NORTH SYDNEY BOYS HIGH SCHOOL

2010 YEAR 9 YEARLY EXAM

STAGE 5.3 MATHEMATICS

Student Name:_____

Class: _____ **Teacher**_____

Number of pages used : _____(Fill at the end of exam)

General Instructions

- Working time – **90 minutes**
- Write on **one side** of your paper only
- Write using blue or black pen.
- **Board approved** calculators are allowed.
- All necessary working should be shown in every question.
- Start each new **question** on a **new page.**
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Question	1	2	3	4	5	6	7	8	9	10	Total	Tota
Mark	/11	/16	/17	/13	/9	/11	/11	/10	/14	/10	/122	/100

Question 1 (11 marks) **Marks**

a) Calculate to 3 significant figures $\dfrac{4\cdot 6+\sqrt{5\cdot 8^2-3}}{8}$ **2**

b) Write $4\cdot 7\times 10^{-3}$ as a basic numeral **1**

c) Increase \$8750 by 6 ½% **2**

d) Expand and simplify $\left(2\sqrt{3}+1\right)^2$ **2**

e) Rationalise the denominator and simplify **4**

i) $\dfrac{4}{3\sqrt{6}}$ ii) $\dfrac{6}{7-\sqrt{5}}$

Question 2 (16 marks)

a) Solve the following equations and inequalities **6**

i) $6x-5=10x+1$

ii) $\dfrac{3x+8}{5}=\dfrac{4x-6}{3}$

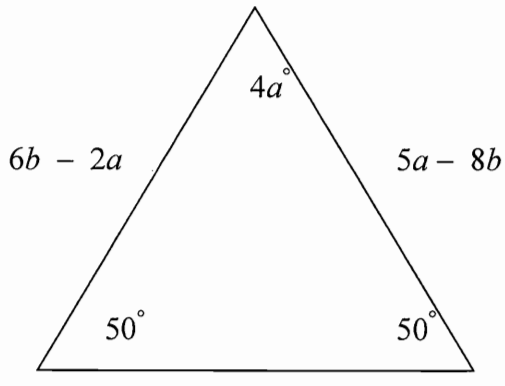
iii) $4-3x\leq 10$

b) Solve simultaneously **6**

i) $y=10-2x$ and $x+y=7$

ii) $10x-7y=36$ and $2x+y=6$

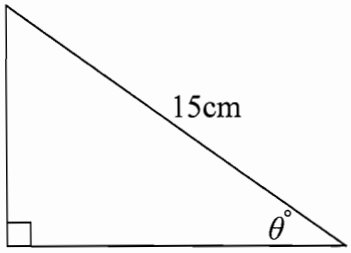
c) Solve for a and b (Showing working) **4**



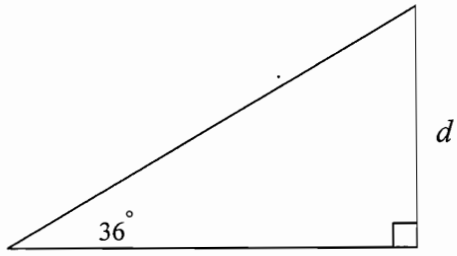
Question 3 (17 marks)

a) Find the value of the pronumerals in the following triangles to the nearest degree, where appropriate. **4**

i)



ii)



b) Find the exact value of **3**

i) $\tan 30^\circ$
ii) $\sin 60^\circ$
iii) $\cos 150^\circ$

c) If $\tan\theta = \frac{5}{7}$, $0^\circ \leq \theta^\circ \leq 90^\circ$ find the exact value of **3**

i) $\sin\theta^\circ$
ii) $\cos\theta^\circ$

d) Sketch the graph of $y = \sin x^\circ$ for $0^\circ \leq x^\circ \leq 360^\circ$
clearly marking any x and y intercepts on your diagram. **2**

e) Grilled Bear was exploring a tropical jungle. He left base camp B on a bearing of 050°T and hiked 13·5km to an old avalanche site A. He then changed course and proceeded on a bearing of 140°T until he reached a cliff top C due east of his starting point.

