NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.1-5.3 MATHEMATICS
2012 YEAR 9
FINAL EXAMINATION

General Instructions

- Working time – **90 minutes**
- You **MUST** start each new question on a **new page.**
- Write using blue or black **pen**
- **Board approved** calculators may be used
- All necessary **working** should be shown in every question
- Marks may not be awarded for carelessly or badly arranged work.

- O 9M1 - Mr Ireland
- O 9M2 - Mr Lam
- O 9M3 - Ms Ziaziaris/Mr Lowe
- O 9M4 - Mr Lin
- O 9M5 – Ms Juhn

Student Name:_____

(To be used by the exam markers only.)

Section	A	B	C	D	E	F	G	H	I	J	Total	Total
Mark	/11	/11	/12	/19	/17	/9	/12	/8	/12	/9	/120	/100

Section A – Number (11 marks) **Marks**

(1) Calculate $2.057 \times 10^{12} - 5.57 \times 10^{11}$, giving your answer in scientific notation. 1

(2) Evaluate $\sqrt{\frac{3^2 + 12^2}{231 - 12^2}}$ correct to 3 significant figures. 2

(3) Simplify $\sqrt{3} + \sqrt{27}$ fully. 2

(4) Express with a rational denominator in simplest form:

(i) $\frac{1}{2\sqrt{7}}$ 1

(ii) $\frac{10}{5 - 3\sqrt{2}}$ 2

(5) Given that $\left(1 + 2\sqrt{3}\right)^2 = a + \sqrt{b}$, find the values of a and b. 2

(6) Express $\frac{1}{8^x}$ as a power of 2. 1

Section B (11 marks) Coordinate geometry and graphs *Start a new page*

(1) Write down the equation of the horizontal line through the point $(-7,2)$ 1

(2) (i) Write down the gradient of the line $3x+4y+5=0$. 1

(ii) Find the equation of the line passing through the point $(1,2)$ that is perpendicular to the line $3x+4y+5=0$. 2

(3) Show that the points $A(-1,1)$, $B(0,4)$ and $C(2,10)$ are collinear. 2

(4) Sketch the region defined by the intersection of the equations $x \geq 0$, $y \geq 0$ and $2x+3y-6<0$. 3

(5) $L(-4,0)$ and $N(x,y)$ are the endpoints of the interval LN. If $M(0,3)$ is the midpoint of LN, then find the coordinates of N. 2

Exam continues overleaf →

Section C (12 marks) Algebra *Start a new page*

(1) Fully factorise these expressions:

(i) $2x^2 + 3x - 2$ 2

(ii) $1 - x^4 y^4$ 2

(iii) $m^3 + 125$ 2

(2) Write $\left(\frac{1}{x} + \frac{y}{x}\right)^{-1}$ as the simplest fraction. 1

(3) Simplify $\frac{1}{x^2 + x} - \frac{1}{x^2 - 1}$ and express in simplest fraction form. 3

(4) Make x the subject, given that $m = \frac{x + yz}{xz}$ 2

Section D (19 marks) Equations and Inequations *Start a new page*

(1) Solve the inequality $5 - 3x < 7$ and graph your solution on a number line. 2

(2) Solve the following equations

(i) $2(x-1) - 3(5-x) = 0$ 2

(ii) $3 + \frac{2}{x} = \frac{5}{2x}$ 2

(iii) $x^2 = 5x$ 2

(iv) $x^2 - 5x + 4 = 0$ 2

(v) $27^{x+1} = 81^{2x}$ 2

(3) Solve these equations simultaneously:

$x + 2y = 4$

$3x + 14y = 36$ 3

(4) Tickets for a castle tour cost \$15 for an adult and \$8 for a child. On a particular tour, 46 tickets were sold and the money received was \$438.

(i) Form a pair of equations which represent this information, 2

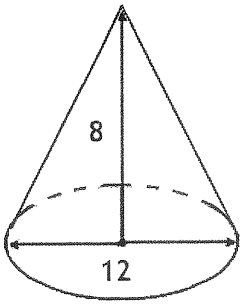
(ii) Solve the equations simultaneously to find how many adult tickets were sold for the tour. 2

Exam continues overleaf →

Section E (17 marks) Area and Volume *Start a new page*

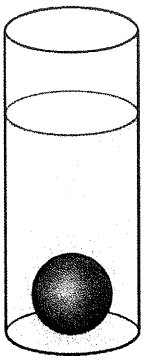
(1) Convert 0·64 m^3 into cm^3 1

(2) A solid cone has base diameter 12cm and vertical height 8cm.
Find its:



