SYDNEY GRAMMAR SCHOOL

NAME _____ MATHS MASTER _____

2025 **Half-Yearly Examination**

Form III Mathematics

Friday 23rd May, 2025
Session A

General Instructions

- Working time — 1 hour 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Eight Questions — 104 Marks

- Each question is worth 13 marks.
- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.

Classes

3A: RCF	3B: KCCT	3C: RR/OMD	3D: WJM/LYL	3E: PJE
3F: NWS	3G: YH	3H: NJL	3I: MCW	3J: PKS

Checklist

- 14 sheets of writing paper per boy
- Candidature: 201 pupils

Writer: MCW

QUESTION ONE (13 marks) Start a new page.

(a) Simplify:

(i) $3a - 2a$ **1**

(ii) $8b^2 + 4 - 5b^2 - 1$ **1**

(b) Expand:

(i) $3\,(x + 2)$ **1**

(ii) $-4\,(x - 3)$ **1**

(c) Factorise:

(i) $2x - 8$ **1**

(ii) $3n^2 + 12n$ **1**

(d) Evaluate $2x + 5y$ when $x = 3$ and $y = -1$. **1**

(e) Solve the following equations:

(i) $\dfrac{a}{4} = -3$ **1**

(ii) $2(b - 3) = 8$ **1**

(f) Write 3.4×10^4 in decimal notation. **1**

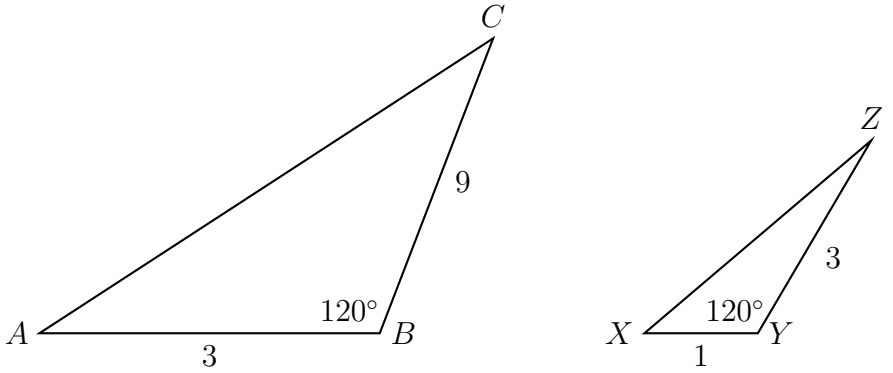
(g) Write $\dfrac{a^2}{a^4}$ with a positive index. **1**

(h) Luke got the following marks out of 100 in his History class tests. **1**

98, 92, 62, 88

What is Luke's median mark?

(i) Which similarity test proves that $\triangle ABC$ is similar to $\triangle XYZ$? **1**

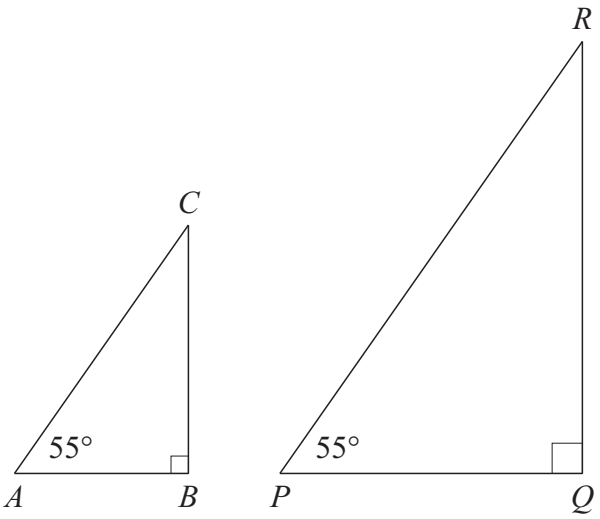


QUESTION TWO (13 marks) Start a new page.

(a) The cost of an adult train ticket is $\$A$, and the cost of a child train ticket is $\$C$. Write an algebraic expression for the cost of a family's train journey if there are 2 adults and 4 children in the family. **1**

(b) Express $\sqrt[3]{6}$ in index notation. **1**

(c) **1**



The diagram above shows two similar triangles ABC and PQR. Copy and complete the ratio statement: $\frac{AB}{\square} = \frac{AC}{\square}$.

(d) State the number of significant figures in 0.0408 metres. **1**

(e) Simplify the following:

(i) $\sqrt{75}$ **1**

(ii) $3\sqrt{7} - 2\sqrt{7}$ **1**

(iii) $\sqrt{2}\left(3\sqrt{2} + 1\right)$ **1**

(f) Convert each of the following into scientific notation.

(i) $3\,200\,000$ **1**

(ii) $0.000\,005$ **1**

(g) Expand and simplify:

(i) $(x - 7)(x + 7)$ **1**

(ii) $2x + 3 - 2(x - 4)$ **2**

(iii) $(3 + x)^2$ **1**

QUESTION THREE (13 marks) Start a new page.