i) Draw a diagram to show this information. **2**

ii) What is the measure of $\angle BAC$? **1**

iii) Calculate how far due east he is from base camp. **2**

Question 4 (13 marks)

a) A and B are the points (4, 8) and (7, – 10) respectively. Find

i) The mid point of AB. **1**

ii) The gradient of AB. **1**

iii) The equation of AB. **2**

b) Draw on a number plane the triangle PQR with vertices P(–9, 9), Q(5, –1) and R(1, –5)

i) Prove that ΔPQR is isosceles. **3**

ii) Find the co-ordinates of S so that PQRS is a parallelogram. **2**

c) Are the points A(4, 4), B(2, 5) and C(0, 6) collinear? Justify your answer. **2**

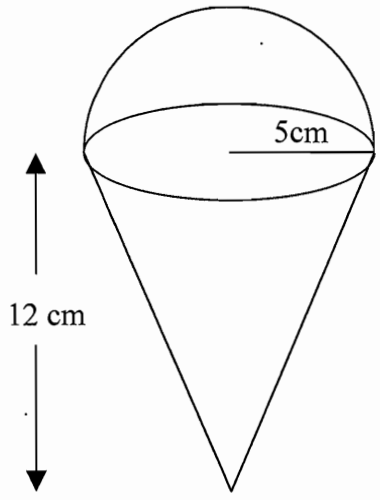
d) Find the equation of the line through (6, 2) parallel to $4x - 5y + 1 = 0$ **2**

Question 5 (9 marks)

a) Ice Creams '**R**' Us sells the MEGA CONE.
It consists of a cone radius 5 cm, 12 cm deep, filled with ice cream with a hemisphere of ice cream on the top.

i) Calculate the volume of ice cream. **4**

ii) Find the surface area of the cone only. **2**



b) A cylindrical water tank with base diameter of 2·5 m is filled with 900L of water. How high does the water level rise? **3**

Question 6 (11 marks)

a) Factorise fully **6**

i) $x^2 + 7x - 30$

ii) $8a^2 - 18m^2$

iii) $6x^2 - 13x - 5$

b) Simplify by expressing your answer with a single denominator **2**

$$\frac{5}{k+1} - \frac{3}{k-2}$$

c) Fully simplify $\frac{a^2 + 5a + 4}{a^2 - 16} \times \frac{a^2 - a - 12}{a^2 - 9}$ **3**

Question 7 (11 marks)

a)

Taxable Income	*Tax on this income*
\$0 – \$6,000	Nil
\$6,001 – \$35,000	15 cents for each dollar over \$6,000
\$35,001 – \$80,000	\$4,350 plus 30c for each \$1 over \$35,000
\$80,001 – \$180,000	\$17,850 plus 38c for \$1 over \$80,000
\$180,001 and over	\$55,850 plus 45c for each \$1 over \$180,000

The above rates DO NOT include the Medicare levy.

Using the above Tax Rate Table find:

i) The tax payable on an income of \$73550. **2**

ii) The Medicare levy of 1·5% on the above income. **1**

iii) The amount of income that would have owe a tax of \$8115 **2**

b) Find the compound interest on \$78,000 invested at 8%p.a. compounding half yearly, invested for 3 years. **3**

c) A motorcyclist wishes to borrow $18000 for a new bike.
This personal loan will cost him 9·5% p.a. simple interest over 5 years. **3**

i) Find the total interest on this loan.

ii) Calculate the total amount to be paid back.

iii) Find the monthly payment.

Question 8 (10 marks)

a) The following 33 marks were recorded on a stem and leaf plot from a recent mathematics test.

3	2
4	3
5	11
6	04668
7	1156778999
8	02234688
9	115569

i) Find the median **1**

ii) Find Q_1, Q_3 and the interquartile range. **3**

iii) Draw a box and whisker plot to show this information. **2**

iv) Describe the skewness of the distribution. **1**

b) Hamish received the following results in three English tests.

