NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.1-5.3 MATHEMATICS

2011 Year 9 Final Examination

General instructions

- Working time – 90 minutes.
- Marks may be deducted for careless or poorly arranged work.
- Commence each new question on a new sheet.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 9M1 – Mr Berry/Mr Lin
- ◯ 9M2 – Mr Lowe/Mr Lin
- ◯ 9M3 – Mr Lam
- ◯ 9M4 – Ms Ziaziaris
- ◯ 9M5 – Mr Weiss/Mr Lin

NAME: ..... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	6	7	8	**Total**	%
MARKS	/12	/12	/12	/12	/17	/12	/13	/10	/100	

Question 1 (12 Marks) Commence a NEW page. **Marks**

(a) Simplify fully: $\frac{7ab}{4} - \frac{5ab}{4}$. **2**

(b) Solve the equation $17 + 6y = 35$. **2**

(c) Expand and simplify:

i. $(x - 8)(x + 2)$. **2**

ii. $(m^2 + 3n)^2$. **2**

(d) Fully factorise:

i. $4a^2 + 10a$. **2**

ii. $x^2 - 36$. **2**

Question 2 (12 Marks) Commence a NEW page. **Marks**

(a) Fully simplify $x^{2a-1} \times x^{a+3}$. **2**

(b) Divide \$2 247 in the ratio $3 : 2 : 1$. **2**

(c) Solve:

i. $\frac{m+6}{3} = \frac{2m+4}{4}$. **2**

ii. $\frac{4}{y-3} + 2 = 1$. **2**

(d) Solve $2x^2 + 3x - 3 = 0$, giving your solutions as exact values. **2**

(e) A light year is the distance light travels in a year. **2**

The speed of light is 2.998×10^5 km per second and there are 365.25 days in a year. Find the number of kilometres in a light year in scientific notation, correct to 3 significant figures.

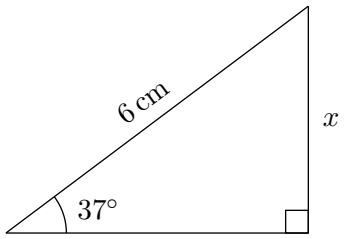
Question 3 (12 Marks) Commence a NEW page. **Marks**

(a) Mrs Short has a taxable income of \$86 770. Using the tax table below, find her tax payable. **2**

Taxable income	**Tax on this income**
\$0 – \$6 000	Nil
\$6 001 – \$37 000	15c for each \$1 over \$6 000
\$37 001 – \$80 000	\$4 650 plus 30c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 550 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 550 plus 45c for each \$1 over \$180 000

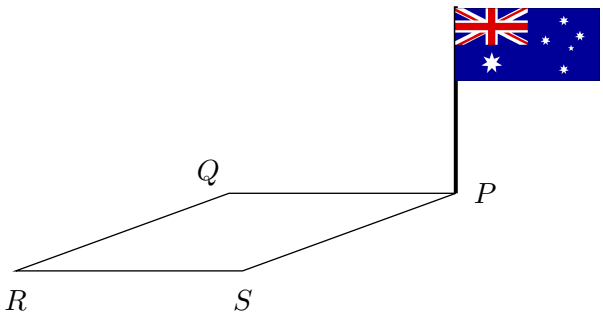
(b) Use your calculator to find $\tan 78.4°$ correct to 3 decimal places. **1**

(c) Find x correct to three significant figures. **2**



(d) If $\sin\theta = -\dfrac{1}{\sqrt{6}}$ and $\tan\theta > 0$, find the exact value of $\cos\theta$. **3**

(e) A flagpole stands at one corner P of a level square field $PQRS$, each side being 120 m long.



