NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2015 YEAR 9 FINAL EXAMINATION

Student Name:_____

Class: _____ **Teacher**_____

General Instructions

- Working time – **90 minutes**
- Write on **one side** of your paper only
- Write using blue or black pen.
- **Board approved** calculators are allowed.
- All necessary working should be shown in every question.
- Start each new **question** on a **new page.**
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Section	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	/9	/9	/21	/15	/7	/11	/17	/14	/7	/9	/119	/100

Question 1 *(9 Marks)*

Marks

(a) A desk is measured to have a width of 160 cm (to the nearest cm). Calculate the:

(i) Absolute error **1**

(ii) Relative error **1**

(b) Round each of the following:

(i) 1.0238 (to 3 sig fig) **1**

(ii) 50001 (to 2 sig fig) **1**

How many significant figures in:

(iii) 0.000012 **1**

(iv) 1.020 **1**

(c) A JPEG image is 50 KB. How many copies could fit on a 2 Terabyte hard drive. Express your answer in scientific notation. **3**

Question 2 *(9 Marks)*

(a) Mr Cheung works as a Mathematics teacher . He earns $ 94 754 p.a.
He pays $100 p.a. for the membership of a professional organisation and $ 1127 p.a in union fees. He spends an additional $700 of his own money on classroom supplies.

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

(i) Calculate Mr Cheung's taxable income **1**

(ii) Use the tax table above to calculate the amount his annual amount of income tax. **1**

(iii) Mr Cheung's employer takes $ 860 per fortnight out of his annual salary for income tax. Calculate the size of Mr Cheung's tax refund/bill. **2**

(You may assume that Mr Cheung has private health insurance and does not need to pay the medicare levy)

(b) Amy invests $ 1500 into a bank account that pays 3.6% p.a. interest compounded monthly.

(i) How much will she have at the end of four years? **2**

(ii) How much interest has she earned? **1**

(iii) What would be the equivalent flat rate of interest? **2**

Question 3 (21 Marks)

(a) Expand and simplify each of the following:

(i) $(2x + 1) - 3(x - 1)$ **2**

(ii) $(x - 2)(2x + 3)$ **2**

(iii) $(x + 2)^3$ **2**

(b) Factorise fully:

(i) $ab^2 - 2abc + a^2b^2$ **1**

(ii) $x^2 + 3x - 28$ **2**

(iii) $3 + 2x - 4x^2$ **2**

(iv) $8p^3 + 27q^6$ **3**

(c) Simplify the following fractions:

(i) $\frac{x}{2} - \frac{y}{7}$ **1**

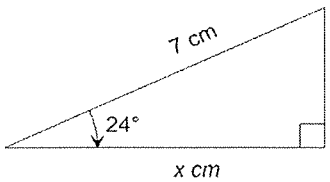
(ii) $\frac{x^2+10x+25}{4x} \div \frac{x^2-25}{2x^2-10x}$ **3**

(iii) $\frac{3}{x+2} - \frac{2x+7}{x^2+5x+6}$ **3**

Question 4 (15 Marks)

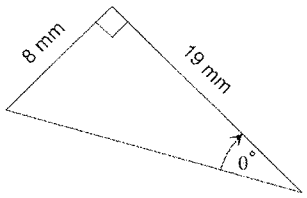
(a) Calculate the value of the pronumerals in each of the following:

(i)



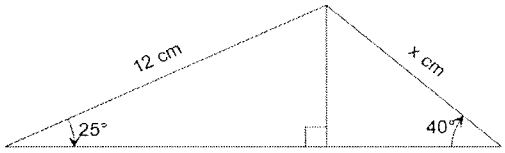
2

(ii)



2

(iii)

