaneously these equations: 3

$$2x-3y=5$$

$$5x+2y=-16$$

3. Solve the inequality $1-2x>7$. 2

4. Change the subject of the formula $A=\frac{mx+1}{x+m}$ to x. 3

Section G (15 marks) **Commence a NEW page** **Mark**

1. Expand and simplify:

   (a) $15x-3(2-3x)$ 2

   (b) $(5x-1)^2$ 1

2. Factorise fully:

   (a) $18-32x^2$ 2

   (b) $x^2-15x-100$ 1

   (c) $3x^2-11x-4$ 2

   (d) a^3+64 2

3. Fully simplify:

   (a) $\frac{x-1}{2}-\frac{x+3}{3}$ 2

   (b) $\frac{x+2}{x^2-x-2}\div\frac{x^2-4}{3x+3}$ 3

Section H (8 marks) **Commence a NEW page** **Mark**

1. Given the scores 1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11, 11

   (a) find the range 1

   (b) find the median 1

   (c) find the upper and lower quartiles 1

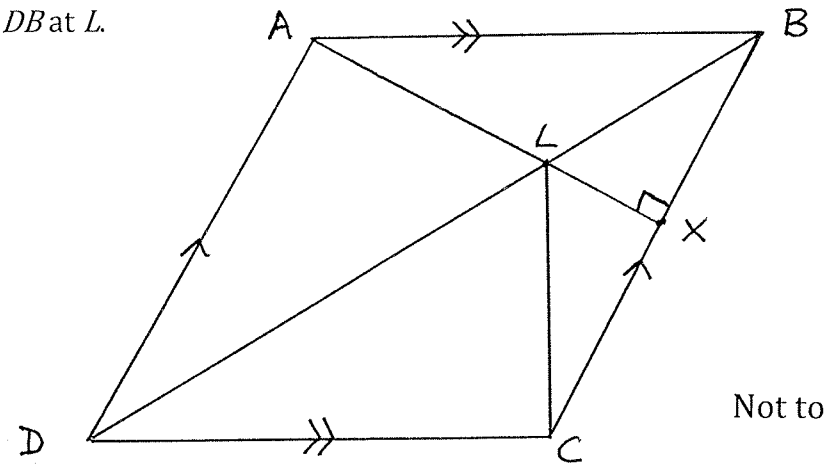
   (d) find the interquartile range 1

   (e) draw a box-and-whisker plot of the data using an appropriate scale 2

2. The mean of a set of 14 scores is 35. After a new score is added, the mean rises to 36. Find the score that was added. 2

Section I (10 marks) **Commence a NEW page** **Mark**

1. In the diagram below, $ABCD$ is a rhombus. AX is perpendicular to BC, and AX meets DB at L.



Not to scale

   (a) Copy the diagram into your answer booklet.

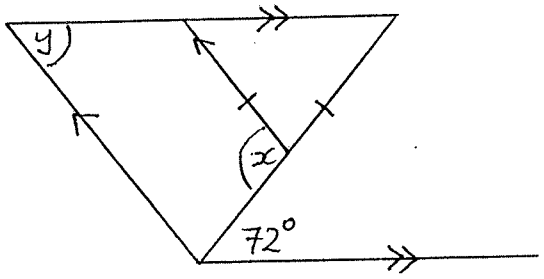
   (b) State why $\angle ADB = \angle CDB$ 1

   (c) Prove that $\Delta ALD \equiv \Delta CLD$ 3

   (d) State why $\angle DAL$ is a right angle 1

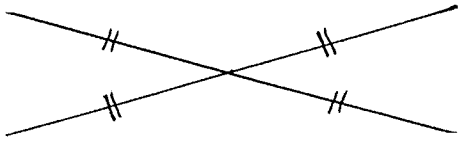
   (e) Hence, or otherwise, find the size of $\angle LCD$ 1

