NORTH SYDNEY BOYS HIGH SCHOOL

# STAGE 5.1-5.3 MATHEMATICS

Year 9 Final Examination

Tuesday, November 11 2014

**General instructions**

- Working time – 90 minutes.
- Write your answers on the booklet(s) supplied.
- Marks may be deducted for careless or poorly arranged work.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklet(s) used in the correct order within this paper and hand to examination supervisors.

**Class** (please ✔)

- ◯ 9MAT.A – Mr Bouckley
- ◯ 9MAT.B – Miss Lee
- ◯ 9MAT.C – Mr Berry
- ◯ 9MAT.D – Mr Lam
- ◯ 9MAT.E – Mr Weiss

**NAME:** ..... **# BOOKLET USED:** .....

Marker's use only.

| **QUESTION** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | **Total** | % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **MARKS** | /9 | /9 | /13 | /17 | /14 | /15 | /13 | /9 | /15 | /114 | |

**Question 1** (9 Marks) Commence a NEW page. **Marks**

(a) In 1990 Alex bought a new car for \$22 500 and after 7 years traded it in for \$7 537.50. What is his average annual rate of loss in value expressed as a percentage of the original price? **2**

(b) i. Samantha earns \$42 750 per year. Use the tax table below to calculate her annual tax payable on the salary. In your calculation, include the Medicare levy of 2%. **2**

| Taxable income | Tax on this income |
|---|---|
| \$0 – \$18 200 | Nil |
| \$18 201 – \$37 000 | 19c for each \$1 over \$18 200 |
| \$37 001 – \$80 000 | \$3 572 plus 32.5c for each \$1 over \$37 000 |
| \$80 001 – \$180 000 | \$17 547 plus 37c for each \$1 over \$80 000 |
| \$180 001 and over | \$54 547 plus 45c for each \$1 over \$180 000 |

ii. Samantha receives a 7.5% increase in her salary. Calculate her new annual salary. **1**

(c) Tristan and Isolde borrow \$33 000 to build a new swimming pool at an simple interest rate of 11.5% p.a., to be repaid over five years.

i. Calculate the total interest to be repaid. **1**

ii. How much must they repay in total? **1**

iii. Determine the monthly repayment amount. **2**

**Question 2** (9 Marks) Commence a NEW page. **Marks**

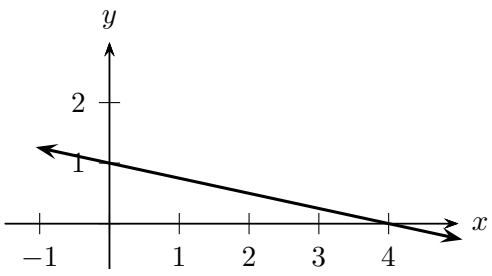
(a) Find the exact distance between the points $(-1, 0)$ and $(2, 5)$. **2**

(b) Find the gradient of the line joining the points $(-2, 4)$ and $(-1, -4)$. **2**

(c) Find the equation of the line in general form, with gradient of 3 and $y$ intercept of 2. **2**

(d) Show that $(-3, -10)$ lies on the line $y = 3x - 1$. **1**

(e) Write the general form of the equation of the following line: **2**



**Question 3** (13 Marks) Commence a NEW page. **Marks**

(a) Express $\dfrac{3}{\sqrt{5}}$ with a rational denominator. **1**

(b) Express $\sqrt{162}$ in the form $a\sqrt{b}$. **1**

(c) Fully simplify $\left(\sqrt{5}-8\right)\left(\sqrt{5}+8\right)$. **2**

(d) If $\left(2+3\sqrt{5}\right)^2 = a+\sqrt{b}$, find the value of $a$ and $b$. **2**

(e) Fully simplify:

i. $\left(5p^2\right)^3 \times 3p$. **1**

ii. $3^x \times 3^{x+1}$. **1**

iii. $6a^4b^3 \div a^3b^5$. **1**

(f) Write $4x^{-3}$ without a negative index. **1**

(g) Evaluate: $\left(\dfrac{1}{4}\right)^{-3}$. **1**

(h) Fully simplify: $\dfrac{2^x+2^{x+1}}{2^x}$. **2**

**Question 4** (17 Marks) Commence a NEW page. **Marks**

(a) Express $3.12 \times 10^4 \times 4.2 \times 10^6$ in scientific notation. **1**