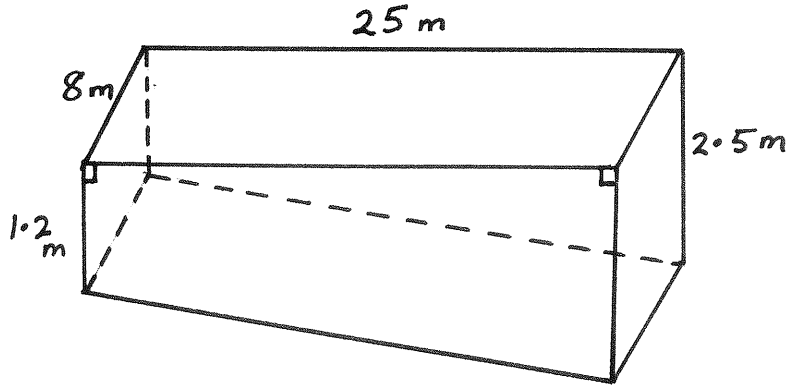
(i) Volume 2
(ii) Total surface area 3

(3)



A cylindrical can with radius 4cm and height 10cm is half full of water.
A metal sphere, radius 3cm, is dropped into the can (see illustration).
How many centimetres does the water rise? 3

(4) The diagram shows an in-ground pool, length 25m and width 8m, that is 1·2 m deep at the shallow end and 2·5m deep at the other end.



NOT TO SCALE

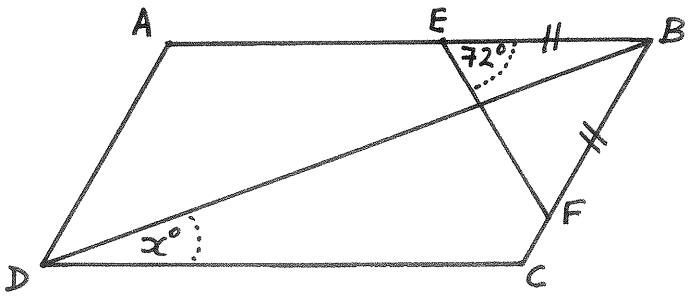
(i) Find the volume of the pool in m^3 2
(ii) Give the capacity of the pool in litres 1
(iii) The pool is filled using a pump with a constant flow rate.
Sketch a graph showing how the water level is changing with time. 2

(5) The base of a right square pyramid is a square of side 10cm. The perpendicular height of the pyramid is 12cm. Calculate the total surface area of the pyramid. 3

Section F (9 marks) Geometry *Start a new page*

(1) Find the values of the pronumerals indicated. No reasons are needed.

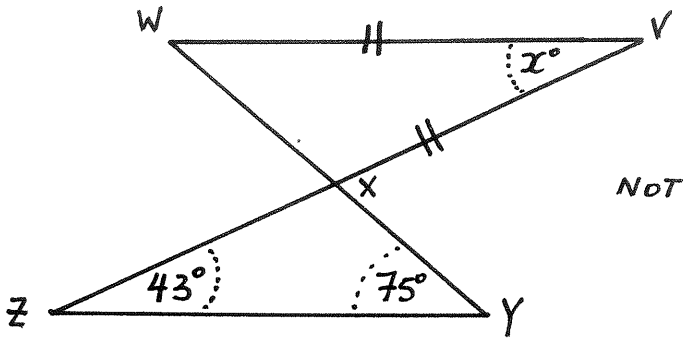
(i) 2



NOT TO SCALE

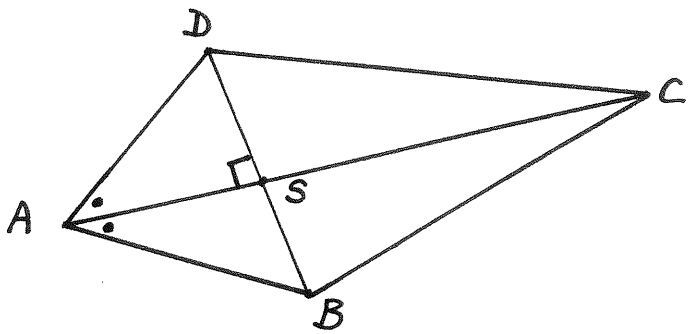
[ABCD is a rhombus]

(ii) 2



NOT TO SCALE

(2) In the diagram (not to scale), ABCD is a quadrilateral. The diagonals AC and BD intersect at right-angles, and $\angle DAS = \angle BAS$



(i) Copy the diagram onto your answer sheet.