If the angle of elevation to the top of the pole from Q is $13°19'$, find:

i. the height of the flagpole, correct to the nearest centimetre. **1**

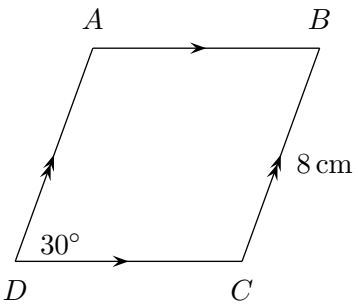
ii. its angle of elevation from R. **3**

Question 4 (12 Marks) Commence a NEW page. **Marks**

(a) Find the radius of a sphere with volume $1\,220\,\text{cm}^3$ correct to 1 decimal place. **2**

(b) A cone has a diameter of 16 cm and perpendicular height of 15 cm. Calculate the curved surface area of the cone in terms of π. **2**

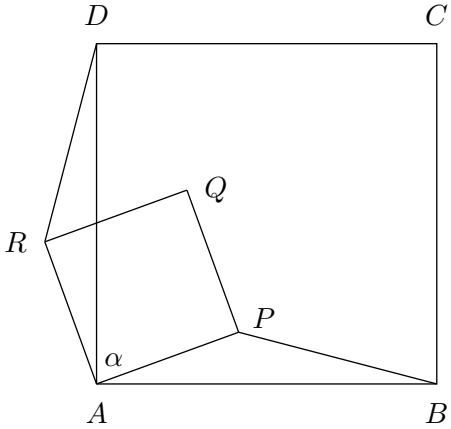
(c) In the diagram, $ABCD$ is a parallelogram with $\angle ADC = 30^\circ$ and $BC = 8\,\text{cm}$.



i. Find the size of $\angle ABC$, giving a reason. **2**

ii. If the diagonals AC and BD are perpendicular, find the perimeter $ABCD$. Give reasons. **2**

(d) $ABCD$ and $APQR$ are squares and $\angle DAP = \alpha$.



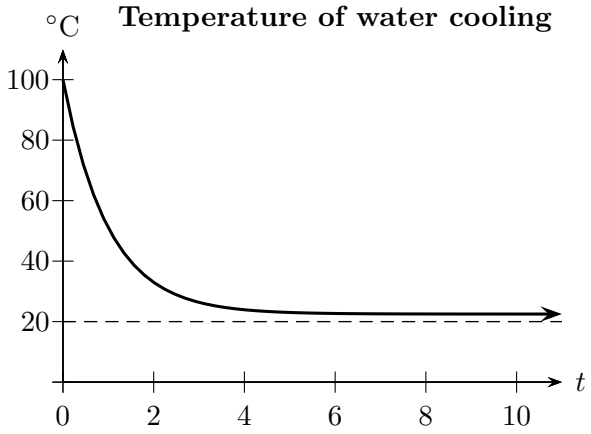
i. Prove $\triangle DRA \equiv \triangle ABP$. **3**

ii. Hence or otherwise, given a reason why $BP = DR$. **1**

Question 5 (17 Marks) Commence a NEW page. **Marks**

(a) Sketch the region represented simultaneously by the inequalities $x + y \leq 1$ and $y \geq 2x$. **3**

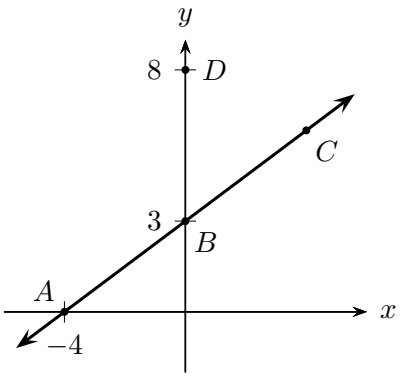
(b) The graph shows the temperature of some hot water which is cooling over time. Use this graph to answer the following questions.



i. Was the water cooling more rapidly at 2 hours or at 8 hours? Give reasons for your answer. **2**

ii. As time passes, the temperature of the water approaches the temperature of the room. Write down the temperature of the room. **1**

(c) The diagram shows line ℓ cutting the x axis at $A(-4, 0)$ and the y axis at $B(0, 3)$. The point D has coordinates $(0, 8)$.