2. Find the values of x and y (no reasons needed): 2



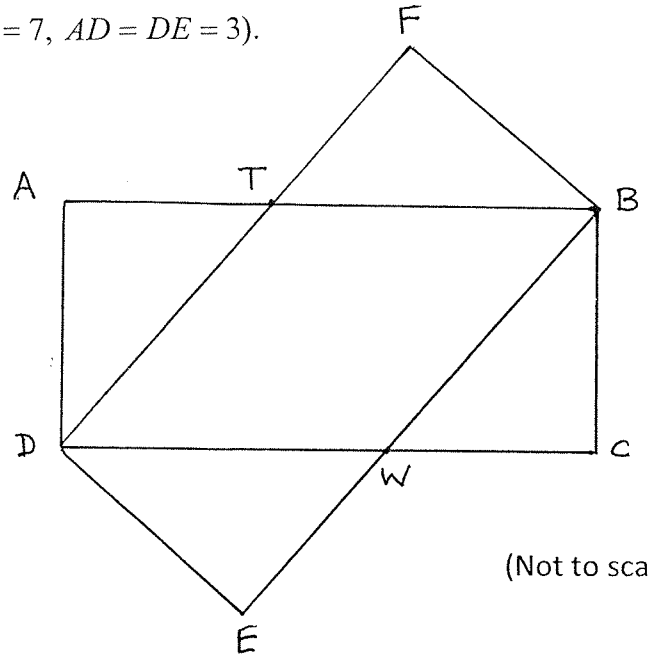
Not to scale

3. These are the diagonals of a quadrilateral. What type is it? Why? 2



Section J (12 marks) **Commence a NEW page** **Mark**

1. $ABCD$ and $DEBF$ are two congruent rectangles with sides 3 and 7 units as in as in the diagram. ($AB = DF = 7$, $AD = DE = 3$).



(Not to scale)

(a) Copy the diagram into your answer booklet.

(b) Show that $\Delta ATD \equiv \Delta FTB$ 2

(c) $AT = FT$ since they are matching sides in congruent triangles.

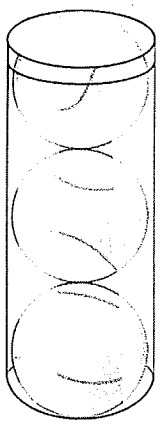
Show that $AT = \frac{20}{7}$ (hint: Let $AT = FT = x$) 2

(d) Find the area of the figure $DWBT$. 2

2. A group of friends ordered home-delivered pizza, which cost \$120. The bill was to be shared equally, but two people had no money, so the others had to pay an extra \$3 each.

By forming and solving a quadratic equation, calculate the number of people in the group. 3

3. Three identical tennis balls <u>just fit</u> into a cylindrical container (*see diagram*).



What fraction of the space within the container do the three balls together occupy? (Clear working is needed for full marks). 3

END OF EXAM

2016 Year 9 Yearly (Task 4) — Suggested Solutions

A

① $\dfrac{9.1^2}{\sqrt{1-0.9^2}} \doteq 189.979169\ldots$ ✓

$= 190$ (2 sig.figs) ✓

② (a) $\sqrt{54} = 3\sqrt{6}$ ✓

(b) $(\sqrt{6}-3)^2 = 6 - 6\sqrt{6} + 9$ ✓

$= 15 - 6\sqrt{6}$ ✓

③ $\dfrac{2}{4-\sqrt{5}} \times \dfrac{4+\sqrt{5}}{4+\sqrt{5}} = \dfrac{8+2\sqrt{5}}{16-5}$ ✓

$= \dfrac{8+2\sqrt{5}}{11}$ ✓

④ $\dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ ✓

⑤ (a) 0.0531 (4 d.p.) ✓

(b) 0.0531 (3 s.f.) ✓

(c) 5.307×10^{-2} ✓

⑥ $2x(2x^2)^2 = 2x \cdot 4x^4 = 8x^5$ ✓

⑦ $\dfrac{2^n \times 4^{n+1}}{8^{n-2}} = \dfrac{2^n \times (2^2)^{n+1}}{(2^3)^{n-2}}$ ✓