(ii) Prove that $\Delta DAS \equiv \Delta BAS$ 4

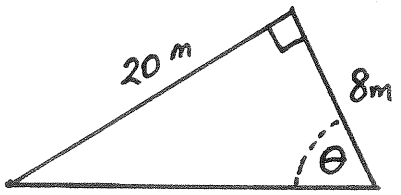
(iii) Hence state the reason that $DA = AB$ 1

Exam continues overleaf →

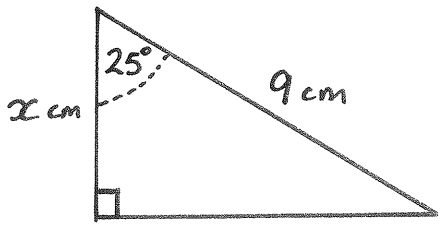
Section G (12 marks) Trigonometry *Start a new page*

(1) Find the values of the pronumerals in the diagrams below, rounding off angles to the nearest minute and lengths to 1 decimal place. 4

(a)



(b)

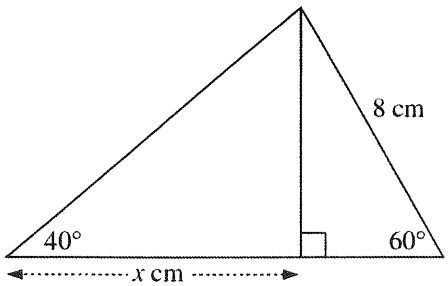


(2) If the bearing of Pete from Mike is $070°$, what is the bearing of Mike from Pete? 1

(3) Find the value of x if $\cos x° = \sin 35°$ 1

(4) An atoll is 9.6km from a ship which bears 156° from the atoll. Show this in a diagram, and calculate to 1 decimal place how far east of the atoll the ship is. 3

(5) Calculate the value of x in the diagram. Answer to 1 decimal place. 3



Section H (8 marks) Statistics *Start a new page*

(1) The following stem-and-leaf plot shows the heights, in centimetres, of 30 year 9 students:

13	6
14	5 7 9
15	1 2 2 4 5 6 6 8 9 9
16	0 0 1 2 3 5 5 7 8 8 9
17	0 2 2 3 9

Given this data,

(i) Find the median 1

(ii) Find the upper and lower quartiles 2

(iii) State the range 1

(iv) Represent your findings on a box-and-whisker plot. 2

(2) The mean for a science test of 30 students was 68%. Lance discovered that his mark of 81% had been recorded as 18%. What is the correct mean for the test? 2

Exam continues overleaf →

Section I (12 marks) Consumer Arithmetic *Start a new page*

(1) Sheriden buys a sound system for $1 210, including 10% GST. What is the value of the GST component? 1

(2) Joey invests $1000 at 8% per year compound interest, compounded quarterly. Calculate the value of her investment after 5 years. 2

(3) The personal income tax rates for 2012-2013 are depicted in the table:

Taxable income	Tax on this income
0 — $18,200	Nil
$18,201 — $37,000	19c for each $1 over $18,200
$37,001 — $80,000	$3,572 plus 32·5c for each $1 over $37,000
$80,001 — $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

Andrew has a gross annual salary of $84,000. He earned $3,600 in dividends and had allowable deductions of $1,580.

(i) Calculate Andrew's taxable income. 1
(ii) How much income tax is Andrew obliged to pay? 2
(iii) If his employer deducted $375 in PAYG tax instalments each week from his gross pay, will Andrew receive a tax refund or have to pay more tax? How much will he receive or owe ? 2

(4) A salesperson earns a $250 per week base salary, plus 2% commission on the first $10 000 of sales and 1·5% on the remainder. What's his total income for a week where his sales total $18,000 ? 2

(5) A furniture store offers a discount of 15% during a sale. A further 5% discount is given for paying bycash.

(i) Find the price paid by Emil if he pays cash for a sofa priced at $2,500. 1
(ii) What single percentage discount would be equivalent to these two discounts? 1

Section J Working mathematically (9 marks)

(1) Ray visits his friend Phil and then returns home by the same route.
He always walks 2 km/h when going uphill, 6 km/h when going downhill, and 3 km/h on level ground. If his total walking time is 6 hours, find the total distance that he walks. 2

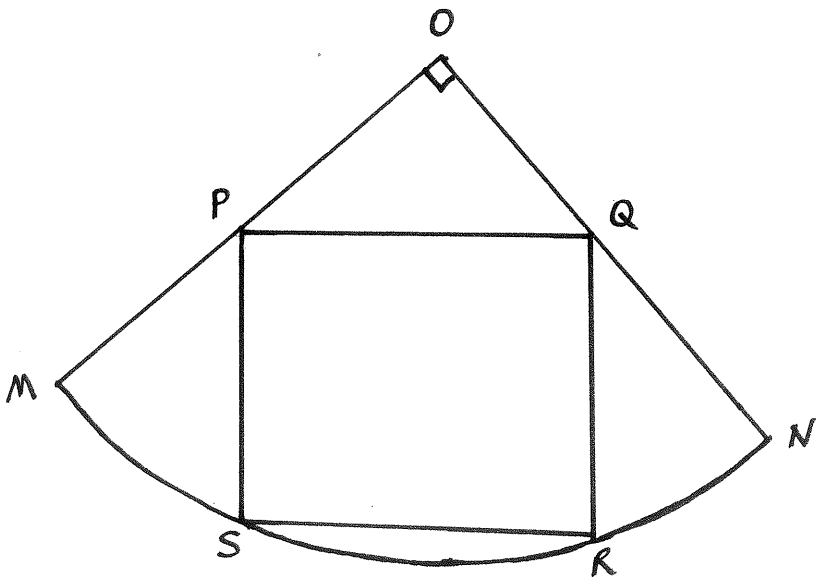
(2) Solve the following simultaneous equations: 3