i. Find the gradient of the line ℓ. **1**

ii. Show that the line ℓ has equation $3x - 4y + 12 = 0$. **2**

iii. The point C lines on ℓ and B is the midpoint of AC. Prove that the coordinates of C are $(4, 6)$. **2**

iv. Show that the exact length of AD is $4\sqrt{5}$ units. **2**

v. Show that AD is perpendicular to DC. **2**

vi. Find the area of $\triangle ADC$. **2**

Question 6 (12 Marks) Commence a NEW page. **Marks**

(a) Simplify fully $\sqrt{45}+\sqrt{5}$. **2**

(b) Express $\dfrac{\sqrt{3}}{\sqrt{3}-1}$ as the simplest fraction with a rational denominator. **2**

(c) Solve the following pair of simultaneous equations: $\begin{cases} y = x - 10 \\ x - 3y = 6 \end{cases}$. **3**

(d) Solve $x + \dfrac{10}{x} = 7$. **2**

(e) Fully simplify the following expression: $\dfrac{1}{x^2 - x - 6} + \dfrac{4}{x^2 + 8x + 12}$. **3**

Question 7 (13 Marks) Commence a NEW page. **Marks**

(a) Boxes of soap powder are each labelled as having a mass of 1 kg. The mass of these packets is distributed normally with a mean of 1.02 kg and a standard deviation of 0.01 kg. **2**

What percentage of boxes have an actual mass between 1 kg and 1.03 kg?

(b) Consider the following scores:

4, 6, 7, 8, 9, 10, 10, 11, 11, 12, 16, 17, 18, 19, 20

i. Find the standard deviation. **1**

ii. What is the median? **1**

iii. What is the lower quartile Q_1? **1**

iv. What is the upper quartile Q_3? **1**

v. Find the interquartile range. **1**

vi. Draw a box-and-whisker plot showing the above information. **2**

(c) Vincent travels to school every day by train. He has found that the probability the train is late by five or more minutes is 7% on Mondays and 13% on Tuesdays.

i. Draw a probability tree diagram for the late trains on Monday and Tuesday. **2**

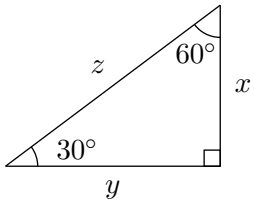
ii. Find the probability that the train is late by five minutes or more on *either* day, but not both. **2**

Question 8 (10 Marks) Commence a NEW page. **Marks**

(a) Find k in terms of n given **3**

$$x^k = \sqrt{\frac{x^{n+1} \times (x^2)^{n+2}}{\sqrt{x} \times x^{n+\frac{1}{2}}}}$$

(b) A piece of wire of length 20 cm is bent in the shape of a triangle with interior angles 30°, 60°and 90°.



i. Show that $y = \sqrt{3}x$ and $z = 2x$. **2**

ii. Hence or otherwise, find the length of the hypotenuse, giving your answer in surd form with a rational denominator. **2**

(c) Sammi thinks of a two-digit number. The sum of the digits is 8. If she reverses the digits the new number is 36 greater than her original number. What was Sammi's original number? **3**

End of paper.

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Suggested Solutions

Question 1

(a) (2 marks)

✓ [1] for $\frac{2ab}{4}$.

✓ [1] for $\frac{ab}{2}$.

$$\frac{7ab}{4} - \frac{5ab}{4} = \frac{2ab}{4_2} = \frac{ab}{2}$$

(b) (2 marks)

✓ [−1] for each error.

$$\begin{aligned} 17 + 6y &= 35 \\ -17 \quad & \quad -17 \\ 6y &= 18 \\ y &= 3 \end{aligned}$$

(c) i. (2 marks)

✓ [−1] for each error.

$$(x-8)(x+2) = x^2 - 6x - 16$$

ii. (2 marks)

✓ [−1] for each error.