	Score	Mean	Standard Deviation
Test 1	78	70	6
Test 2	65	59	8
Test3	89	85	5

i) Which was his best result and why. **2**

ii) It was decided to standardise each of these results
to a mean of 80 with a standard deviation of 9.
What would his result for test 1 then be recorded as? **1**

Question 9 (14 marks)

a) Simplify i) $\dfrac{12a^5}{4a^7}$ **1**

ii) $(4p^2q)^3$ **1**

iii) $\dfrac{6m^3n^2}{5m^5} \times \dfrac{10n^6}{9mn^7}$ **3**

b) Express the following without either a negative or fractional index.

i) $\left(\dfrac{p}{q}\right)^{-1}$ **1**

ii) $m^{\frac{a}{b}}$ **2**

iii) $x^{-\frac{1}{2}}$ **2**

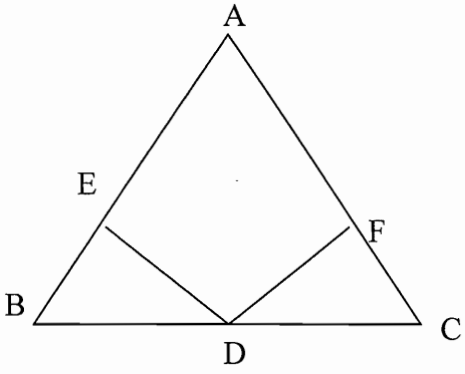
c) Solve for x $2^{3x-5} = 8^{5x}$ **2**

d) Simplify $\dfrac{a^2 + b^2}{a^{-2} + b^{-2}}$ **2**

TURN OVER TO PAGE 7 FOR QUESTION 10

Question 10 (10 marks)

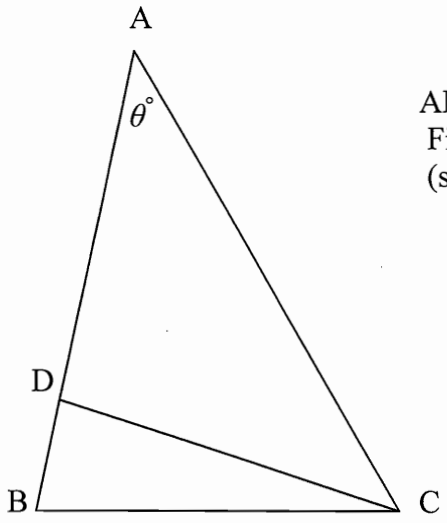
a)



D is the mid point of BC,
DE⊥AB, DF⊥AC, DE = DF. **4**

Prove ∠EBD = ∠FCD

b) **3**



AD = DC = CB, AB = AC.
Find the value of θ.
(show working)

c) **3**

Four numbers are written in ascending order.
The average of the first and second numbers is 50.
The average of the first and third numbers is 60.
The average of the third and fourth numbers is 80.
Find the average of the second and fourth numbers.
(To obtain full marks appropriate working must be shown.)

END OF EXAMINATION

2010 YEAR 9 YEARLY

Q1

a) $1.26691\ldots$ — 1

1.27 — 1/2

b) 0.0047 — 1

c) 8750×1.065 — 1

$= \$9318.75$ — 1

d) $(2\sqrt{3}+1)^2$

$= 12 + 2\sqrt{3} + 2\sqrt{3} + 1$ — 1

$= 4\sqrt{3} + 13$ — 1

e) i) $\frac{4}{3\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{18}$ — 1

$= \frac{2\sqrt{6}}{9}$ — 1

ii) $\frac{6}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}}$ — 1

$= \frac{42+6\sqrt{5}}{49-5} = \frac{42+6\sqrt{5}}{44}$ — 1

$= \frac{21+3\sqrt{5}}{22}$

Q2

a) i) $6x - 5 = 10x + 1$

$-4x = 6$

$x = -1\frac{1}{2}$ — 2

ii) $\frac{3x+8}{5} = \frac{4x-6}{3}$

$3(3x+8) = 5(4x-6)$

$9x + 24 = 20x - 30$ — 1

$-11x = -54$

$x = \frac{54}{11}$ — 1/2

iii) $4 - 3x \leq 10$

$-3x \leq 6$

$x \geq -2$

1 for correct sign

1 for -2

/2

b) i) $y = 10 - 2x$ (1)