$$\frac{1}{x}+\frac{1}{y}=10$$

$$\frac{1}{x}-\frac{1}{y}=-6$$

(3) Solve the equation $\sqrt{x+1}-\sqrt{x-1}=1$ 2

(4) A square is inscribed in a quadrant of a circle of radius 1 unit, as shown.



NOT TO SCALE

Find the area of the square, in square units. 2

END OF EXAM

2012 - Y9 Yearly - Suggested Solutions

SECTION A

(1) 1.5×10^{12} ✓

(2) $\sqrt{\dfrac{3^2+12^2}{231-12^2}} = 1.326\,129\,96\ldots$ ✓

$= 1.33$ (3 s.f.) ✓

(3) $\sqrt{3}+\sqrt{27} = \sqrt{3}+3\sqrt{3}$ ✓

$= 4\sqrt{3}$ ✓

(4) (i) $\dfrac{1}{2\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{7}}{14}$ ✓

(ii) $= \dfrac{10}{5-3\sqrt{2}} \cdot \dfrac{5+3\sqrt{2}}{5+3\sqrt{2}}$ ✓

$= \dfrac{50+30\sqrt{2}}{25-18} = \dfrac{50+30\sqrt{2}}{7}$ ✓

(5) $(1+2\sqrt{3})^2 = 1+4\sqrt{3}+12$

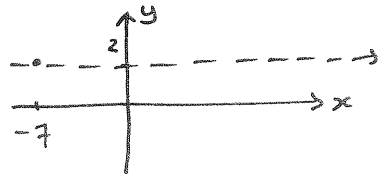
$= 13+4\sqrt{3}$

$= 13+\sqrt{48}$ $\therefore$ $a = 13$ ✓

$b = 48$ ✓

(6) $\dfrac{1}{8^x} = 2^{-3x}$ ✓

SECTION B

(1) 

$\therefore \quad y = 2$ ✓

(2) (i) $3x + 4y + 5 = 0$ has gradient $m = -\frac{3}{4}$ ✓

(ii) $m_\perp = \frac{4}{3}$ ✓

$\therefore \quad y - 2 = \frac{4}{3}(x-1)$ ✓

Thus $3y - 6 = 4x - 4$, $\therefore \quad 4x - 3y + 2 = 0$

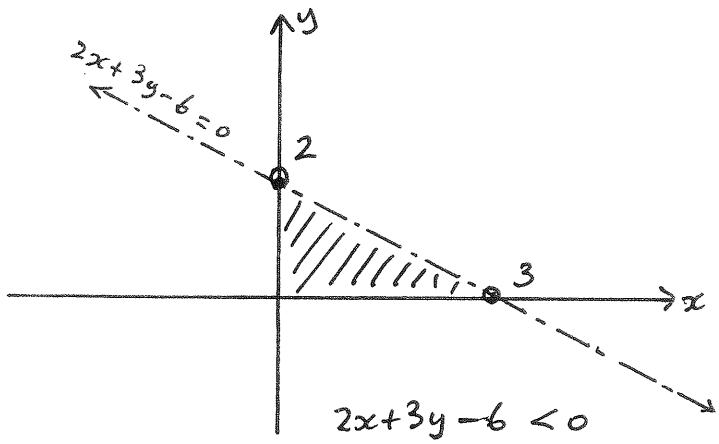
(or, $y = \frac{4}{3}x + \frac{2}{3}$)

(3) $m_{AB} = \frac{4-1}{0+1} = 3$

$m_{BC} = \frac{10-4}{2-0} = 3$ } ✓

Since $m_{AB} = m_{BC}$ and point B is common, $\therefore$ ✓ (Reason needed)

A, B, C are collinear.