$= \dfrac{2^n \times 2^{2n+2}}{2^{3n-6}}$

$= 2^8$ (i.e. 256) ✓

14

2016 Y9 T.4

B ① (a) taxable income = 75400 + 8200 − 4900

= $ 78 700 ✓

(b) tax owed = 3572 + 0.325 (78700 − 37000) ✓

= $ 17 124.50 ✓

(c) Levy = 0.02 × 78700

= $ 1574 ✓

(d) Total owed = 17124.50 + 1574

= $ 18698.50

· Amount paid = 26 × 710

= $ 18 460

∴ tax bill = 18698.50 − 18460 ✓

= $ 238.50 ✓

② Number of months = 5 × 12 = 60

Monthly rate = $\frac{0.036}{12}$ = 0.003

$A = 28000\,(1.003)^{60}$ ✓

∴ A = $ 33513.05 is the amount at end. ✓

∴ interest = A − 28000 = $ 5513.05 ✓

③ Original price = $\frac{\$60}{0.76}$ ✓

= $ 78.95 ✓

11

2016 Y9 T.4

C

① Volume = big cone − little cone

$$= \frac{\pi \times 8^2 \times 4}{3} - \frac{\pi \times 4^2 \times 2}{3} \quad \checkmark\checkmark$$

$$V = \frac{224\pi}{3}\ cm^3 \quad (\doteq 234.6\ cm^3) \quad \checkmark$$

② Looking from above, we see that

SA = SA of rect. prism + curved SA of cylinder

$$\therefore SA = 2\times16\times11 + 2\times9\times11 + 2\times9\times16 + \left(2\times\pi\times\frac{9}{2}\right)\times10 \quad \checkmark\checkmark$$

$$= 352 + 198 + 288 + 90\pi$$

$$\therefore SA = 838 + 90\pi\ cm^2 \quad \checkmark$$

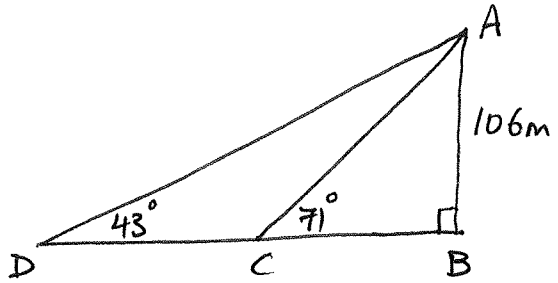
$$(\doteq 1120.7\ cm^2)$$

/6

2016 49 T.4

D ① $\tan\theta = \frac{9}{5}$ $\therefore \theta \doteq 60^\circ 56' 43''$

$\therefore \theta = 60^\circ 57'$ (nearest minute) ✓✓

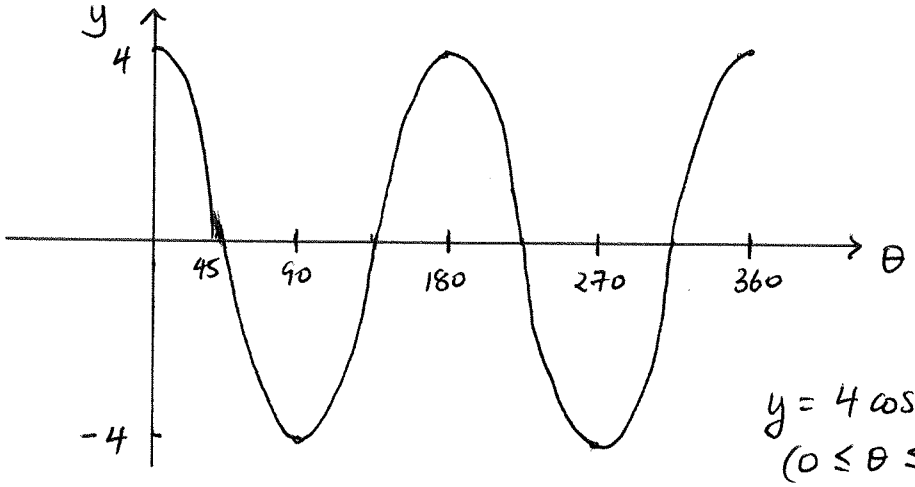
② 

$\frac{106}{DB} = \tan 43^\circ$ $\therefore DB = \frac{106}{\tan 43}$

Likewise, $CB = \frac{106}{\tan 71}$ ✓

$\therefore CD = DB - CB = \frac{106}{\tan 43} - \frac{106}{\tan 71} \doteq 77.1723\ldots$ ✓