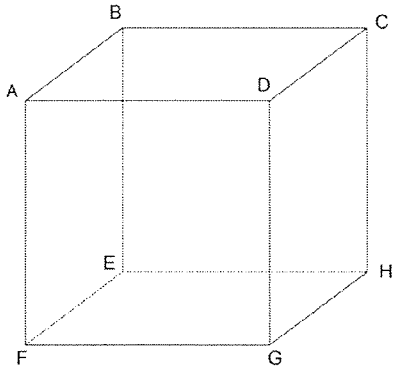


3

(b) Sketch the graph of $y = cos\frac{x}{2}$ for $0 \leq x \leq 360°$ **3**

(c) If $\sin x = -\frac{2}{5}$, find the possible values for $\tan x$ **2**

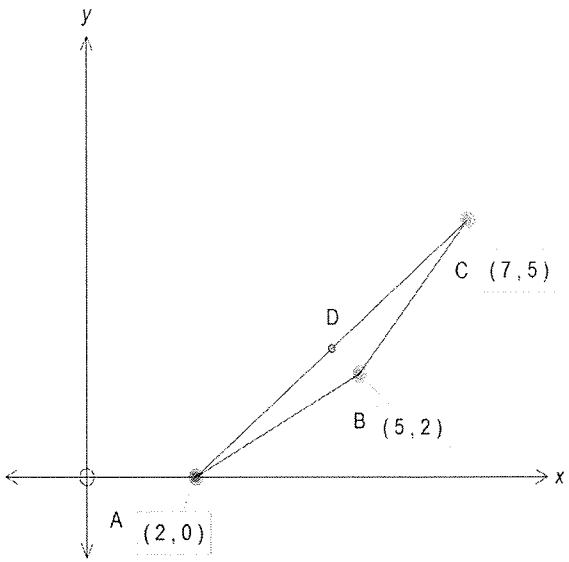
(d) ABCDEFGH is a cube.



Find the size of $\angle CFH$ **3**

Question 5 (7 Marks)

The points A(2,0), B(5,2) and C(7,5) are joined to form a triangle as shown below.
D is the midpoint of AC.



(i) Find the length of AC **1**

(ii) Find the coordinates of D **1**

(iii) Find the gradient of DB and prove that D is perpendicular to AC **2**

(iv) BD is extended to E so that $BD = DE$. Find the coordinates of the point E. **1**

(v) Find the area of quadrilateral ABCE **2**

Question 6 (11 Marks)

(a) Simplify each of the following:

(i) $2x^2 \times 3x$ **1**

(ii) $\frac{(5x^3)^2}{15x}$ **2**

(b) Express the following without any fractional or negative indices:

(i) x^{-2} **1**

(ii) $2^{\frac{2}{3}}$ **1**

(iii) $(\frac{x}{y})^{-1}$ **1**

(c) Simplify: $\sqrt{75} - \sqrt{12}$ **2**

(d) Express $5\sqrt{7}$ as a single surd **1**

(e) Rationalise the denominator and fully simplify:

$$\frac{3}{\sqrt{5} - \sqrt{2}}$$ **2**

Question 7 (17 Marks)

(a) Solve each of the following

(i) $5x - 1 = 24$ **1**

(ii) $2(x + 5) = 3(x - 2)$ **2**

(iii) $\frac{x}{4} - 1 = \frac{2x}{3} + 1$ **3**

(b) Solve and sketch on a number line:

$3x - 4 < x + 2$ **2**

(c) Solve the following equations:

(i) $x^2 = 5x$ **1**

(ii) $x^2 + x = 20$ **2**

(iii) $2x^2 - 3x - 4 = 0$ **3**

(d) Solve the pair of simultaneous equations.

$3x + 4y = 5$
$2x + 7y = 12$ **3**

Question 8 (14 Marks)

(a)

4 , 5 , 5 , 7 , 9 , 9 , 9 , 9 , 12 , 15 , 16 , 25

For the numbers above find the:

(i) mean **1**

(ii) median **1**

(iil) mode **1**

(iv) represent the data on a box and whiskers graph **3**

(b) David is playing Starcraft and records his average APM (*Actions Per Minute*) each match.