(4) 

- Correct region ✓
- dotted line ✓
- intercepts ✓

(5) $\frac{-4+x}{2} = 0 \quad \therefore \quad x = 4$

$\frac{0+y}{2} = 3 \quad \therefore \quad y = 6$ Thus $N = (4, 6)$ ✓✓

SECTION C

(1) (i) $2x^2 + 3x - 2 = (2x - 1)(x + 2)$ ✓ ✓

(ii) $1 - x^4y^4 = (1 - x^2y^2)(1 + x^2y^2)$ ✓

$= (1 - xy)(1 + xy)(1 + x^2y^2)$ ✓

(iii) $m^3 + 125 = (m + 5)(m^2 - 5m + 25)$ ✓✓

(2) $\left(\frac{1}{x} + \frac{y}{x}\right)^{-1} = \left(\frac{1+y}{x}\right)^{-1} = \frac{x}{1+y}$ ✓

(3) $\frac{1}{x^2 + x} - \frac{1}{x^2 - 1} = \frac{1}{x(x+1)} - \frac{1}{(x+1)(x-1)}$ ✓

$= \frac{x - 1 - x}{x(x+1)(x-1)}$ ✓

$= \frac{-1}{x(x+1)(x-1)}$ ✓

(or, $\frac{1}{x - x^3}$).

(4) $m = \frac{x + yz}{xz}$

$\therefore mxz = x + yz$ ✓

$x(mz - 1) = yz$

$\therefore x = \frac{yz}{mz - 1}$ ✓

12

2012 4U yearly

SECTION D

(1) $5 - 3x < 7 \quad \therefore -3x < 2$

$\therefore x > -\frac{2}{3}$ ✓



✓

(2) (i) $2(x-1) - 3(5-x) = 0$

$2x - 2 - 15 + 3x = 0$ ✓

$5x = 17$

$x = \frac{17}{5}$ ✓

(ii) $3 + \frac{2}{x} = \frac{5}{2x}$

$\frac{6x+4}{2x} = \frac{5}{2x}$ ✓

$6x = 1$

$x = \frac{1}{6}$ ✓

(iii) $x^2 = 5x$

$x(x-5) = 0$

$x = 0$ or 5 ✓✓

(iv) $x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$ ✓

$x = 1$ or 4 ✓

(v) $27^{x+1} = 81^{2x}$

$(3^3)^{x+1} = (3^4)^{2x}$ ✓

$\therefore 3x + 3 = 8x$

$3 = 5x$

$x = \frac{3}{5}$ ✓

SECTION D – continued

(3) $x + 2y = 4$ ------ (1)

$3x + 14y = 36$ ------ (2)

$3\times(1) \rightarrow 3x + 6y = 12$ ------ (3)

$(2)-(3) \rightarrow 8y = 24$

$y = 3$

$x = 4 - 2(3) = 4 - 6$

$= -2$

(alt: substitution)

Correct working ✓

$\therefore x = -2$ ✓
$y = 3$ ✓

(4) Let there be x adult tickets sold, and y childrens tickets sold.

(i) $x + y = 46$ ------ (1) ✓

$15x + 8y = 438$ ------ (2) ✓

(ii) From equation (1), $y = 46 - x$

Substituting into (2),

$15x + 8(46-x) = 438$ ✓

$7x + 368 = 438$

$7x = 70$

$x = 10$

$\therefore$ 10 adult tickets were sold. ✓

(alt.: elimination)

2012 Y9 Yearly

SECTION E

(1) $0.64\ m^3 = 640\ 000\ cm^3$ ✓

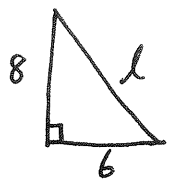
(2) (i) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 8$ ✓

$V = 96\pi\ cm^3$ ✓

$(\doteqdot 301.6\ cm^3$ to 1 d.p.)

(ii) $SA = \pi r^2 + \pi r l$



$l = \sqrt{8^2+6^2} = 10$ ✓

$\therefore SA = \pi \times 6^2 + \pi \times 6 \times 10$ ✓

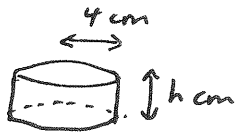
$SA = 96\pi\ cm^2$ ✓

$(\doteqdot 301.6\ cm^2$ to 1 d.p.).

(3) $V_{sphere} = \frac{4}{3} \cdot \pi \cdot 3^3$

$= 36\pi\ cm^3$ ✓

Let h = height the water rises, in cm:



Then $\pi \cdot 4^2 \cdot h = 36\pi$ ✓

$h = 2.25\ cm.$

The water rises 2.25 cm. ✓

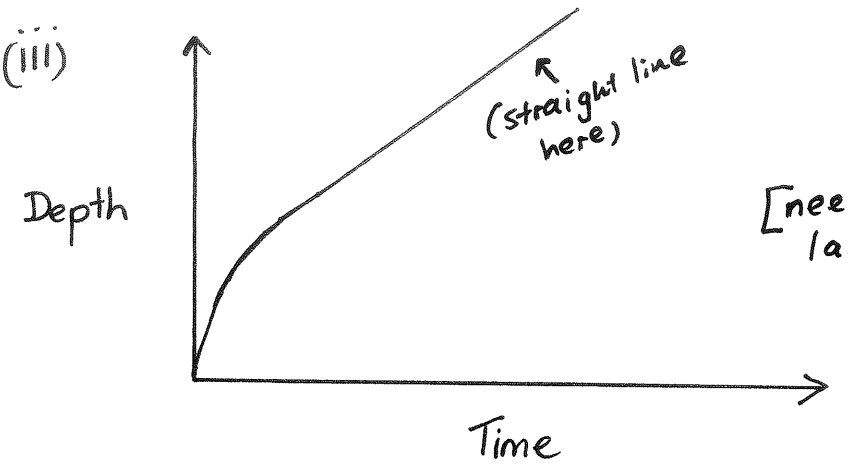
SECTION E – continued

(4) (i) $V = \left[\frac{1.2 + 2.5}{2} \times 25\right] \times 8 \ m^3$ ✓

$V = 370 \ m^3$ ✓

(ii) 370 000 L ✓

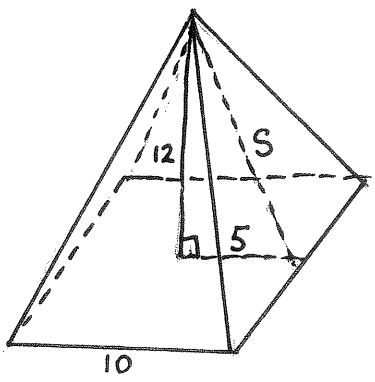
(iii)



✓✓

[needs correct shape; labelled axes]

(5)

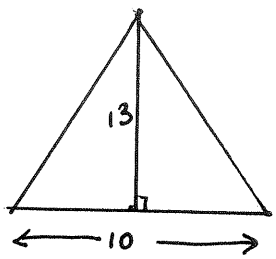


S = slant height $= \sqrt{12^2 + 5^2}$

$S = 13$ ✓

S.A. $= 4 \times \frac{10 \times 13}{2} + 10^2$ ✓

S.A. $= 360 \ cm^2$ ✓



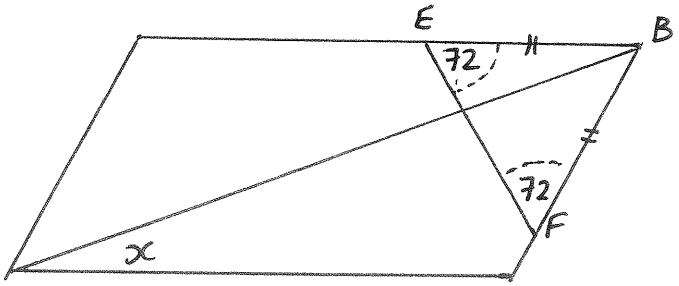
A triangular face

17

SECTION F

(1)

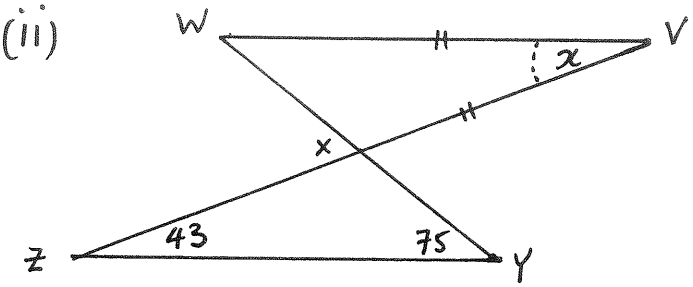
(i)



$\angle EBF = 180 - (72+72) = 36^\circ$ ✓

$\therefore x = \frac{1}{2} \times 36^\circ = 18^\circ$ ✓

(ii)



(NOT TO SCALE!)

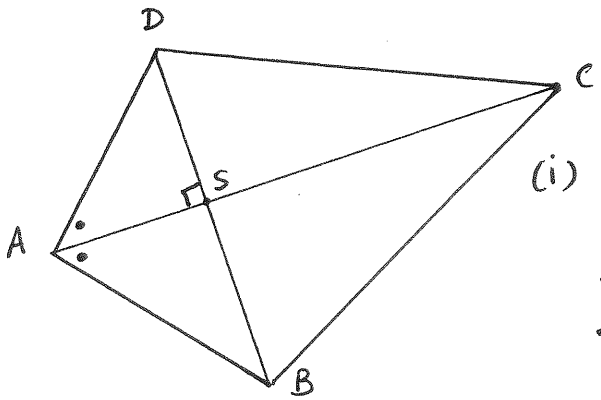
WV = XV.