✓ [0] for $m^4 + 9n^2$.

$$\left(m^2 + 3n\right)^2 = m^4 + 6m^2n + 9n^2$$

(d) i. (2 marks)

✓ [−1] for each error.

$$4a^2 + 10a = 2a(2a+5)$$

ii. (2 marks)

✓ [−1] for each error.

$$x^2 - 36 = (x-6)(x+6)$$

Question 2

(a) (2 marks)

✓ [−1] for each error.

$$x^{2a-1} \times x^{a+3} = x^{2a+a-1+3} = x^{3a+2}$$

(b) (2 marks)

✓ [1] for finding 1 part.
✓ [1] for final answer.

$$1 \text{ part} = \frac{2\,247}{6} = 374.5$$
$$\therefore 3 : 2 : 1 = \$1\,123.5 : \$749 : \$374.5$$

(c) i. (2 marks)

✓ [1] for correctly cross multiplying.
✓ [1] for final answer.

$$\frac{m+6}{3} = \frac{2m+4}{4} = \frac{\cancel{2}(m+2)}{\cancel{4}_2}$$
$$2(m+6) = 3(m+2)$$
$$\underset{-2m}{2m} + \underset{-6}{12} = \underset{-2m}{3m} + \underset{-6}{6}$$
$$\therefore m = 6$$

ii. (2 marks)

✓ [−1] for each error.

$$\frac{4}{y-3} + \underset{-2}{2} = \underset{-2}{1}$$
$$\underset{\left(\underset{\div -1}{\times (y-3)}\right)}{\frac{4}{y-3}} = \underset{\left(\underset{\div -1}{\times (y-3)}\right)}{-1}$$
$$\underset{+3}{-4} = y - \underset{+3}{3}$$
$$y = -1$$

(d) (2 marks)

✓ [1] for recalling quadratic formula.
✓ [1] for (both) final answers.

$$2x^2 + 3x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{9 - 4(2)(-3)}}{2 \times 2}$$
$$= \frac{-3 \pm \sqrt{33}}{4}$$

(e) (2 marks)

✓ [1] for multiplying by $60 \times 60 \times 24 \times 365.25$.
✓ [1] for final answer.

$$\frac{2.998 \times 10^5 \text{ km}}{1 \text{ sec}} \underset{\times 60 \times 60 \times 24 \times 365.25}{\times 60 \times 60 \times 24 \times 365.25}$$
$$= \frac{9.46 \times 10^{12} \text{ km}}{1 \text{ year}}$$

Question 3

(a) (2 marks)

✓ [1] for tax payable on marginal rate.
✓ [1] for final answer.

$$\begin{aligned} &17\,550 + 0.37(86\,770 - 80\,000) \\ &= \$20\,054.90 \end{aligned}$$

(b) (1 mark)

$$\tan 78.4^\circ = 4.872$$

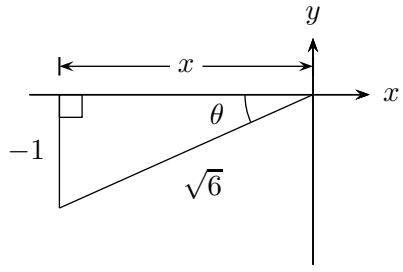
(c) (2 marks)

✓ [1] for identifying sin ratio.
✓ [1] for final answer.

$$\frac{x}{6} = \sin 37^\circ$$
$$x = 6\sin 37^\circ = 3.61$$

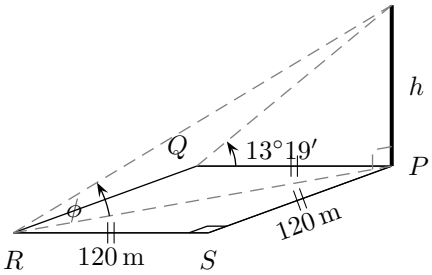
(d) (3 marks)

✓ [1] for identifying 3rd quadrant.
✓ [1] for finding missing side.
✓ [1] for final answer.



$$x^2 + 1^2 = \left(\sqrt{6}\right)^2$$
$$x^2 = 5$$
$$\therefore x = \sqrt{5}$$
$$\therefore \cos\theta = -\frac{\sqrt{5}}{\sqrt{6}} = -\sqrt{\frac{5}{6}}$$

(e) i. (1 mark)



$$\frac{h}{120} = \tan 13^\circ 19'$$
$$h = 120\tan 13^\circ 19' = 28.4\,\text{m}$$

ii. (3 marks)

✓ [1] for finding RP.
✓ [1] for $\frac{h}{PR} = \tan\phi$.
✓ [1] for final answer.