The results are presented in the table below

APM	cc	Frequency	fx	cf
0 - <50		2		
50 - <100		6		
100 - <150		8		
150 - <200		4		
200 - <250		3		
250 - <300		2		

(i) Copy and complete the table in your answer book **3**

(ii) Calculate David's mean APM **1**

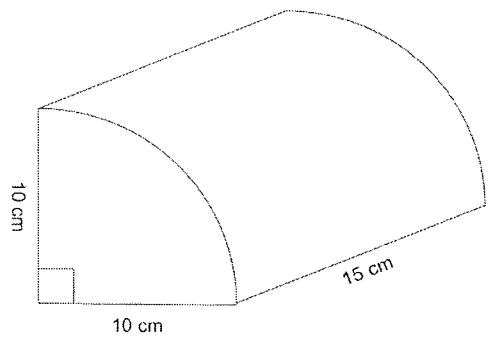
(iii) Represent this information with a cumulative frequency histogram and polygon **3**

(iv) Use the graph from part (iii) to estimate David's median APM. **1**

Question 9 (7 Marks) **Marks**

(a) A sphere has a surface area of $36\pi\ cm^2$. Find its volume. **2**
Express your answer as an exact value.

(b)



The figure above is a quarter-cylinder

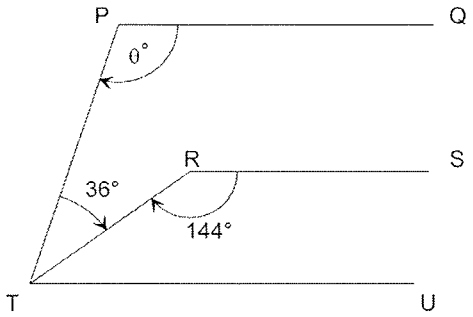
Calculate the:

(i) Volume **2**

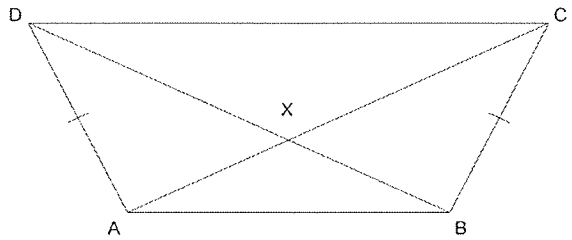
(ii) Surface area **3**

Question 10 (9 Marks)

(a) Evaluate the pronumeral. Give full reasoning **2**



(b) The diagram below shows a quadrilateral ABCD. The diagonals AC and BD are equal and intersect at X. The sides AD and BC are equal.



(i) Show that $\Delta ABC \equiv \Delta BAD$ **3**

(ii) Hence show that ΔABX isosceles **1**

(iii) Thus show that ΔCDX is also isosceles. **2**

(iv) Show that $AB \parallel DC$ **1**

End of Examination

Question 1

(a) 0.5 cm ✓

(b) (i) 1.02 ✓
(ii) 50 000 ✓
(iii) 2 ✓
(iv) 4 ✓

(c) $(2 \times 10^{12}) \div (5 \times 10^{3}) = 4 \times 10^{8}$ ✓sci not ✓division ✓answ.

Question 2

(a)(i) \$ 92827 ✓

(ii) \$ 22292.99 ✓

(iii) $(\$860 \times 26) - \$22292.99$ ✓
$= \$67.01$ ✓

(b)(i) $1500(1.003)^{48} = \$1731.95$ (2 d.p.) ✓rate ✓time ✓answ.

(ii) \$231.95 ✓

(iii) $\frac{\$231.95...}{4 \times \$1500} \times 100\% = 3.866\%$ (3 d.p.) ✓working ✓answ.

Question 3

(a) (i) $2x + 1 - 3x + 3$

$= -x + 4$ ✓ ✓

(ii) $2x^2 + 3x - 4x - 6$ ✓

$= 2x^2 - x - 6$ ✓

(b) (i) $ab(b - 2c + ab)$ ✓

(ii) $(x + 7)(x - 4)$ ✓✓

(iii) $(2x + 3)(1 - 2x)$ ✓ any method ✓ answ.

