NORMANHURST BOYS HIGH SCHOOL

STAGE 5.1-5.3 MATHEMATICS

2019 Year 9 Yearly Examination

Monday, October 28 2019

General instructions

- Working time – 80 minutes.
- Write your answers in the space provided.
- Marks may be deducted for careless or poorly arranged work.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.

Class (please ✔)

- ◯ 9MAT.N – Mr Sekaran
- ◯ 9MAT.B – Dr Lawson
- ◯ 9MAT.H – Mrs Dupuche
- ◯ 9MAT.S – Mrs Gan

NAME: ..... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	6	7	**Total**	%
MARKS	/14	/14	/13	/13	/11	/13	/8	/86	

Question 1 (14 Marks) Commence a NEW page. **Marks**

(a) Fully simplify: $\frac{7ab}{4} - \frac{5ab}{4}$. **2**

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(b) Expand and fully simplify:

i. $(x-8)(x+2)$. **2**

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ii. $\left(m^2+3n\right)^2$. **2**

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(c) Fully factorise:

i. $4a^2+10a$. **2**

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ii. x^2-36. **2**

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(d) Fully simplify the following expression: $\frac{1}{x^2-x-6} + \frac{4}{x^2+8x+12}$. **4**

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Question 2 (14 Marks) Commence a NEW page. **Marks**

(a) Solve the equation $17 + 6y = 35$. **2**

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(b) Solve:

i. $\dfrac{m+6}{3} = \dfrac{2m+4}{4}$. **2**

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ii. $\dfrac{4}{y-3} + 2 = 1$. **2**

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(c) Solve and sketch the solution on the number line: **3**

$$3x + 5 > 6x - 4$$

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Question 2 continues overleaf...

Question 2 continued from the previous page...

(d) Given $R = \sqrt{\dfrac{V}{\pi h}}$, find R when $V = 500$ and $h = 12$, correct to two decimal places. **2**

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(e) Solve for x: $x + \dfrac{10}{x} = 7$. **3**

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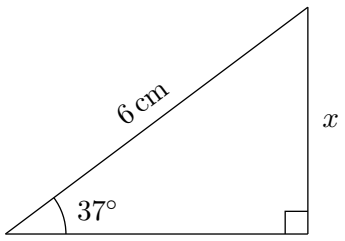
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Question 3 (13 Marks) Commence a NEW page. **Marks**

(a) Use your calculator to find $\tan 78.4°$ correct to 3 decimal places. **1**

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(b) Find x correct to three significant figures. **2**

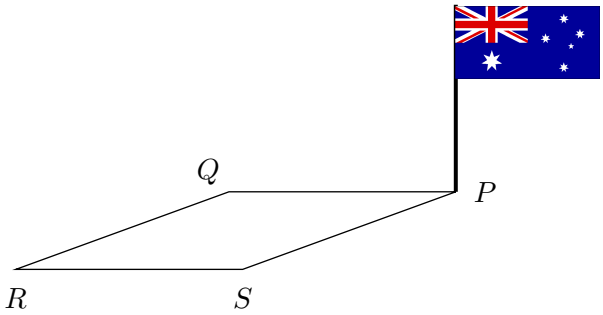


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(c) A flagpole stands at one corner P of a level square field $PQRS$, each side being 120 m long.



If the angle of elevation to the top of the pole from Q is $13°19'$, find:

i. the height of the flagpole, correct to the nearest centimetre. **1**

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ii. its angle of elevation from R. **3**

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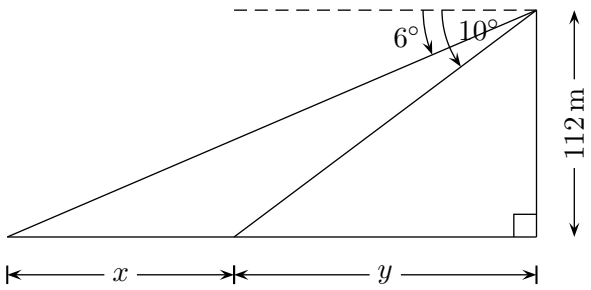
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Question 3 continues overleaf...

Question 3 continued from previous page...