(b) Expand and simplify:

i. $(x+3)(x-8)$. **2**

ii. $(x-y)^2-(x+y)^2$. **2**

(c) Fully factorise:

i. $21xy-3x$. **1**

ii. $3d^2-3$. **2**

(d) Fully simplify:

i. $\dfrac{2x+5}{6}+\dfrac{x-4}{3}$. **2**

ii. $\dfrac{x}{3}-\dfrac{x+1}{5}$. **2**

iii. $\dfrac{3x-6}{4}\times\dfrac{8x}{x^2-2x}$. **2**

iv. $\dfrac{x^2-y^2}{x^2+x}\div\dfrac{x-y}{x+1}$. **3**

**Question 5** (14 Marks) Commence a NEW page. **Marks**

(a) Solve for $x$:

i. $\dfrac{3x+2}{5} - \dfrac{x-4}{3} = 6$ **3**

ii. $\dfrac{5-2x}{3} < 4.$ **2**

(b) Change the subject to $d$: $S = \dfrac{n}{2}(2a + (n-1)d)$ **2**

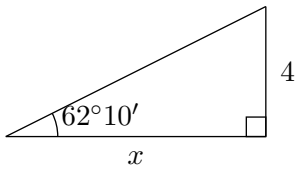
(c) Solve simultaneously: $\begin{cases} 3x - 2y = 4 \\ 2x + 3y = 10 \end{cases}$ **3**

(d) A mathematics test had 20 questions. Some questions are worth 2 marks, whilst others were worth 3 marks. The test had a total of 52 marks. **4**

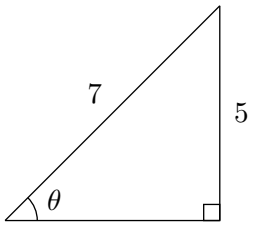
By establishing a pair of simultaneous equations and solving them, find the number of each type of question in the test.

**Question 6** (15 Marks) Commence a NEW page. **Marks**

(a) Find the value of $x$, correct to one decimal place. **2**




(b) Find the value of $\theta$, correct to the nearest minute. **2**




(c) Sketch the graph of $y = 3\sin 2x$ for $0° \leq x \leq 360°$. **3**

(d) Find the exact value of

i. $\sin 315°$. **2**

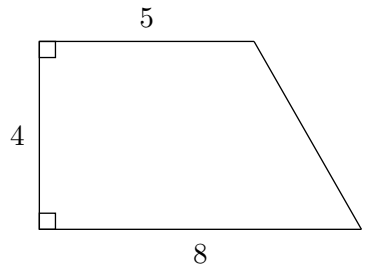
ii. $\cos^2 60°$. **2**

(e) Solve $\cos^2\theta = \dfrac{1}{2}$ for $0° \leq \theta \leq 360°$. **3**

(f) In which quadrant does $\theta$ lie in, if $\cos\theta < 0$ and $\tan\theta > 0$? **1**

**Question 7** (16 Marks) Commence a NEW page. **Marks**

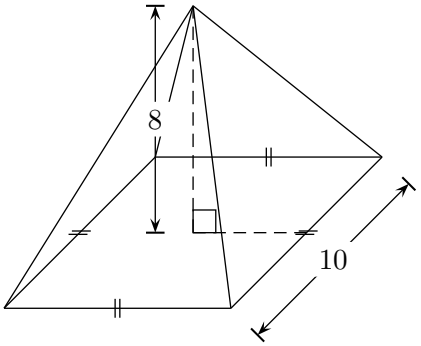
(a) For the following shape, find the:




i. area **1**

ii. perimeter **2**

(b) For the following square pyramid, find the:




i. volume **1**

ii. surface area **3**

(c) Solve for $x$:

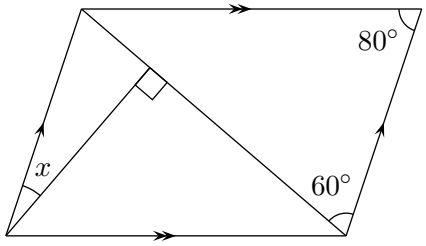
i. $x^2 - 11x + 10 = 0$ **2**

ii. $3x^2 + 11x - 4 = 0$ **2**

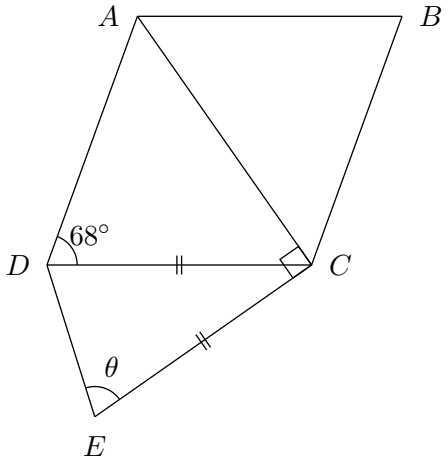
iii. $(x-3)^2 = 16$. **2**

**Question 8** (9 Marks) Commence a NEW page. **Marks**

(a) Find the value of $x$ in the diagram. No reasons are required. **2**

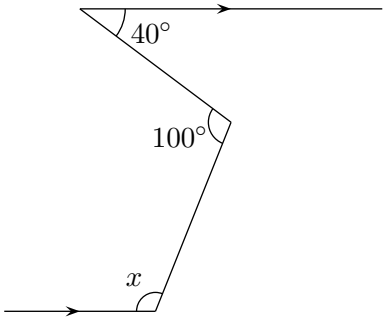



(b) $ABCD$ is a rhombus. $DC = EC$. $\angle ADC = 68°$ and $\angle ACE = 90°$. **2**

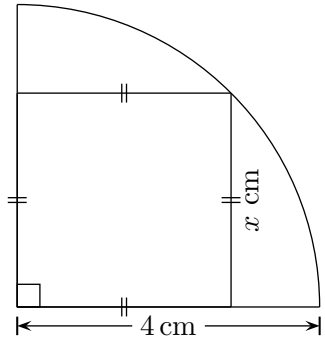



Find the value of $\theta$. No reasons are required.

(c) Find the value of $x$ in the diagram. No reasons are required. **2**




(d) A square of side length $x$ is a drawn in a quadrant of a circle with radius 4 cm. **3**




Find the value of $x$.

**Question 9** (12 Marks) Commence a NEW page. **Marks**

(a) For the following set of scores:

2 7 5 5 7 3 6 7

Find the:

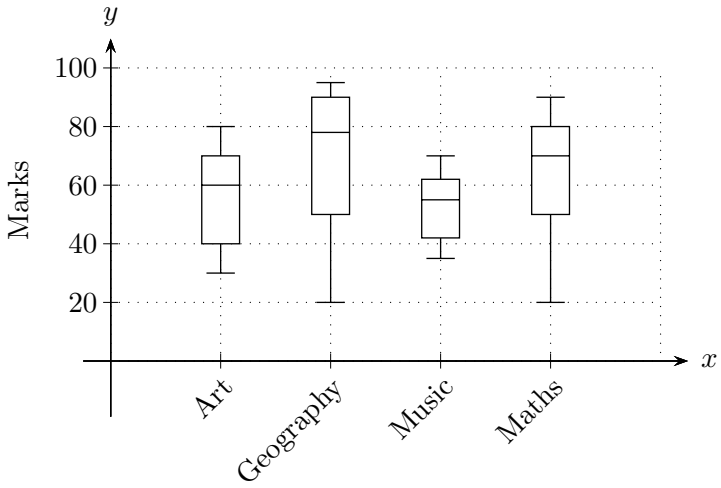
i. mean **1**

ii. median **1**

iii. mode **1**

iv. range **1**

(b) The following box and whisker plots show examination results in four subjects.




i. Which subject had the largest range? **1**

ii. Which subject has the highest median? **1**

iii. Which subject has an upper quartile of 80? **1**

iv. Which subject has an interquartile range of 20? **1**

(c) In an epidemic, a group of patients had a certain skin complaint. The duration of the rash from the onset to the cure is shown below.

| Duration (days) | Class centre | # of patients |
|---|---|---|
| 5–9 | | 20 |
| 10–14 | | 60 |
| 15–19 | | 35 |
| 20–24 | | 14 |
| 25–29 | | 8 |
| 30–34 | | 1 |
| 35–39 | | 2 |

i. Copy and complete this table. **2**

ii. Calculate the mean. **2**

iii. What is the modal class for this data? **1**

iv. If you were a doctor and a new patient asked how long the rash would last, what would you reply? Give reasons for your answer. **2**