$CD = 77\text{m}$ (nearest m). ✓

③ 

$y = 4\cos 2\theta$
$(0 \le \theta \le 360)$ ✓✓✓

④ Quadrant 4 ✓

⑤ (a) $\sin 60 = \frac{\sqrt{3}}{2}$ ✓

(b) $\cos 225 = -\cos 45 = -\frac{1}{\sqrt{2}}$ ✓✓

12

2016 Y9 T.4

E ①

$a(2) - 3(1) + 4 = 0$ ✓

$2a - 3 + 4 = 0 \quad \therefore 2a = -1$

$\therefore a = -\frac{1}{2}$ ✓

② (a) $m = 5$

$\therefore y - 7 = 5(x - 3)$

$y - 7 = 5x - 15$

$\therefore y = 5x - 8$ ✓✓

(b) $\tan\theta = 5$ ✓

$\therefore \theta = 78^\circ 41'$ ✓

③ $m_{PQ} = \frac{7-1}{-4-0} = \frac{6}{-4} = -\frac{3}{2}$ ✓

$m_{QR} = \frac{-14-7}{10+4} = \frac{-21}{14} = -\frac{3}{2}$ ✓

$\therefore$ since $m_{PQ} = m_{QR}$, and point Q is common, ✓

P, Q, R are collinear.

④ $m_{AB} = \frac{4+2}{-3-1} = \frac{6}{-4} = -\frac{3}{2}$ ✓

$\therefore m_{\perp} = \frac{2}{3}$ ✓

$\therefore$ line is $y = \frac{2}{3}x$

Thus, in general form, $2x - 3y = 0$. ✓

12

2016 Y9 T.4.

F ① (a) $\frac{5}{x-2} = \frac{2}{x}$

$\therefore 5x = 2x - 4$

$3x = -4$

$x = -\frac{4}{3}$ ✓✓

(b) $2(4x-1) = 5x + 6$

$\therefore 8x - 2 = 5x + 6$

$3x = 8$

$x = \frac{8}{3}$ ✓✓

(c) $\frac{2x+1}{3} = \frac{x}{2} - 1$

$\therefore 2(2x+1) = 3x - 6$

$4x + 2 = 3x - 6$

$x = -8$ ✓✓

(d) $x^2 = 10x$

$x(x-10) = 0$

$\therefore x = 0$ or 10 ✓✓

(e) $x^2 = 3x + 1$

$\therefore x^2 - 3x - 1 = 0$

$\therefore x = \frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2}$

$\therefore x = \frac{3 \pm \sqrt{13}}{2}$ ✓✓

(f) $(x-3)^2 = 2$

$\therefore x - 3 = \pm\sqrt{2}$ $\therefore x = 3 \pm \sqrt{2}$ ✓✓

F-continued

(2) $2x - 3y = 5$ --- (1)

$5x + 2y = -16$ --- (2)

$2 \times (1) \rightarrow 4x - 6y = 10$ ---(3)

$3 \times (2) \rightarrow 15x + 6y = -48$ --- (4)

$(3) + (4) \rightarrow 19x = -38$

$\therefore x = -2$

Sub. in (1) $\rightarrow 3y = 2x - 5$

$= 2(-2) - 5$

$= -9$

$\therefore y = -3$ $\therefore$ $x = -2, y = -3$ ✓✓

(3) $1 - 2x > 7$

$-2x > 6$

$\therefore x < -3$ ✓✓

(4) $A = \dfrac{mx + 1}{x + m}$ ✓

$\therefore Ax + Am = mx + 1$

$Ax - mx = 1 - Am$ ✓

$x(A - m) = 1 - Am$

$\therefore x = \dfrac{1 - Am}{A - m}$ ✓

(or, $x = \dfrac{Am - 1}{m - A}$)