(d) Find the length of x and y to two decimal places. **3**



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(e) A ship sails from port A on a bearing of 124°T towards port B, which is 80 km east of port A.

i. Sketch the diagram that represents the information. **1**

ii. How far has the ship sailed, correct to the nearest kilometre? **2**

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Question 4 (13 Marks) Commence a NEW page. **Marks**

(a) Fill in the missing word/expression to make the statement true.

i. The sum of interior angles of an n sided polygon is ..... degrees. **1**

ii. Each angle of an ..... triangle is 60°. **1**

iii. Two triangles are ..... if all of their corresponding sides are equal. **1**

iv. Opposite sides of a parallelogram are ..... and **2**

(b) Find the radius of a sphere with volume $1\,220\,\text{cm}^3$ correct to 1 decimal place. **2**

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(c) A cone has a diameter of 16 cm and perpendicular height of 15 cm. Calculate the curved surface area of the cone in terms of π. **2**

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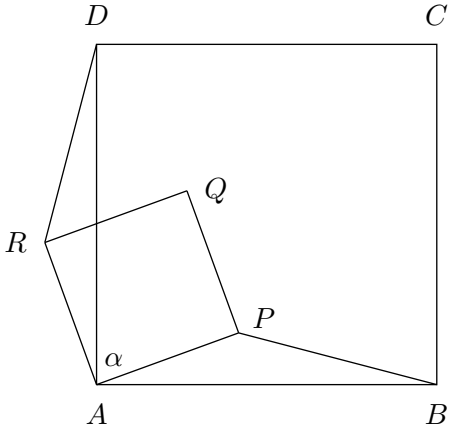
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Question 4 continues overleaf...

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(d) $ABCD$ and $APQR$ are squares and $\angle DAP = \alpha$.



i. Prove $\triangle DRA \equiv \triangle ABP$. Give full reasoning. **3**

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ii. Hence or otherwise, state the reason why $BP = DR$. **1**

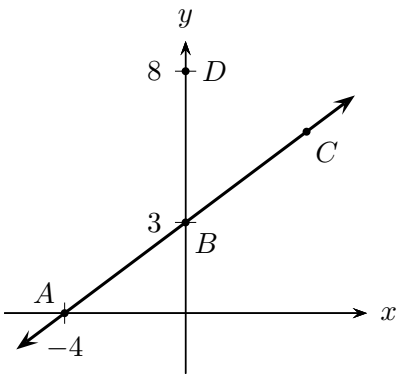
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Examination continues overleaf...

Question 5 (11 Marks) Commence a NEW page. **Marks**

The diagram shows line ℓ cutting the x axis at $A(-4,0)$ and the y axis at $B(0,3)$. The point D has coordinates $(0,8)$.



(a) Find the gradient of the line ℓ. **1**

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(b) Show that the line ℓ has equation $3x - 4y + 12 = 0$. **2**

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(c) The point C lies on ℓ and B is the midpoint of AC. Prove that the coordinates of C are $(4,6)$. **2**

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Question 5 continues overleaf...

Question 5 continued from previous page...

(d) Show that the exact length of AD is $4\sqrt{5}$ units. **2**

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(e) Show that AD is perpendicular to DC. **2**

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(f) Find the area of $\triangle ADC$. **2**

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Question 6 (13 Marks) Commence a NEW page. **Marks**

(a) Fully simplify:

i. $x^{2a-1} \times x^{a+3}$. **2**

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ii. $\dfrac{18ab^2c}{6abc^3}$. **2**

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iii. $\left(5n^2\right)^3$. **2**

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(b) Fully simplify: $\sqrt{45} + \sqrt{5}$. **2**

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(c) Express $\dfrac{\sqrt{3}}{\sqrt{3}-1}$ as the simplest fraction with a rational denominator. **2**

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(d) Find k in terms of n given **3**

$$x^k = \sqrt{\frac{x^{n+1} \times (x^2)^{n+2}}{\sqrt{x} \times x^{n+\frac{1}{2}}}}$$

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Examination continues overleaf...

Question 7 (8 Marks) Commence a NEW page. **Marks**

(a) Max has completed four of the five tasks for the year in English and is presently averaging 73%. **2**

Max wishes to improve and would like his average by the end of the year, to be 80%. Is this possible? Clearly explain your answer.

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(b) Consider the following scores:

4, 6, 7, 8, 9, 10, 10, 11, 11, 12, 16, 17, 18, 19, 20

i. What is the median? **1**

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ii. What is the lower quartile Q_1? **1**

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iii. What is the upper quartile Q_3? **1**

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iv. Find the interquartile range. **1**

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v. Draw a box-and-whisker plot showing the above information. **2**

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Suggested Solutions

Question 1

(a) (2 marks)
- ✓ [1] for $\frac{2ab}{4}$.
- ✓ [1] for $\frac{ab}{2}$.

$$\frac{7ab}{4} - \frac{5ab}{4} = \frac{\cancel{2}ab}{\cancel{4}_2} = \frac{ab}{2}$$

(b) i. (2 marks)
- ✓ [−1] for each error.