**End of paper.**

# Suggested Solutions

**Question 1** (9 marks)

(a) (2 marks)

✓ [−1] for each new error introduced.

$$\begin{aligned}\text{Loss} &= \$22\,500 - 7\,537.5\\ &= 14\,962.5\\ \text{Loss as \% of cost} &= \frac{14\,962.5}{22\,500}\\ &= 0.665\\ \text{Over 7 years} &= \frac{0.665}{7}\\ &= 0.095 = 9.5\%\end{aligned}$$

(b) i. (2 marks)

$$\begin{aligned}\text{PAYG} &= 3\,572\\ &\quad + 0.325(42\,750 - 37\,000)\\ &= \$5\,440.75\\ \text{M/care} &= 42\,750 \times 0.02\\ &= \$855\\ \text{Payable} &= \$5\,440.75 + \$855\\ &= \$6\,295.75\end{aligned}$$

ii. (1 mark)

$$\begin{aligned}\text{New salary} &= 42\,750 \times 1.075\\ &= \$45\,956.25\end{aligned}$$

(c) i. (1 mark)

$$P = 33\,000 \quad r = 0.115 \quad n = 5 \text{ yrs}$$
$$\begin{aligned}I &= Prn\\ &= 33\,000 \times 0.115 \times 5 = 18\,975\end{aligned}$$

ii. (1 mark)

$$\begin{aligned}\text{Total repaid} &= P + I\\ &= 33\,000 + 18\,975\\ &= \$51\,975\end{aligned}$$

iii. (2 marks)

$$\text{Monthly repayment} = \frac{\$51\,975}{60} = \$866.25$$

**Question 2** (9 marks)

(a) (2 marks)

✓ [1] for working.

✓ [1] for final answer.

$$\begin{aligned}&\quad (-1,0) \qquad (2,5)\\ d &= \sqrt{(2-(-1))^2 + (5-0)^2}\\ &= \sqrt{34}\end{aligned}$$

(b) (2 marks)

✓ [1] for working.

✓ [1] for final answer.

$$\begin{aligned}&\quad (-2,4) \qquad (-1,-4)\\ m &= \frac{4-(-4)}{-2-(-1)}\\ &= -8\end{aligned}$$

(c) (2 marks)

$$\begin{aligned}m = 3 \qquad b = 2\\ y = 3x + 2\\ 3x - y + 2 = 0\end{aligned}$$

(d) (1 mark)

When $x = -3$,

$$\begin{aligned}y &= 3(-3) - 1\\ &= -9 - 1 = -10\end{aligned}$$

Hence $(-3, -10)$ lies on the line $y = 3x - 1$.

(e) (2 marks)

✓ [1] for gradient-intercept form

✓ [1] for final answer.

$$m = -\frac{1}{4} \qquad b = 1$$
$$\therefore \underset{\times 4}{y} = \underbrace{-\frac{1}{4}x + 1}_{\times 4}$$
$$\begin{aligned}4y &= -x + 4\\ x + 4y - 4 &= 0\end{aligned}$$

**Question 3** (13 marks)

(a) (1 mark)

$$\frac{3}{\sqrt{5}} \frac{\times\sqrt{5}}{\times\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

(b) (1 mark)

$$\sqrt{162} = \sqrt{2 \times 81} = 9\sqrt{2}$$

(c) (2 marks)

✓ [1] for proper expansion of difference of two squares.

✓ [1] for final answer.

$$\left(\sqrt{5} - 8\right)\left(\sqrt{5} + 8\right) = 5 - 64 = -59$$

(d) (2 marks)

✓ [1] for correct expansion.