(c) (i) $\frac{7x - 2y}{14}$ ✓

(ii) $\frac{(x+5)^{\cancel{2}}}{4\cancel{x}} \times \frac{2\cancel{x}(x-5)}{(\cancel{x}-5)(\cancel{x}+5)}$ ✓ factors ✓ cancelling

$= \frac{x+5}{2}$ ✓ answ.

Question 4

(a) (i) $\cos 24^\circ = \frac{x}{7}$ ✓

$x = 7\cos 24^\circ$

$= 6.4\text{cm}$ (1 d.p.). ✓

(ii) $\tan\theta = 8/14$ ✓

$\theta = \tan^{-1}(8/14)$

$\doteq 22^\circ 50'$ (nearest minute) ✓

(iii) let h be the perpendicular height

$\sin 24^\circ = \frac{h}{12}$ ✓

$\therefore h = 12\sin 24^\circ$

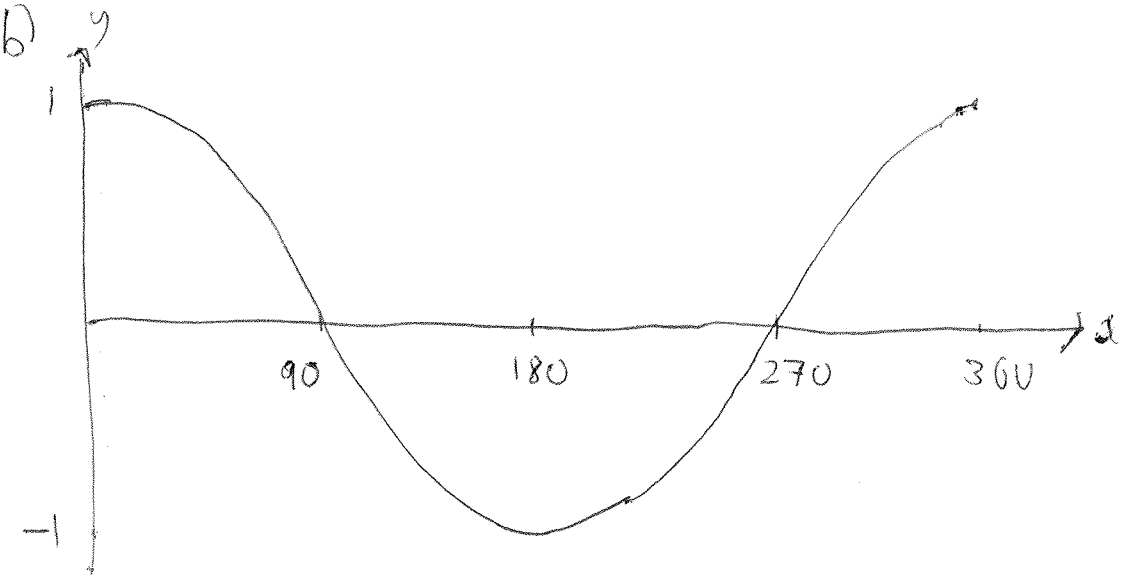
$\sin 40^\circ = \frac{h}{x}$ ✓

$\therefore x = \frac{h}{\sin 40^\circ}$

$= \frac{12\sin 24^\circ}{\sin 40^\circ}$ ✓

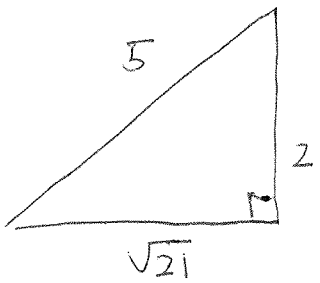
$= 7.593\text{mm}$ (3 d.p.)