In $\triangle PSR$,

$$\begin{aligned} PR^2 &= PS^2 + RS^2 = 120^2 + 120^2 \\ &= 2 \times 120^2 \\ \therefore PR &= 120\sqrt{2} \\ \tan\phi &= \frac{h}{PR} \\ &= \frac{\cancel{120}\tan 13^\circ 19'}{\cancel{120}\sqrt{2}} \\ &= 0.167\cdots \\ \therefore \phi &= 9^\circ 30' \end{aligned}$$

Question 4

(a) (2 marks)

✓ [1] for recalling formula for volume of sphere.

✓ [1] for final answer.

$$V = \frac{4}{3}\pi r^3$$
$$1\,220 = \frac{4}{3}\pi r^3$$
$$r^3 = \frac{1\,220 \times 3}{4}$$
$$\therefore r \approx 6.628\,\text{cm}$$

(b) (2 marks)

✓ [1] for correctly calculating slanted height.

✓ [1] for final answer.

$$s^2 = 8^2 + 15^2 = 289$$
$$\therefore s = 17$$
$$SA = \pi r s = \pi \times 8 \times 17$$
$$= 136\pi$$

(c) i. (2 marks)

✓ [1] for final answer.

✓ [1] for correct reason.

$\angle ABC = 30°$
(opposite $\angle$ in parallelogram)

ii. (2 marks)

✓ [1] for final answer.

✓ [1] for correct reason.

- $P = 8 \times 4 = 32\,\text{cm}$.
  (Diagonals of a parallelogram bisect each other, and since they are $\perp$, $\therefore$ the parallelogram is a rhombus. Sides of a rhombus are equal.)

(d) i. (3 marks)

✓ [−1] for each missing/incorrect step in the proof.

- $AB = AD$
  (sides of a square $ABCD$)
- $\angle PAB = \angle RAD$
  ($\angle$ in square; complementary to α in squares $ABCD$ and $APQR$ respectively)
- $AP = AR$
  (sides of a square $APQR$)

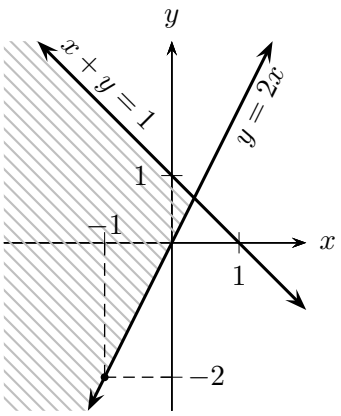
$\therefore \triangle ABP \equiv \triangle DRA$ (SAS)

ii. (1 mark)
$BP = DR$
(corresponding sides of congruent $\triangle ABP$ & $\triangle DRA$)

Question 5

(a) (3 marks)

✓ [1] for sketching $x + y = 1$ correctly.
✓ [1] for sketching $y = 2x$ correctly.
✓ [1] for correctly shaded region.



(b) i. (2 marks)

✓ [1] for 2 hrs.
✓ [1] for reason.

2 hrs – graph is more steep.

ii. (1 mark) 20°C

(c) i. (1 mark) $m = \dfrac{3-0}{0-(-4)} = \dfrac{3}{4}$

ii. (2 marks)

✓ [1] for each of m and b.

$$\underset{\times 4}{y} = \underset{\times 4}{\frac{3}{4}x} + \underset{\times 4}{3}$$
$$4y = 3x + 12$$
$$\therefore 3x - 4y + 12 = 0$$

(use gradient from previous part, read off y intercept)

iii. (2 marks)

✓ [1] for using midpoint formula correctly
✓ [1] for solving for x and y.
✓ [0] for simply restating question etc.