✓ [1] for values of $a$ and $b$.

$$\begin{aligned}\left(2 + 3\sqrt{5}\right)^2 &= 4 + 12\sqrt{5} + 45 \\ &= 49 + 12\sqrt{5} \\ &= 49 + \sqrt{144 \times 5} \\ \therefore a = 49 \qquad & b = 720\end{aligned}$$

(e) i. (1 mark)

$$125p^6 \times 3p = 375p^7$$

ii. (1 mark)

$$3^x \times 3^{x+1} = 3^{2x+1}$$

iii. (1 mark)

$$6a^4b^3 \div a^3b^5 = \frac{6a^4b^3}{a^3b^5} = \frac{6a}{b^2}$$

(f) (1 mark)

$$4x^{-3} = \frac{4}{x^3}$$

(g) (1 mark)

$$\left(\frac{1}{4}\right)^{-3} = 4^3 = 64$$

(h) (2 marks)

$$\frac{2^x + 2^{x+1}}{2^x} = \frac{2^x + (2 \times 2^x)}{2^x} = \frac{\cancel{2^x}(1+2)}{\cancel{2^x}} = 3$$

**Question 4** (17 marks)

(a) (1 mark)

$$3.12 \times 10^4 \times 4.2 \times 10^6 = 1.310\,4 \times 10^{11}$$

(b) i. (2 marks)

$$(x+3)(x-8) = x^2 - 5x - 24$$

ii. (2 marks)

$$\begin{aligned}(x-y)^2 - (x+y)^2 &= x^2 - 2xy + y^2 \\ &\quad - \left(x^2 + 2xy + y^2\right) \\ &= -4xy\end{aligned}$$

(c) i. (1 mark)

$$21xy - 3x = 3x(7y - 1)$$

ii. (2 marks)

$$3d^2 - 3 = 3\left(d^2 - 1\right) = 3(d-1)(d+1)$$

(d) i. (2 marks)

$$\begin{aligned}\frac{2x+5}{6} + \frac{x-4}{3} &= \frac{2x + 5 + 2(x-4)}{6} \\ &= \frac{4x - 3}{6}\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}\frac{x}{3} - \frac{x+1}{5} &= \frac{5x - 3(x+1)}{15} \\ &= \frac{2x - 3}{15}\end{aligned}$$

iii. (2 marks)

$$\begin{aligned}\frac{3x-6}{4} \times \frac{8x}{x^2 - 2x} &= \frac{\cancel{3(x-2)}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}x}{\cancel{x(x-2)}} \\ &= 6\end{aligned}$$

iv. (3 marks)

$$\begin{aligned}\frac{x^2 - y^2}{x^2 + x} \div \frac{x-y}{x+1} &= \frac{\cancel{(x-y)}(x+y)}{\cancel{x(x+1)}} \times \frac{\cancel{x+1}}{\cancel{x-y}} \\ &= \frac{x+y}{x}\end{aligned}$$

**Question 5** (14 marks)

(a) i. (3 marks)

✓ [1] for combining fractions.
✓ [1] for correct numerator.
✓ [1] for final answer.

$$\begin{aligned}
\frac{3x+2}{5}\underset{\times 3}{\times\frac{3}{3}} - \frac{x-4}{3}\underset{\times 5}{\times\frac{5}{5}} &= 6 \\
\underset{\times 15}{\frac{3(3x+2)-5(x-4)}{15}} &= \underset{\times 15}{6} \\
9x+6-5x+20 &= 90 \\
4x+26 &= 90 \\
4x &= 64 \\
x &= 16
\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}
\underset{\times 3}{\frac{5-2x}{3}} &< \underset{\times 3}{4} \\
\underset{-5}{5-2x} &< \underset{-5}{12} \\
\underset{\div(-2)}{-2x} &< \underset{\div(-2)}{7} \\
x &> -\frac{7}{2}
\end{aligned}$$

(b) (2 marks)

$$\begin{aligned}
S &= \frac{n}{2}\left(2a+(n-1)d\right) \\
\frac{2S}{n} &= 2a+d(n-1) \\
\frac{2S}{n}-2a &= d(n-1) \\
\therefore d &= \frac{\frac{2S}{n}-2a}{n-1} = \frac{2S-2an}{n(n-1)}
\end{aligned}$$

(c) (3 marks)

✓ [1] for multiplying up by 2 and 3 respectively.
✓ [1] for significant progress towards $y = \frac{22}{13}$.
✓ [1] for $x = \frac{32}{13}$.