$x + y = 7$ (2)

(1) into (2)

$x + 10 - 2x = 7$ — 1

$-x = -3$

$x = 3$

$\therefore y = 4$ — 1

$(3, 4)$ — 1/3

b) ii) $10x - 7y = 36$ ①

$2x + y = 6$ ②

$5 \times$ ② $10x + 5y = 30$ ③ — 1

① − ③

$-12y = 6$ — 1

$y = -\frac{1}{2}$

$\therefore x = 3\frac{1}{4}$

$(3\frac{1}{4}, -\frac{1}{2})$ — 1/3

c) $4a + 100 = 180$ — 1

$4a = 80$

$a = 20$ — 1

$\therefore 6b - 2a = 5a - 8b$

$14b = 7a$ — 1

$14b = 140$

$b = 10$ — 1/4

Q3

a) i) $\cos\theta = \frac{9}{15}$ — 1

$\theta = 53^\circ$ — 1/2

ii) $\tan 36^\circ = \frac{d}{12}$

$d = 12 \times \tan 36^\circ$ — 1

$= 8.71$ dp — 1/2

Q3 b) i) $\frac{1}{\sqrt{3}}$ — 1

ii) $\frac{\sqrt{3}}{2}$ — 1

iii) $-\frac{\sqrt{3}}{2}$ — 1/3

c)

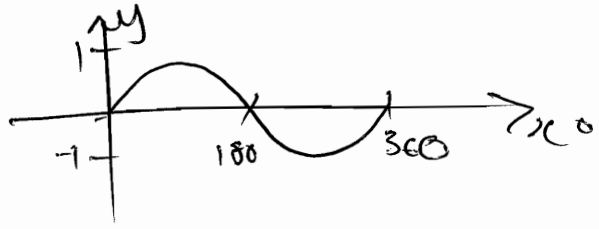


1

i) $\frac{5}{\sqrt{74}}$ — 1

ii) $\frac{7}{\sqrt{74}}$ — 1/3

d)

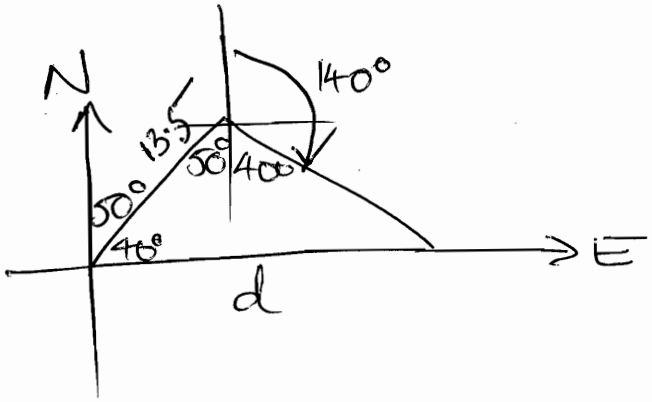


1. for correctly placed shape

1 for Scale on BOTH axes

/2

e) i)



2 must show length + angles as per question

ii) 90° — 1

iii) $\cos 40° = \frac{13.5}{d}$ — 1

$d = 17.6\text{km}$ — 1/5

Q4 a) i)