$$B(0,3) = \left(\frac{-4+x}{2}, \frac{0+y}{2}\right)$$
$$\frac{-4+x}{2} = 0 \quad \Big| \quad \frac{0+y}{2} = 3$$
$$-4 + x = 0 \quad \Big| \quad y = 6$$
$$x = 4$$
$$\therefore C(4,6)$$

iv. (2 marks)

$$A(-4,0) \qquad D(0,8)$$
$$AD = \sqrt{(-4)^2 + 8^2}$$
$$= \sqrt{4^2 + 8^2}$$
$$= \sqrt{16 + 64} = \sqrt{80}$$
$$= 4\sqrt{5}$$

v. (2 marks)

✓ [−1] for each error.

$$m_{AD} = \frac{8-0}{0-(-4)} = \frac{8}{4} = 2$$
$$m_{DC} = \frac{8-6}{0-4} = \frac{2}{-4} = -\frac{1}{2}$$
$$m_{AD} \times m_{DC} = -1$$
$$\therefore AD \perp DC$$

vi. (2 marks)

✓ [1] for length of DC.
✓ [1] for final answer.

$$DC = \sqrt{(4-0)^2 + (6-8)^2}$$
$$= \sqrt{4^2 + 2^2} = \sqrt{20}$$
$$= 2\sqrt{5}$$
$$A_{\triangle ADC} = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 4\sqrt{5} \times 2\sqrt{5}$$
$$= 4 \times 5 = 20\,\text{m}^2$$

Question 6

(a) (2 marks)

✓ [1] for simplifying $\sqrt{45}$.
✓ [1] for final answer.

$$\sqrt{45}+\sqrt{5}=3\sqrt{5}+\sqrt{5}=4\sqrt{5}$$

(b) (2 marks)

✓ [1] for multiplying by conjugate in numerator & denominator.
✓ [1] for final answer.

$$\begin{aligned}\frac{\sqrt{3}}{\sqrt{3}-1}\frac{\times(\sqrt{3}+1)}{\times(\sqrt{3}+1)}&=\frac{\sqrt{3}\left(\sqrt{3}+1\right)}{3-1}\\&=\frac{\sqrt{3}\left(\sqrt{3}+1\right)}{2}\end{aligned}$$

(c) (3 marks)

✓ [1] for correct method.
✓ [1] for each of x and y.

$$\begin{cases}y=x-10 & (1)\\ x-3y=6 & (2)\end{cases}$$

Substitute (1) into (2)

$$\begin{aligned}x-3(x-10)&=6\\-2x+30&=6\\2x&=24\\x&=12\\\therefore y&=2\end{aligned}$$

(d) (2 marks)

✓ [1] for each correct root.

$$\begin{aligned}\underset{\times x}{x}+\frac{10}{\underset{\times x}{x}}&=\underset{\times x}{7}\\x^2+10&=7x\\x^2-7x+10&=0\\(x-5)(x-2)&=0\\\therefore x&=2,5\end{aligned}$$

(e) (3 marks)

✓ [1] for correct factorisations in denominators.
✓ [1] for correct common denominator.
✓ [1] for final answer.

$$\begin{aligned}&\frac{1}{x^2-x-6}+\frac{4}{x^2+8x+12}\\&=\frac{1}{(x-3)(x+2)}+\frac{4}{(x+6)(x+2)}\\&=\frac{(x+6)+4(x-3)}{(x-3)(x+2)(x+6)}\\&=\frac{x+6+4x-12}{(x-3)(x+2)(x+6)}\\&=\frac{5x-6}{(x-3)(x+2)(x+6)}\end{aligned}$$

Question 7

(a) (2 marks)

✓ [1] for identifying $\mu - 2\sigma$ and $\mu + \sigma$ locations.