$$\begin{cases} 3x-2y=4 & (1) \\ 2x+3y=10 & (2) \end{cases}$$

(1) × 2, (2) × 3:

$$\begin{cases} 6x-4y=8 & (3) \\ 6x+9y=30 & (4) \end{cases}$$

(4) − (3):

$$\begin{aligned}
13y &= 22 \\
y &= \frac{22}{13}
\end{aligned}$$

Substitute into (1):

$$\begin{aligned}
3x-2\left(\frac{22}{13}\right) &= 4 \\
3x-\frac{44}{13} &= 4 \\
3x &= 4+\frac{44}{13} = \frac{96}{13} \\
\therefore x &= \frac{32}{13}
\end{aligned}$$

(d) (4 marks)

✓ [1] setting up pair of simultaneous equations.
✓ [1] for attempts to eliminate or substitute.
✓ [1] each for correct $x$ and $y$.

Let $x$ be the number of 2 mark questions, $y$ be the number of 3 mark questions.

$$\begin{cases} x+y=20 & (1) \\ 2x+3y=52 & (2) \end{cases}$$

(1) ×2,

$$2x+2y=40 \qquad ((3))$$

(2) − (3):

$$y = 12$$

Substitute to (1):

$$\begin{aligned}
\therefore x+12 &= 20 \\
x &= 8
\end{aligned}$$

**Question 6** (15 marks)

(a) (2 marks)

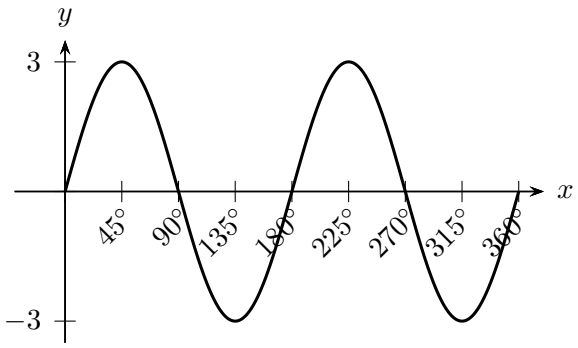
$$\frac{4}{x} = \tan 62°10'$$
$$\therefore x = \frac{4}{\tan 62°10'} = 2.1$$

(b) (2 marks)

$$\sin\theta = \frac{5}{7}$$
$$\theta = 45°35'$$

(c) (3 marks)

✓ [1] for shape
✓ [1] for amplitude
✓ [1] for period




(d) i. (2 marks)

$$\sin 315° = -\sin 45° = -\frac{1}{\sqrt{2}}$$

ii. (2 marks)

$$\cos^2 60° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

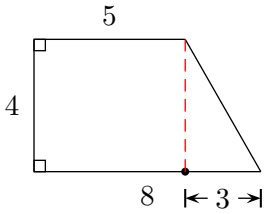
(e) (3 marks)

$$\cos^2\theta = \frac{1}{2}$$
$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$
$$\theta = 45°, 135°, 225°, 315°$$

(f) (1 mark) 3rd

**Question 7** (16 marks)

(a) i. (1 mark)



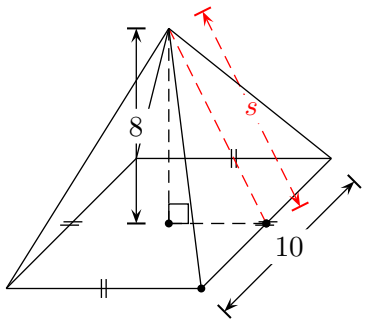

$$A = \frac{1}{2}h(a+b) = \frac{4}{2}(5+8) = 26$$

ii. (2 marks)

Length of slanted side: 5

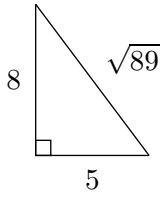
$$P = 4+5+8+5 = 22$$

(b) i. (1 mark)




$$V = \frac{1}{3}Ah = \frac{1}{3}\times 10^2 \times 8 = \frac{800}{3}$$

ii. (3 marks)




$$s = \sqrt{8^2+5^2}$$
$$= \sqrt{89}$$
$$A_{\text{side triangle}} = \frac{1}{2}bh = \frac{1}{2}\sqrt{89}\times 10$$
$$= 5\sqrt{89}$$
$$SA = 4A_{\text{side triangle}} + A_{\text{base}}$$
$$= 20\sqrt{89} + 100$$