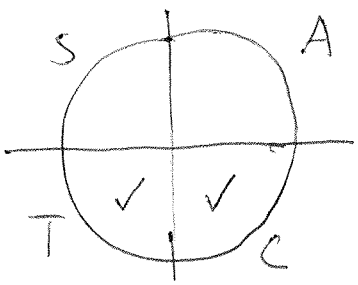
(b)



✓ shape
✓ period & amplit.

(c)





✓ working

$$\tan x = \pm \frac{2}{\sqrt{21}}$$ ✓

(d) $HF = \sqrt{1^2 + 1^2}$ (pythagorus)

$= \sqrt{2}$ ✓

$CF = \sqrt{(\sqrt{2})^2 + 1^2}$ (pythagorus)

$= \sqrt{3}$

$\therefore \sin \angle CFH = \frac{1}{\sqrt{3}}$ ✓

$\therefore \angle CFH = 35^\circ 16'$ (nearest minute) ✓

Question 5

(i) $\sqrt{50}$ ✓

(ii) $(9/2, 5/2)$ ✓

(iii) $m_{DB} = \dfrac{5/2 - 2}{9/2 - 5}$

$= \dfrac{1/2}{-1/2}$

$= -1$ ✓

$m_{AC} = \dfrac{5-0}{7-2}$

$= 1$

$m_{AC} \times M_{DB} = 1 \times (-1)$

$= -1$ ✓

$\therefore$ DB $\perp$ AC.

(iv) $(4, 3)$ ✓

(v) $A = \frac{1}{2} \times \sqrt{2} \times \sqrt{50}$ ✓

$= 5$ units^2. ✓

Question 6

(a) (i) $6x^3$ ✓

(ii) $25x^6 / 15x$ ✓

$= \frac{5x^5}{3}$ ✓

(b) (i) $\frac{1}{x^2}$ ✓

(ii) $\sqrt[3]{2^2}$ ✓

(iii) $\frac{y}{x}$ ✓

(c) $5\sqrt{3} - 2\sqrt{3}$ ✓

$= 3\sqrt{3}$ ✓

(d) $\sqrt{175}$ ✓

(e) $\frac{3}{\sqrt{5} - \sqrt{2}} \times \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}}$ ✓

$= \frac{3(\sqrt{5} + \sqrt{2})}{3}$

$= \sqrt{5} + \sqrt{2}$ ✓

Question 7

(a) (i) $2x + 10 = 3x - 6$

$\therefore x = 16$ ✓ ✓

(iii) $\frac{x}{4} - 1 = \frac{2x}{3} + 1 \quad (\times 12)$ ✓ ×12 or common denom

$3x - 12 = 8x + 12$

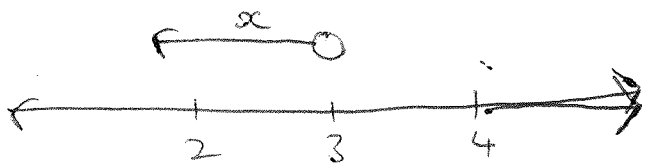
$5x = -24$ ✓

$x = -24/5.$ ✓

(b) $3x - 4 < x + 2$

$2x < 6$

$x < 3$ ✓



✓

(c) (i) $x^2 - 5x = 0$

$x(x - 5) = 0$

$x = 0 \quad \text{or} \quad x = 5$ ✓

(ii) $x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$ ✓

$x = -5 \quad \text{or} \quad x = 4.$ ✓

(d)(i) $2x^2 - 3x = 4$

$x^2 - \frac{3}{2}x = 2$

$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 2 + \left(\frac{3}{4}\right)^2$ ✓