$$MP = \left(\frac{4+7}{2}, \frac{8-10}{2}\right)$$

$$= \left(\frac{11}{2}, -1\right)$$ | 1

ii) $m = \frac{-10-8}{7-4} = \frac{-18}{3} = -6$ | 1

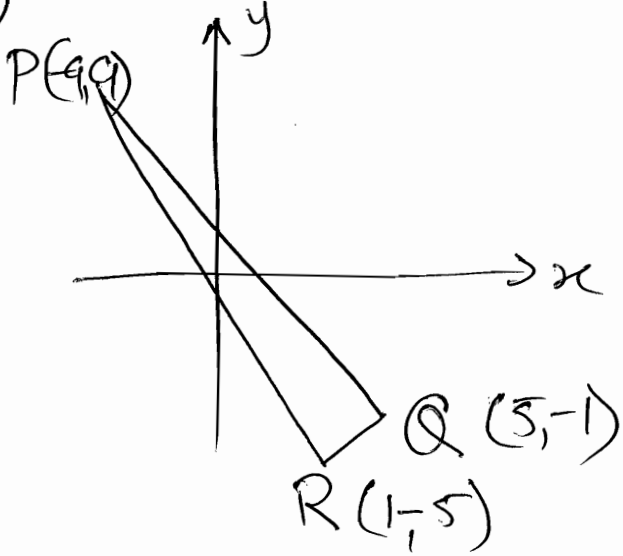
iii) $y - 8 = -6(x - 4)$ | 1

$y - 8 = -6x + 24$

$y = -6x + 32$ | 1

or $6x + y - 32 = 0$

b)



1 for plotting

$$PQ = \sqrt{(-9-5)^2 + (9-1)^2}$$

i) $= \sqrt{14^2 + 10^2}$ | 1

$$PR = \sqrt{(-9-1)^2 + (9-5)^2}$$

$= \sqrt{10^2 + 14^2}$ | 1

$\therefore PQ = PR$

ii) $S(-13, 5)$ | 2 /5

Q4 c)

$m_{AB} = \frac{5-4}{2-4} = -\frac{1}{2}$ — 1

$m_{BC} = \frac{6-5}{0-2} = -\frac{1}{2}$

$\therefore$ // from A to B & B to C — 1

$\therefore$ A, B, C are collinear — 1/2

d) $4x - 5y + k = 0$

Subst (6, 2) — 1

$4 \times 6 - 5 \times 2 + k = 0$

$14 + k = 0$

$\therefore k = -14$ — 1/2

$4x - 5y - 14 = 0$

OR.

$5y = 4x + 1$

$y = \frac{4x}{5} + \frac{1}{5}$

$\therefore m = \frac{4}{5}$. — 1

$y - 2 = \frac{4}{5}(x - 6)$

$5y - 10 = 4x - 24$ — 1.

$\boxed{4x - 5y + 14 = 0}$

Q5.

a) i) $V = \frac{1}{2} \times \frac{4}{3}\pi \times 5^3 + \frac{\pi \times 25 \times 12}{3}$ 1 1

$= \frac{250\pi + 300\pi}{3}$

$= \frac{550\pi}{3}$

$\doteq 575.96 \text{ cm}^3$ } 1 + 1 units. /4.

↑ this is the only place on the paper where units must be shown to get full marks.

ii) $l = 13$ 1

$A = \pi r l = \pi \times 5 \times 13$

$= 65\pi$

$\doteq 204.2 \text{ cm}^2$ } 1/2.

b) $V = 900L = .9m^3$

$V = \pi r^2 h$

$= \pi \times 1.25^2 \times h = .9$ 1

$h = \frac{.9}{\pi \times 1.25^2}$ 1

$= .183m$

$= 18.3cm$ } 1/3

Q6 a i)

$(x+10)(x-3)$ | 2

ii) $2(4a^2-9m^2)$ | 1

$2(2a-3m)(2a+3m)$ | 1/2

iii) $(3x+1)(2x-5)$ | 1+1/2

b) $\frac{5(k-2)-3(k+1)}{(k+1)(k-2)}$ | 1

$= \frac{2k-13}{(k+1)(k-2)}$ | 1/2

c) $\frac{(a+1)(a+4)}{(a+4)(a+4)} \times \frac{(a+3)(a-4)}{(a-3)(a+3)}$ | 2 for correct factors

$= \frac{a+1}{a-3}$ | 1/3

Q7

a) i) $73550 - 35000$

$= 38550$

Tax $= 38550 \times .3 + 4350$ | 1

$= 15915$ | 1/2

Q7a ii)

$73550 \times .015$

$= \$1103.25$ — 1

a) iii) $8115 - 4350$

$= 3765$ — 1

$\Rightarrow \frac{3765}{.3} = 12550$

Income $= 12550 + 35000$

$= \$47550$ — 1/2.

b) $A = 78000 \times 1.04^6$

$= \$98694.8$ — 2

$\therefore I = \$20694.88$ — 1/3

c) i) $I = 18000 \times .095 \times 5$

$= \$8550$ — 1

ii.) $\$26550$ — 1.

iii.) $\$442.50$ — 1./3

Q8.