$\angle WXV = \angle ZXY = 180 - (43+75)$

$= 62^\circ$

$\therefore x = 180 - 2 \times 62 = 56^\circ$ ✓✓

(2)



(i) In ΔDAS, ΔBAS

AS is common ✓

$\angle DAS = \angle BAS$ (given) ✓

$\angle DSA = \angle BSA$ ✓

$= 90^\circ$ (Supplementary $\angle$'s on straight line DB, & $\angle DSA = 90^\circ$ by data)

$\therefore \Delta DAS \equiv \Delta BAS$ (AAS) ✓

(ii) DA = AB (corresponding sides in congruent triangles). ✓

9

2012 4U Yearly

SECTION G

(1) (a) $\tan\theta = \frac{20}{8}$ ✓

$\therefore \theta = 68^\circ 12'$ (nearest minute) ✓

(b) $\frac{x}{9} = \cos 25^\circ$ ✓

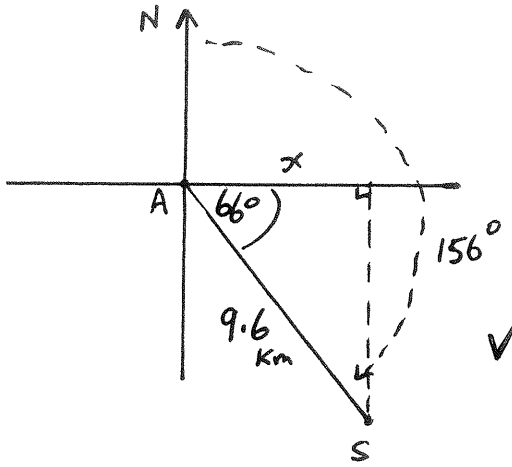
$\therefore x = 9\cos 25^\circ = 8.2$ cm (1 d.p.) ✓

[−1 incorrect rounding in (a) or (b)]

(2) 250° ✓

(3) $x = 55$ ✓

(4)



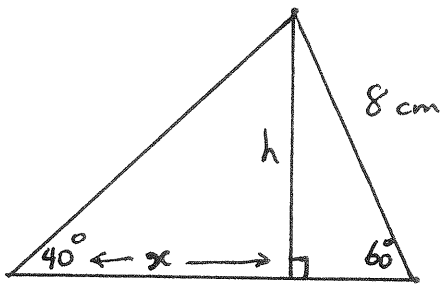
✓ (must show North!)

$\frac{x}{9.6} = \cos 66^\circ$ ✓

$x = 9.6 \times \cos 66^\circ$

$= 3.9$ km ✓

(5)



$\frac{h}{8} = \sin 60^\circ$

$h = 8\sin 60^\circ$ ($\doteq 6.9282$) ✓

Now, $\frac{h}{x} = \tan 40^\circ$

$\therefore x = \frac{h}{\tan 40^\circ}$ ✓

$\therefore x = \frac{8\sin 60^\circ}{\tan 40^\circ}$

$= 8.256711\ldots$ ✓

$x = 8.3$ cm (1 d.p.)

12

SECTION H

(1)

13	6
14	5 7 9
15	1 2 2 (4) 5 6 6 8 9 9
16	0 0 1 2 3 5 5 7 (8) 8 9
17	0 2 2 3 9

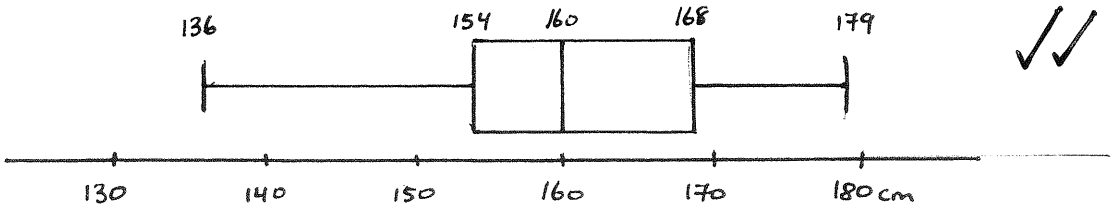
(i) median $= \frac{160+160}{2} = 160$ ✓

(ii) $Q_L = 154$ ✓

$Q_u = 168$ ✓

(iii) range $= 179 - 136 = 43$ ✓

(iv)



✓✓

(2) $81 - 18 = 63$

$\therefore$ correct mean $= \frac{(30 \times 68) + 63}{30}$ ✓ (working)

$= 70.1\%$ ✓

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SECTION I

(1) GST paid = $\frac{\$1210}{11}$ = \$110 ✓

(2) Rate/quarter = 2%,
number of quarters = 5×4 = 20
$A = 1000(1.02)^{20}$ ✓
= \$1485.95 (nearest cent) ✓