$$(x-8)(x+2) = x^2 - 6x - 16$$

ii. (2 marks)
- ✓ [−1] for each error.
- ✓ [0] for $m^4 + 9n^2$.

$$\left(m^2 + 3n\right)^2 = m^4 + 6m^2n + 9n^2$$

(c) i. (2 marks)
- ✓ [−1] for each error.

$$4a^2 + 10a = 2a(2a+5)$$

ii. (2 marks)
- ✓ [−1] for each error.

$$x^2 - 36 = (x-6)(x+6)$$

(d) (4 marks)
- ✓ [1] for each correct factorisations in the denominators.
- ✓ [1] for correct common denominator.
- ✓ [1] for final answer.

$$\begin{aligned}
&\frac{1}{x^2 - x - 6} + \frac{4}{x^2 + 8x + 12} \\
&= \frac{1}{(x-3)(x+2)} + \frac{4}{(x+6)(x+2)} \\
&= \frac{(x+6) + 4(x-3)}{(x-3)(x+2)(x+6)} \\
&= \frac{x + 6 + 4x - 12}{(x-3)(x+2)(x+6)} \\
&= \frac{5x - 6}{(x-3)(x+2)(x+6)}
\end{aligned}$$

Question 2

(a) (2 marks)
- ✓ [−1] for each error.

$$\begin{aligned}
\underset{-17}{17} + 6y &= \underset{-17}{35} \\
6y &= 18 \\
y &= 3
\end{aligned}$$

(b) i. (2 marks)
- ✓ [1] for correctly cross multiplying.
- ✓ [1] for final answer.

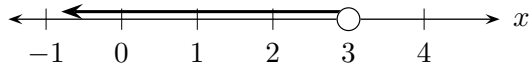
$$\begin{aligned}
\frac{m+6}{3} = \frac{2m+4}{4} &= \frac{\cancel{2}(m+2)}{\cancel{4}_2} \\
2(m+6) &= 3(m+2) \\
\underset{-2m}{2m} + \underset{-6}{12} &= \underset{-2m}{3m} + \underset{-6}{6} \\
\therefore m &= 6
\end{aligned}$$

ii. (2 marks)
- ✓ [−1] for each error.

$$\begin{aligned}
\frac{4}{y-3} + \underset{-2}{2} &= \underset{-2}{1} \\
\underset{\left(\substack{\times(y-3)\\ \div -1}\right)}{\frac{4}{y-3}} &= \underset{\left(\substack{\times(y-3)\\ \div -1}\right)}{-1} \\
\underset{+3}{-4} &= y - \underset{+3}{3} \\
y &= -1
\end{aligned}$$

(c) (3 marks)
- ✓ [1] for proper manipulation.
- ✓ [1] for algebraic expression of x.
- ✓ [1] for number line.

$$\begin{aligned}
\underset{-3x}{3x} + 5 &> \underset{-3x}{6x} - 4 \\
5 &> 3x - 4 \\
\therefore 3x &< 9 \\
x &< 3
\end{aligned}$$



(d) (2 marks)

✓ [1] for correct substitution.
✓ [1] for final answer.

$$\sqrt{\frac{500}{\pi \times 12}} = 3.64$$

(e) (3 marks)

✓ [1] for substantial working to obtain zero as subject.
✓ [1] for each correct root.

$$\underset{\times x}{x} + \frac{10}{\underset{\times x}{x}} = \underset{\times x}{7}$$
$$x^2 + 10 = 7x$$
$$x^2 - 7x + 10 = 0$$
$$(x-5)(x-2) = 0$$
$$\therefore x = 2, 5$$

Question 3

(a) (1 mark)

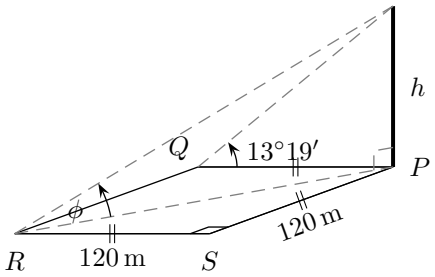
$$\tan 78.4^\circ = 4.872$$

(b) (2 marks)

✓ [1] for identifying sin ratio.
✓ [1] for final answer.

$$\frac{x}{6} = \sin 37^\circ$$
$$x = 6\sin 37^\circ = 3.61$$

(c) i. (1 mark)



$$\frac{h}{120} = \tan 13^\circ 19'$$
$$h = 120\tan 13^\circ 19' = 28.40\,\text{m}$$

ii. (3 marks)

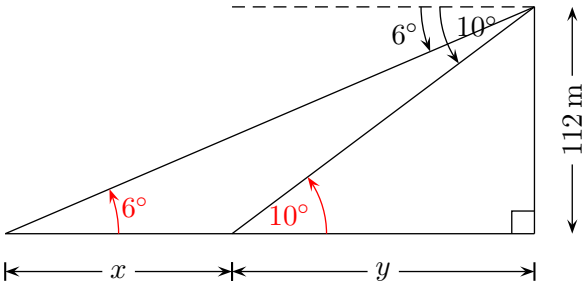
✓ [1] for finding RP.
✓ [1] for $\frac{h}{PR} = \tan\phi$.
✓ [1] for final answer.