(a) A random sample of Form III pupils were asked how many times they open the application TalkChat on a Saturday. The results are recorded and shown in the frequency table below.

Number of times TalkChat is opened	0	1	2	3	4
Number of pupils	8	10	7	4	3

(i) Calculate the mean number of times pupils opened TalkChat. **1**

(ii) Find the standard deviation of the number of times pupils opened TalkChat. Give your answer correct to two decimal places. **1**

(b) Factorise:

(i) $m^2 - 16$ **1**

(ii) $x^2 - 10x + 25$ **1**

(c) Express the following with positive indices and simplify.

(i) $\dfrac{1}{x^{-2}}$ **1**

(ii) $(2a)^{-3}$ **1**

(d) Simplify:

(i) $8a^6b^5 \div 4ab^3$ **1**

(ii) $2x^0 + (4y)^0$ **1**

(iii) $\dfrac{(a^3b^2)^2}{(a^2b)^3}$ **2**

(e) Solve the following equations:

(i) $4x - 5 = 2x - 9$ **2**

(ii) $-5.9 + 0.8x = -1.7$ **1**

QUESTION FOUR (13 marks) Start a new page.

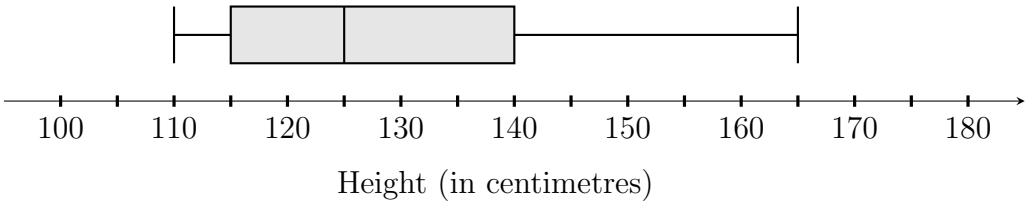
(a) Simplify the fraction below by rationalising the denominator. Show clear working. **2**

$$\frac{2}{\sqrt{5}-1}$$

(b) Next year, Sam's father will be double her age. Form an equation and solve it to find Sam's current age, given that her father is currently 25 years older than Sam. **2**

(c) A 6 m ladder is leaning against a vertical wall. The foot of the ladder is 3.6 m from the base of the wall. Use Pythagoras' Theorem to find how far up the wall the ladder reaches. **2**

(d) Data about the heights of sunflowers is gathered and the results are displayed in the box plot below.



(i) What is the median height of the sunflowers? **1**

(ii) Approximately, what percentage of the sunflowers are between 125 cm and 140 cm tall? **1**

(iii) Calculate the interquartile range for the heights of the sunflowers. **1**

(e) Express the formula $b = 3ac^2 + 4$ with a as the subject. **1**

(f) Factorise the following expressions:

(i) $x^2 - 9x + 20$ **1**

(ii) $2x^2 - 4x - 30$ **2**

QUESTION FIVE (13 marks) Start a new page.

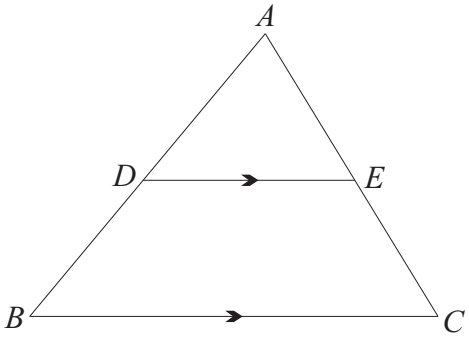
(a) Rectangle A and Rectangle B are similar shapes. Rectangle A has a width of 4 cm and a height of 6 cm. Rectangle B has a width of 10 cm. What is the height of Rectangle B? **1**

(b) (i) Solve the inequation $\dfrac{x+2}{3} \geq \dfrac{5x-1}{4}$. **2**

(ii) Represent your solution on a number line. **1**

(c) The mass of Earth is approximately 5.972×10^{24} kg and the mass of a hydrogen atom is approximately 1.673×10^{-27} kg.
Calculate, correct to four significant figures, how many times bigger the mass of Earth is than the mass of a hydrogen atom. **1**

(d) **3**



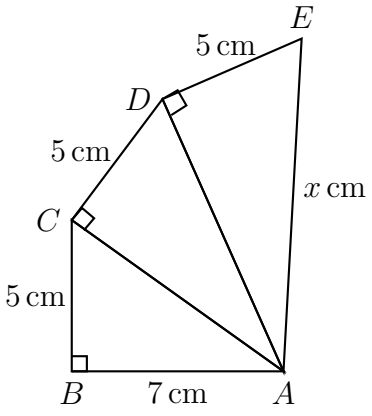
In the triangle ABC, points D and E lie on sides AB and AC of respectively, such that DE is parallel to BC, as shown in the diagram above.
Prove that $\triangle ADE$ is similar to $\triangle ABC$, giving clear geometric reasons.