20

2016 Y9 T.4

G ① (a) $15x - 3(2-3x) = 15x - 6 + 9x$ ✓

$= 24x - 6$ ✓

(b) $(5x-1)^2 = 25x^2 - 10x + 1$ ✓

② (a) $18 - 32x^2 = 2(9 - 16x^2)$ ✓

$= 2(3-4x)(3+4x)$ ✓

(b) $x^2 - 15x - 100 = (x-20)(x+5)$ ✓

(c) $3x^2 - 11x - 4 = (3x+1)(x-4)$ ✓✓

(d) $a^3 + 64 = (a+4)(a^2 - 4a + 16)$ ✓✓

③ (a) $\frac{x-1}{2} - \frac{x+3}{3} = \frac{3(x-1) - 2(x+3)}{6}$ ✓

$= \frac{3x - 3 - 2x - 6}{6}$

$= \frac{x-9}{6}$ ✓

(b) $\frac{x+2}{x^2-x-2} \div \frac{x^2-4}{3x+3} = \frac{\cancel{x+2}}{(x-2)\cancel{(x+1)}} \times \frac{3\cancel{(x+1)}}{(x-2)\cancel{(x+2)}}$ ✓✓

$= \frac{3}{(x-2)^2}$ ✓

/15

2016 Y9 T.4

H

① 1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11, 11

(a) range = 11 − 1 = 10 ✓

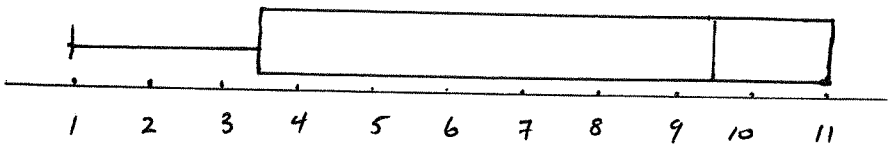
(b) median = $\frac{9+10}{2}$ = 9.5 ✓

(c) $Q_L = \frac{2+5}{2} = 3.5$

$Q_u = \frac{11+11}{2} = 11$ ✓

(d) interquartile range = 11 − 3.5 = 7.5 ✓

(e)



✓✓

② Original total = 14 × 35 = 490

New mean is 36.

Let new score be x.

$\therefore \frac{490 + x}{15} = 36$

$490 + x = 540$

$\therefore x = 50$

new score is 50. ✓✓

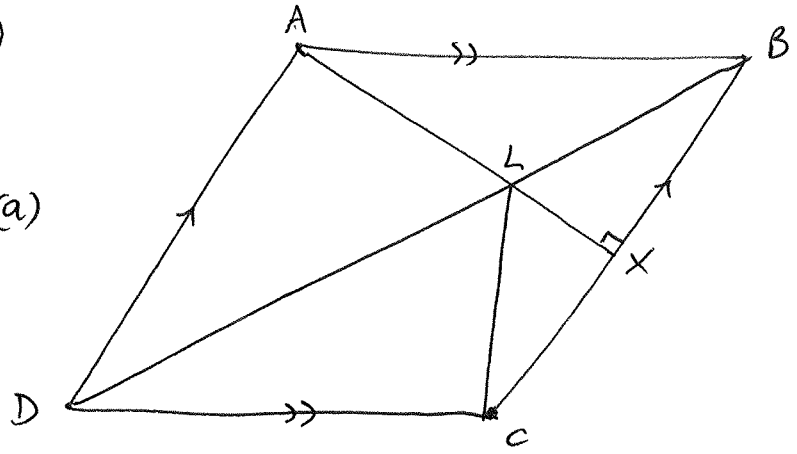
/8

2016 Y9 T.4

I

(1)

(a)



(b) $\angle ADB = \angle CDB$ (diagonals bisect vertex angles in a rhombus). ✓

(c) In $\triangle ALD$, $\triangle CLD$

$AD = CD$ (all sides of rhombus are equal) ✓

$\angle ADL = \angle CDL$ (proven in (a))

DL is common ✓

$\therefore \triangle ALD \equiv \triangle CLD$ (SAS) ✓

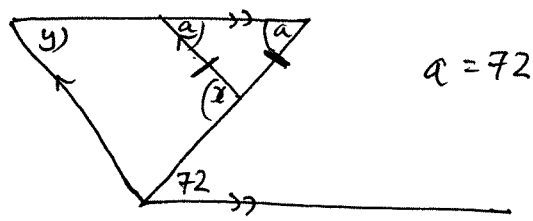
(d) $\angle DAL = \angle DAX$ (same angle)