(c) i. (2 marks)

$$x^2 - 11x + 10 = 0$$
$$(x-10)(x-1) = 0$$
$$x = 1, 10$$

ii. (2 marks)

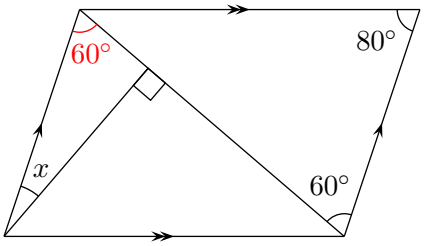
$$3x^2 + 11x - 4 = 0$$
$$(3x - 1)(x + 4) = 0$$
$$x = \frac{1}{3}, -4$$

iii. (2 marks)

$$(x - 3)^2 = 16$$
$$x - 3 = \pm 4$$
$$x = 3 \pm 4 = -1 \text{ or } 7$$

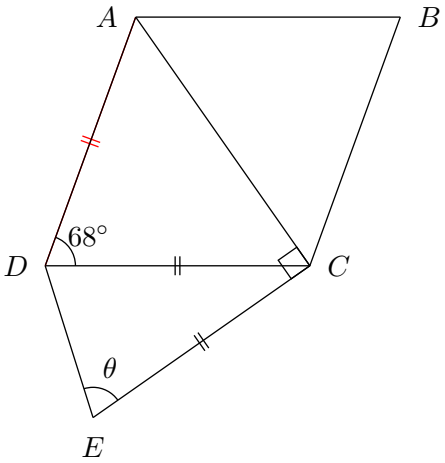
**Question 8** (9 marks)

(a) (2 marks)



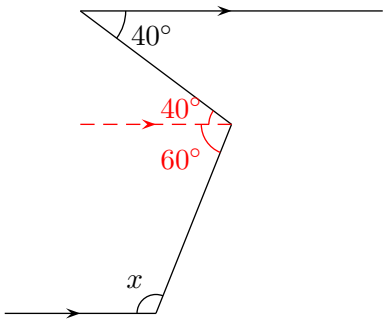

$$x = 30^\circ$$

(b) (2 marks)




$$\angle DAC = \angle DCA = \frac{180^\circ - 68^\circ}{2} = 56^\circ$$
$$\therefore \angle DCE = 90^\circ - 56^\circ = 34^\circ$$
$$\theta = \frac{180^\circ - \angle DCE}{2} = 73^\circ$$

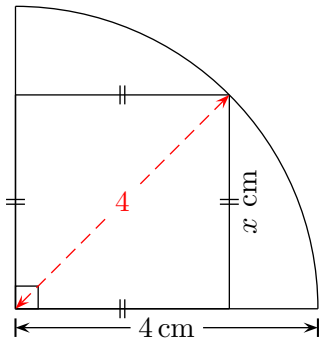
(c) (2 marks)




$$x = 180^\circ - 60^\circ = 120^\circ$$

(d) (3 marks)

✓ [1] for identifying radius as the hypotenuse.
✓ [1] for Pythagoras' Theorem.
✓ [1] for final answer.




Using Pythagoras' Theorem on the right triangle,

$$x^2 + x^2 = 4^2$$
$$2x^2 = 16$$
$$x^2 = 8$$
$$x = 2\sqrt{2}$$

**Question 9** (9 marks)

(a) i. (1 mark)

$$\overline{x} = \frac{42}{8} = \frac{21}{4}$$

ii. (1 mark)

$$\widetilde{x} = 5.5$$

iii. (1 mark) Mode $= 7$

iv. (1 mark) Range $= 5$

(b) i. (1 mark) - Geography

ii. (1 mark) - Geography

iii. (1 mark) - Maths

iv. (1 mark) - Music

(c) i. (2 marks)

| Duration (days) | Class centre | # of patients |
|---|---|---|
| 5–9 | 7 | 20 |
| 10–14 | 12 | 60 |
| 15–19 | 17 | 35 |
| 20–24 | 22 | 14 |
| 25–29 | 27 | 8 |
| 30–34 | 32 | 1 |
| 35–39 | 37 | 2 |

ii. (2 marks)

$$\overline{x} = 14.9$$

iii. (1 mark) Modal class: 10-14

iv. (2 marks)

- 10-14 days.
- Timeframe accounts for nearly half of all patients.