In $\triangle PSR$,

$$PR^2 = PS^2 + RS^2 = 120^2 + 120^2$$
$$= 2 \times 120^2$$
$$\therefore PR = 120\sqrt{2}$$
$$\tan\phi = \frac{h}{PR}$$
$$= \frac{120\tan 13^\circ 19'}{120\sqrt{2}}$$
$$= 0.167\cdots$$
$$\therefore \phi = 9^\circ 30'$$

(d) (3 marks)

✓ [1] for $y = \frac{112}{\tan 10^\circ} \approx 635.18\,\text{cm}$
✓ [1] for $\tan 6^\circ = \frac{112}{x+y}$
✓ [1] for final result.

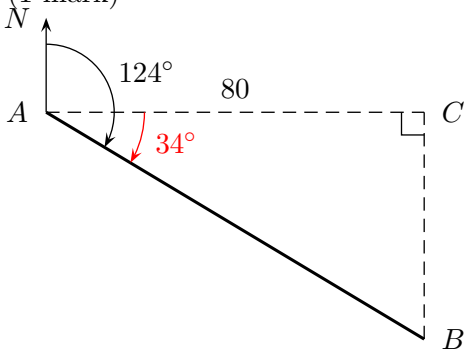


$$\tan 10^\circ = \frac{112}{y}$$
$$\therefore y = \frac{112}{\tan 10^\circ}$$

Also,

$$\tan 6^\circ = \frac{112}{x+y}$$
$$x + y = \frac{112}{\tan 6^\circ}$$
$$\therefore x = \frac{112}{\tan 6^\circ} - y$$
$$= \frac{112}{\tan 6^\circ} - \frac{112}{\tan 10^\circ}$$
$$= 430.425\cdots$$
$$\approx 430.43\,\text{km (2 d.p.)}$$

(e) i. (1 mark)



ii. (2 marks)

$$\cos 34^\circ = \frac{80}{AB}$$
$$AB = \frac{80}{\cos 34^\circ} \approx 96\,\text{km}$$

Question 4

(a) i. (1 mark) - $(n-2) \times 180°$

ii. (1 mark) - equilateral

iii. (1 mark) - congruent

iv. (2 marks) - equal, parallel

(b) (2 marks)

✓ [1] for recalling formula for volume of sphere and substituting into the formula and substitution.

✓ [1] for final answer.

$$V = \frac{4}{3}\pi r^3$$
$$1\,220 = \frac{4}{3}\pi r^3$$
$$r^3 = \frac{1\,220 \times 3}{4}$$
$$\therefore r \approx 6.6\,\text{cm (1 d.p.)}$$

(c) (2 marks)

✓ [1] for correctly calculating slanted height.

✓ [1] for final answer.

$$s^2 = 8^2 + 15^2 = 289$$
$$\therefore s = 17$$
$$SA = \pi r s = \pi \times 8 \times 17$$
$$= 136\pi$$

(d) i. (3 marks)

✓ [−1] for each missing/incorrect step in the proof.

- $AB = AD$
  (sides of a square $ABCD$)
- $\angle PAB = \angle RAD$
  ($\angle$ in square; complementary to α in squares $ABCD$ and $APQR$ respectively)
- $AP = AR$
  (sides of a square $APQR$)

$\therefore \triangle ABP \equiv \triangle DRA$ (SAS)

ii. (1 mark)
$BP = DR$
(corresponding sides of congruent $\triangle ABP$ & $\triangle DRA$)

Question 5

(a) (1 mark) $m_\ell = \dfrac{3-0}{0-(-4)} = \dfrac{3}{4}$

(b) (2 marks)

✓ [1] for each of m and b.

$$\begin{aligned} y &= \frac{3}{4}x + 3 \\ \times 4 & \quad \times 4 \quad \times 4 \\ 4y &= 3x + 12 \\ \therefore 3x - 4y + 12 &= 0 \end{aligned}$$

(use gradient from previous part, read off y intercept)

(c) (2 marks)

✓ [1] for using midpoint formula correctly

✓ [1] for solving for x and y.