$= \angle AXB$ (alternate angles, $AD \parallel BC$) ✓

$= 90^\circ$

(e) $\angle LCD = 90^\circ$ (matching angle in congruent triangle) ✓

(2)



$a = 72 \therefore x = 2 \times 72$

$\therefore x = 144^\circ$ ✓

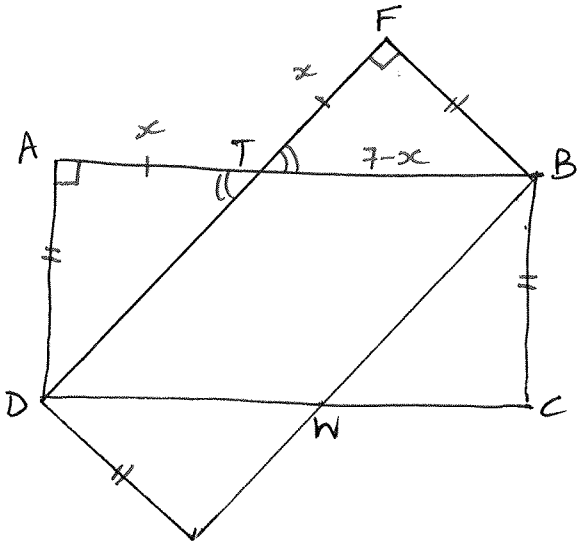
$y = 72^\circ$ ✓

(3) Rectangle; diagonals bisect and are equal. ✓✓

10

J

(1) (a)



(b) In ΔATD, ΔFTB,

$\angle TAD = \angle TFB$

$= 90^\circ$ (angles in ~~right angle~~ rectangle)

$AD = FB$ (matching sides in congruent rectangles)

$\angle ATD = \angle FTB$ (vertically opposite angles)

$\therefore \Delta ATD \equiv \Delta FTB$ (AAS) ✓✓

(c) Let $AT = FT = x$.

$\therefore TB = 7 - x$ (as $AB = 7$)

Also, $FB = DE = 3$

So by Pythagoras in ΔFTB,

$x^2 + 3^2 = (7-x)^2$

$x^2 + 9 = x^2 - 14x + 49$

$14x = 40 \quad \therefore x = \frac{40}{14}$ ie $AT = \frac{20}{7}$ ✓✓

(d) DWBT is a parallelogram,

$\therefore$ Area DWBT $= TB \times BC = \left(7 - \frac{20}{7}\right) \times 3$ ✓✓

Area $= \frac{87}{7} = 12\frac{3}{7}\ u^2$

$(\doteq 12{\cdot}43\ u^2.)$

J (2) Let number of people be n.

$\therefore$ $n-2$ people paid.

They each paid $\frac{120}{n-2}$.

This, we are told, \$3 more than what they'd have had to pay if all n paid up.

i.e. $\frac{120}{n-2} = \frac{120}{n} + 3$ ✓

Thus $120n = 120(n-2) + 3 \cdot n \cdot (n-2)$

$120n = 120n - 240 + 3n^2 - 6n$

$\therefore 3n^2 - 6n - 240 = 0$

$n^2 - 2n - 80 = 0$

$(n-10)(n+8) = 0$

$\therefore n = 10$ or $n = -8$

But $n > 0 \therefore n = 10$ $\therefore$ 10 people in the group. ✓✓

(3) Let radius of balls = radius of cylinder = r

So $\therefore$ cylinder's height is $6r$.

$\therefore$ Volume of balls $= 3 \times \frac{4}{3}\pi r^3$

$= 4\pi r^3$

& Volume cylinder $= \pi \cdot r^2 \cdot 6r$

$= 6\pi r^3$

$\therefore \frac{V_{balls}}{V_{cylinder}} = \frac{4\pi r^3}{6\pi r^3} = \frac{2}{3}$ ✓✓✓

12