✓ [1] for final answer.

$$P(1 < x < 1.03) = 0.475 + 0.34 = 0.815$$

(b) i. (1 mark) $\sigma = 4.83$

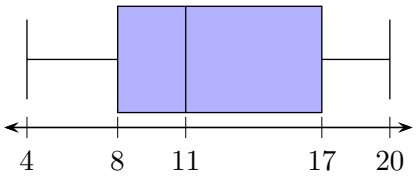
ii. (1 mark) $\widetilde{x} = 11.$

iii. (1 mark) $Q_1 = 8$

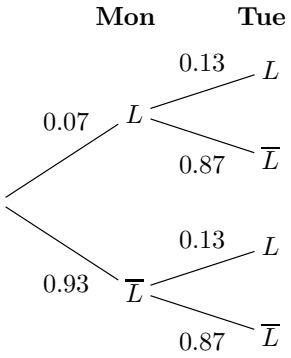
iv. (1 mark) $Q_3 = 17$

v. (1 mark) $IQR = 9$

vi. (2 marks)



(c) i. (2 marks)



ii. (2 marks)

$$\begin{aligned} P(L\overline{L}, \overline{L}L) &= (0.07 \times 0.87) + (0.93 \times 0.13) \\ &= 0.1818 \end{aligned}$$

Question 8

(a) (3 marks)

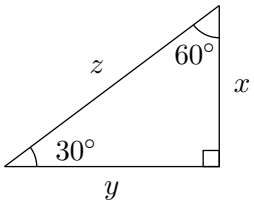
✓ [−1] for each error.
✓ [3] if students have simplified all correctly down to x^{n+2}.

$$\begin{aligned} x^k &= \sqrt{\frac{x^{n+1} \times (x^2)^{n+2}}{\sqrt{x} \times x^{n+\frac{1}{2}}}} \\ &= \left(\frac{x^{n+1} \times x^{2n+4}}{x^{\frac{1}{2}} \times x^{n+\frac{1}{2}}} \right)^{\frac{1}{2}} \\ &= \left(\frac{x^{3n+5}}{x^{n+1}} \right)^{\frac{1}{2}} \\ &= \left(x^{2n+4} \right)^{\frac{1}{2}} \\ &= x^{n+2} \end{aligned}$$

$$\therefore k = n + 2$$

(b) i. (2 marks)

✓ [1] for proving each relationship correctly.



$$\tan 60^\circ = \sqrt{3} = \frac{y}{x}$$

$$\therefore y = \sqrt{3}x$$

Also,

$$\cos 60^\circ = \frac{1}{2} = \frac{x}{z}$$

$$\therefore z = 2x$$

ii. (2 marks)

✓ [1] for obtaining $x = \frac{20}{3+\sqrt{3}}$.
✓ [1] for final answer with rationalised denominator.

$$\begin{aligned} x + y + z &= 20 \\ x + \sqrt{3}x + 2x &= 20 \\ x(1 + \sqrt{3} + 2) &= 20 \\ x &= \frac{20}{3 + \sqrt{3}} \end{aligned}$$

As z is the hypotenuse,

$$\begin{aligned} z = 2x &= \frac{40}{3 + \sqrt{3}} \frac{\times(3-\sqrt{3})}{\times(3-\sqrt{3})} \\ &= \frac{40\left(3 - \sqrt{3}\right)}{\cancel{9 - 3}6} \\ &= \frac{20\left(3 - \sqrt{3}\right)}{3} \end{aligned}$$

(c) (3 marks)

✓ [1] for observing the original number is $10x + y$, if x is the first digit and y is the second.
✓ [1] for correctly constructing second worded equation.
✓ [1] for obtaining $x = 2$.

- Let the number have digits x and y, x being the first digit, i.e. $\left(x \times 10^1\right) + \left(y \times 10^0\right) = 10x + y$.
- Switching the digits results in $10y + x$.

$$\begin{cases} x + y = 8 & (1) \\ 10y + x = 36 + (10x + y) & (2) \end{cases}$$

Simplifying (2),

$$\begin{aligned} 10y + x &= 36 + 10x + y \\ 9x + 36 &= 9y \\ \therefore x + 4 &= y \end{aligned}$$

Substitute to (1),

$$\begin{aligned} x + (x + 4) &= 8 \\ 2x + 4 &= 8 \\ \therefore x = 2 \ \therefore y &= 6 \end{aligned}$$

The original number is 26.