$(x - 3/4)^2 = 2 + 9/16$

$x - 3/4 = \pm\sqrt{41/16}$ ✓

$\therefore x = \frac{3 \pm \sqrt{41}}{4}$ ✓

(ii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✓

$= \frac{-1 \pm \sqrt{1 - 4(5)(-3)}}{2 \times (-3)}$ ✓

$= \frac{-1 \pm \sqrt{61}}{-6}$

$= \frac{1 \pm \sqrt{61}}{6}$ ✓

(e) $3x + 4y = 5 \quad \ldots ①$

$2x + 7y = 12 \quad \ldots ②$

$6x + 8y = 10 \quad \ldots ③$

$6x + 21y = 36 \quad \ldots ④$

$-13y = -26$

$y = 2$

✓ method

✓ single equation

sub into ①

$3(x) + 4(2) = 5$

$3x = 5 - 8$

$3x = -3$

$x = -1$

$\therefore \boxed{\begin{array}{l} x = -1 \\ y = 2 \end{array}}$

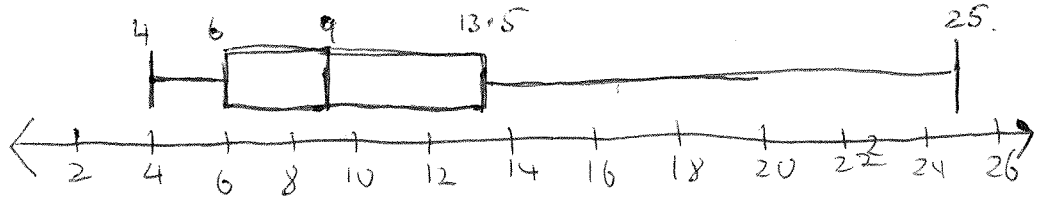
✓ answ.

Question 8.

(a)(i) $\frac{125}{12}$ ✓

(ii) 9 ✓

(iii) 9 ✓

(iv) 

✓5 pts
✓shape
✓no. line

(b)

APM	cc	Freq	fx	cf
0 - <50	25	2	50.	2
50 - <100	75	6	450	8
100 - <150	125	8	1000	16
150 - <200	175	4	700	20
200 - <250	225	3	675	23
250 - <300	275	2	550	25