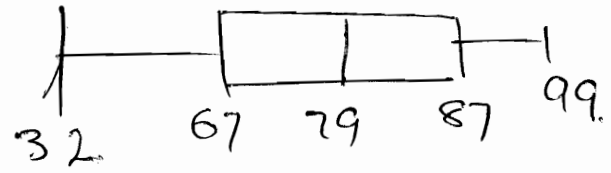
a) i) 79 — 1

ii) $Q_1 = 67$ — 1

$Q_2 = 87$ — 1

IQR = 20 — 1/3.

iii)



2

iv) Negative — 1

b) i) Test 1 $z = 1\frac{1}{3}\sigma$ above mean

Test 2 $z = 1\sigma$ " "

Test 3 $z = -8$ " "

∴ Test 1 is best result

1 ans

1 reason

/2

ii) $80 + 9 \times \frac{1}{3} = 92\%$ — 1.

Q9

a) i) $3a^{-2}$ or $\frac{3}{a^2}$ — 1

ii) $64p^6q^3$ or $4^3p^6q^3$ — 1

iii) $\frac{6m^3n^2}{5m^5} \times \frac{10n^6}{9mn^7}$ — −1 for each error

$= \frac{4n}{3m^3}$ — /3

b) i) $\frac{q}{p}$ — 1

ii) $\sqrt[b]{m^a}$ or $(\sqrt[b]{m})^a$ — 2

iii) $\frac{1}{\sqrt{x}}$ — 2 (1 root, 1 reciprocal)

c) $2^{3x-5} = 2^{15x}$

$3x - 5 = 15x$ — 1

$12x = -5$

$x = \frac{-5}{12}$ — 1 /2

d) $\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{a^2+b^2}{\frac{a^2+b^2}{a^2b^2}}$ — 1

$= a^2b^2$ — 1 /2

Q10

a) In $\triangle BED$ and $\triangle DFC$

$\angle BED = \angle DFC = 90°$ (given)

$BD = DC$. (mid point of BC)

$DE = DF$ (given)

$\therefore \triangle BED \equiv \triangle DFC$ (RHS)

$\therefore \angle EBD = \angle FCD$

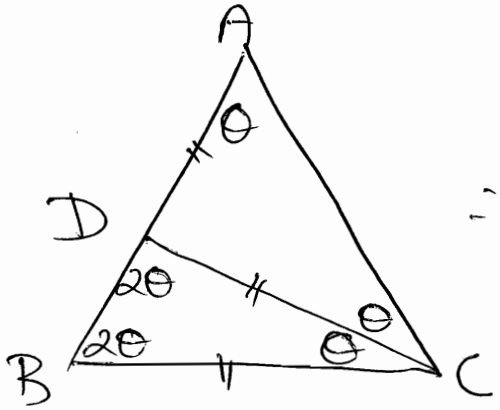
(corres $\angle$s corg $\triangle$s)

2.

1

1/4

b)



$\angle ACD = \theta$

$\therefore \angle BDC = 2\theta$

$\angle DBC = 2\theta$.

$AB = AC =$

SG $\angle DEB = \theta$

1 answer

2 working.

1/3

Note question said to show work

$\therefore 5\theta = 180°$

$\theta = 30°$

c) a b c d.

$\frac{a+b}{2} = 50$ $\frac{a+c}{2} = 60$

$\frac{c+d}{2} = 80$

$a+b = 100$

$a+c = 120$

$c+d = 160$

$\therefore b = 100 - a$

$d = 160 - c$

1 answer

2 working

NOTE Q requested working to be shown

$b + d = 260 - (a+c)$

$= 260 - 120$

$= 140$

$\therefore \frac{b+d}{2} = 70$