(3) (i) Taxable income = 84000 + 3600 − 1580
= \$86 020 ✓

(ii) income tax due = 17547 + 0.37[86020 − 80000] ✓
= \$19 774.40 ✓

(iii) PAYG deducted = 52 × \$375
= \$19500

So Andrew owes 19774.40 − 19500
= \$274.40 ✓✓

(4) Total income = 250 + 0.02 × 10000 + 0.015 × 8000
= 250 + 200 + 120
= \$570 ✓✓

(5) (i) Emil pays 0.95 × 0.85 × 2500
= \$2018.75 ✓

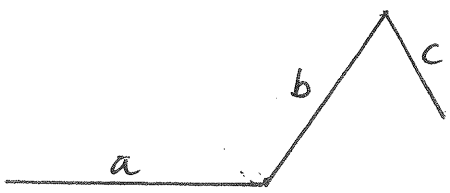
(ii) $\frac{2500 - 2018.75}{2500} = 0.1925$
i.e. 19.25% discount. ✓

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2012 4U Yearly

SECTION J

(1)



Let the level, uphill and downhill portions be a, b and c km respectively, on the way to Phil's.

Then time taken $= \frac{a}{3} + \frac{b}{2} + \frac{c}{6}$ hours.

The return trip takes $\frac{c}{2} + \frac{b}{6} + \frac{a}{3}$ hours.

Total time is thus $\frac{2a}{3} + \frac{2b}{3} + \frac{2c}{3} = 6$ hours

ie $\frac{2}{3}(a+b+c) = 6$ hours $\therefore a+b+c = 9$

$\therefore$ total distance $= 18$ km

working: ✓
answer: ✓

(2) $\frac{1}{x} + \frac{1}{y} = 10$ ---- (1)

$\frac{1}{x} - \frac{1}{y} = -6$ ---- (2)

(1)+(2) $\rightarrow$ $\frac{2}{x} = 4$

$\therefore x = \frac{1}{2}$

$\therefore \frac{1}{y} = 10 - 2 = 8$

$\therefore y = \frac{1}{8}$

working: ✓

So $x = \frac{1}{2}$, $y = \frac{1}{8}$

✓ ✓

SECTION J – continued

(3) $\sqrt{x+1} - \sqrt{x-1} = 1$

Squaring both sides, $x+1 - 2\sqrt{x+1}\cdot\sqrt{x-1} + x-1 = 1$

$\therefore \quad 2x - 2\sqrt{x^2-1} = 1$

$\therefore \quad 2\sqrt{x^2-1} = 2x-1$ useful working: ✓

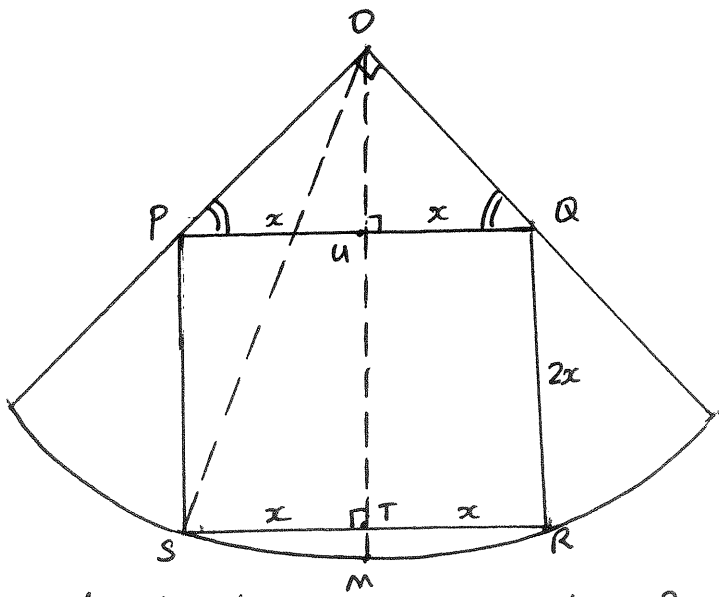
Square again:

$4(x^2-1) = 4x^2 - 4x + 1$

$-4 = -4x + 1$

$\therefore \quad x = \frac{5}{4}$ ✓

(4)



Let side lengths of square be $2x$

By symmetry, and since $\angle OPQ = \angle OQP = 45^\circ$,

$\therefore \quad OU = x.$

Thus $OT = OU + UT = x + 2x = 3x$

$\therefore$ in the right ΔOTS, $OT^2 + ST^2 = OS^2$

$\therefore \quad (3x)^2 + x^2 = 1$

$10x^2 = 1$

$x^2 = \frac{1}{10}$

useful working: ✓

$\therefore$ Area of square $PQRS = (2x)^2 = 4x^2 = \frac{4}{10}$ units^2. ✓

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