✓✓✓ 1 each column

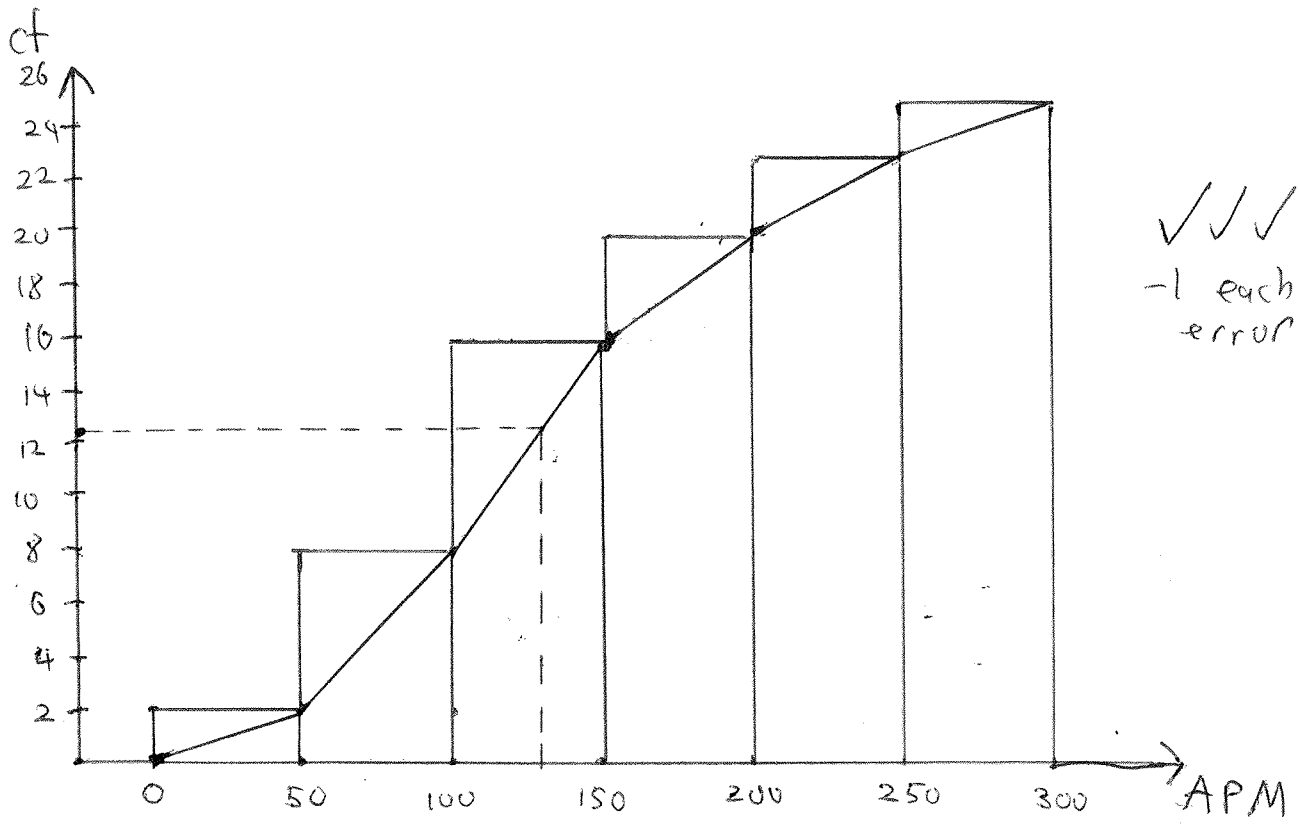
$\Sigma f = 25 \quad \Sigma fx = 3425$

(ii) $\bar{x} \div \frac{\Sigma fx}{\Sigma f}$

$\div \frac{3425}{25}$

$\div 137$ ✓

(iii)



✓✓✓
-1 each error

(iv) 125 (approx) ✓

Question 9

(a) $S.A = 4\pi r^2$

$36\pi = 4\pi r^2$ ✓

$\therefore r^2 = 9$

$r = 3$ cm (as $r > 0$).

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3^3$

$= 36\pi \text{ cm}^3$ ✓

(b)(i) $V = Ah$

$= \frac{1}{4} \times \pi \times 10^2 \times 15$ ✓

$= 375\pi \text{ cm}^3$ ✓

(ii) $SA = A_{front} + A_{back} + A_{left} + A_{bottom} + A_{right}$ ✓

$= 2\left(\frac{1}{4} \times \pi \times 10^2\right) + 2(10 \times 15) + \frac{1}{4} \times 2 \times \pi \times 10 \times 15$ ✓

$= 50\pi + 75 + 75\pi$

$= (125\pi + 75) \text{ cm}^2$. ✓

Question 10

(a) $\angle RTU = 36°$ (co-int $\angle$s $RS // TU$)

$\therefore \angle PTU = 72°$ (adjacent $\angle$s) ✓

$\therefore \theta = 108°$ (co-int $\angle$s, $TU // PQ$). ✓

(b) (i) In $\triangle ABC$ & $\triangle BAD$

AB is common ✓

$AD = BC$ (given) ✓

$AC = BD$ (given)

$\therefore \triangle ABC \equiv \triangle BAD$ (SSS). ✓

(ii) $\angle ABD = \angle BAC$ (matching sides in congruent $\triangle$s) ✓

$\therefore \triangle ABC$ is isosceles (base angles are equal)

(iii) $AC = BD$ (given)

$AX = BX$ (sides in isos $\triangle ABX$)

$\therefore AC - AX = BD - BX$ ✓

$\therefore CX = DX$

$\therefore \triangle CDX$ is isosceles (2 equal sides) ✓

(iv) $\angle AXB = \angle DXC$ (vertically opposite).

$\therefore \angle ABX = \angle CDX$ ✓

(both base angles of isosceles triangles with the same angle between the two equal sides)

$\therefore AB // DC$ (alternate angles are equal).