✓ [0] for simply restating question etc.

$$B(0,3) = \left(\frac{-4+x}{2}, \frac{0+y}{2}\right)$$

$$\begin{array}{l|l} \dfrac{-4+x}{2} = 0 & \dfrac{0+y}{2} = 3 \\ -4+x = 0 & y = 6 \\ x = 4 & \end{array}$$

$$\therefore C(4,6)$$

(d) (2 marks)

$$\begin{aligned} A(-4,0) \qquad & D(0,8) \\ AD &= \sqrt{(-4)^2 + 8^2} \\ &= \sqrt{4^2 + 8^2} \\ &= \sqrt{16+64} = \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

(e) (2 marks)

✓ [−1] for each error.

$$\begin{aligned} m_{AD} &= \frac{8-0}{0-(-4)} = \frac{8}{4} = 2 \\ m_{DC} &= \frac{8-6}{0-4} = \frac{2}{-4} = -\frac{1}{2} \\ m_{AD} \times m_{DC} &= -1 \\ \therefore AD &\perp DC \end{aligned}$$

(f) (2 marks)

✓ [1] for length of DC.

✓ [1] for final answer.

$$\begin{aligned} DC &= \sqrt{(4-0)^2 + (6-8)^2} \\ &= \sqrt{4^2 + 2^2} = \sqrt{20} \\ &= 2\sqrt{5} \\ A_{\triangle ADC} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4\sqrt{5} \times 2\sqrt{5} \\ &= 4 \times 5 = 20 \text{ units}^2 \end{aligned}$$

Question 6

(a) i. (2 marks)

✓ [−1] for each error.

$$x^{2a-1} \times x^{a+3} = x^{2a+a-1+3} = x^{3a+2}$$

ii. (2 marks)

✓ [−1] for each error.

$$\frac{18\cancel{a}b^{\cancel{2}}\cancel{c}}{6\cancel{a}\cancel{b}c^{\cancel{2}}} = \frac{3b}{c^2}$$

$$\begin{aligned} x^k &= \sqrt{\frac{x^{n+1} \times \left(x^2\right)^{n+2}}{\sqrt{x} \times x^{n+\frac{1}{2}}}} \\ &= \left(\frac{x^{n+1} \times x^{2n+4}}{x^{\frac{1}{2}} \times x^{n+\frac{1}{2}}}\right)^{\frac{1}{2}} \\ &= \left(\frac{x^{3n+5}}{x^{n+1}}\right)^{\frac{1}{2}} \\ &= \left(x^{2n+4}\right)^{\frac{1}{2}} \\ &= x^{n+2} \\ \therefore k &= n+2 \end{aligned}$$

(b) (2 marks)

✓ [−1] for each error.

$$\left(5n^2\right)^3 = 125n^6$$

(c) (2 marks)

✓ [1] for simplifying $\sqrt{45}$.

✓ [1] for final answer.

$$\sqrt{45} + \sqrt{5} = 3\sqrt{5} + \sqrt{5} = 4\sqrt{5}$$

(d) (2 marks)

✓ [1] for multiplying by conjugate in numerator & denominator.

✓ [1] for final answer.

$$\begin{aligned} \frac{\sqrt{3}}{\sqrt{3}-1} \frac{\times(\sqrt{3}+1)}{\times(\sqrt{3}+1)} &= \frac{\sqrt{3}\left(\sqrt{3}+1\right)}{3-1} \\ &= \frac{\sqrt{3}\left(\sqrt{3}+1\right)}{2} \end{aligned}$$

(e) (3 marks)

✓ [−1] for each error.

✓ [3] if students have simplified all correctly down to x^{n+2}.

Question 7

(a) ✓ [1] for mathematical reasoning.
✓ [1] for conclusion from reasoning.

- Let each score be x_1, x_2, x_3 etc. His current average:

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = 73$$
$$\therefore x_1 + x_2 + x_3 + x_4 = 292$$

- Add in the fifth exam result, and new average is 80%:

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 80$$

But knowing $x_1 + x_2 + x_3 + x_4 = 292$,

$$\frac{292 + x_5}{5} = 80$$
$$x_5 + 292 = 400$$
$$x_5 = 108$$

- He needs 108% for his fifth result, which is not possible.

(b) i. (1 mark) $\widetilde{x} = 11$.

ii. (1 mark) $Q_1 = 8$

iii. (1 mark) $Q_3 = 17$

iv. (1 mark) $IQR = 9$

v. (2 marks)