(e) The formula for the area of a trapezium is given by: **2**

$$A = \frac{1}{2}(a+b)h$$

where a and b are the lengths of the parallel sides and h is the height. Rearrange the formula to make b the subject.

(f) **3**



Find an exact value for x in the diagram above, in its simplest form.

QUESTION SIX (13 marks) Start a new page.

(a) Solve the following equations:

(i) $3(2x-4)+5=2(x+7)+1$ **2**

(ii) $\dfrac{2x+7}{4}-\dfrac{3x-1}{5}=1$ **3**

(b) Simplify the following: **2**

$$\left(\frac{81x^{-4}y^{\frac{1}{2}}}{9x^2y^{-\frac{5}{2}}}\right)^{\frac{1}{2}}$$

(c) For $a=\sqrt{3}-1$ and $b=\sqrt{5}+2$, find in simplest form $\dfrac{1}{a}+\dfrac{1}{b^2}$. **3**

(d) The length of a pig pen exceeds its width by 4 m. If the length increased by 3 m and the width decreased by 1 m, the area would be unaltered. Let x metres be the width of the pen.

(i) Write an expression in x for the original area of the pig pen. **1**

(ii) Form an equation in x and solve it to find the new dimensions of the pig pen. **2**

QUESTION SEVEN (13 marks) Start a new page.

(a) Given the formula $A = \sqrt{b^2 - 2c}$, find c when $A = 4$ and $b = 6$. **2**

(b) Factorise:

(i) $112a - 28a^3$ **2**

(ii) $a^2 - 2ab + b^2 - 1$ **2**

(c) The sides of a right-angled triangle are a, $2a + b$ and $a + 2b$, where $0 < a < b$.

(i) Express a in terms of b. **2**

(ii) Find the area of the triangle, given that the perimeter is $2 + \sqrt{3}$ metres. **2**

(d) A cyclist travelled from town X to town Y, which is mostly downhill, in an hour and a half. On the return journey, which is mostly uphill, the cyclist took two hours. You may assume that each journey is ridden at a constant speed. If the difference in speed due to the terrain is 4 km/h, calculate the distance between towns X and Y. **3**

QUESTION EIGHT (13 marks) Start a new page.

(a) Is $2^{72} > 3^{48}$? Justify your answer without the use of a calculator. **1**

(b) Suppose that p, q, r, and t are positive integers and r is not a square. Given that the following expression is rational: **3**

$$\frac{p}{q+\sqrt{r}} + \frac{t}{\sqrt{r}},$$

prove that:

$$q^2t = r(p+t)$$

(c) Lily competes in an Iron Woman race over a total distance of seven kilometres, which comprises three different legs: an ocean swim, a paddle on a surf board and a beach run. She tackles each leg of the event at a different constant speed. Her speed for the paddling section is 20% quicker than her swimming speed and she runs twice as fast as she paddles. The paddling course is two kilometres long, the swim is over a distance of one and a half kilometres. **3**

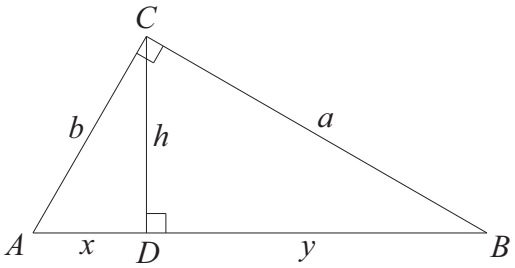
Her race time is recorded as 1 hour 51 minutes. Form and solve a linear equation to find her times for each of the separate legs.

(d) Simon has a drink bottle which contains a certain amount of pure cordial and the rest is water.

(i) Suppose the concentration of the drink is one-quarter pure cordial. A certain proportion of the drink is then removed and replaced with water only. What proportion of the drink has to be removed in order to reduce the concentration of pure cordial to one-fifth? **1**

(ii) Suppose the concentration of the drink is x% pure cordial. What proportion of the drink, if any, has to be removed and replaced with water, in order to ensure the concentration of pure cordial is at most one-tenth? Give your answer in terms of x. **2**

(e) **3**



In the diagram above, prove that $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{h^2}$

——— END OF PAPER ———

B L A N K P A G E

Form III Half-Yearly 2025 Solutions.

Question 1

(a)(i) a ✓

(ii) $3b^2 + 3$ ✓

(b)(i) $3x + 6$ ✓

(ii) $-4x + 12$ ✓

(c)(i) $2(x - 4)$ ✓

(ii) $3n(n + 4)$ ✓

(d) $2(3) + 5(-1)$

$= 1$ ✓

(e)(i) $\frac{a}{4} = -3$ $\quad a = -12$ ✓

(ii) $2b - 6 = 8$

$2b = 14$

$b = 7$ ✓

(f) 34,000 ✓

(g) $\frac{1}{a^x}$ ✓

(h) 62 88 92 98

$\frac{88+92}{2} = 90$ marks is the median ✓

(i) SAS ✓

Question 2.

(a) $\frac{1}{2}(2A + 4C)$ ✓

(b) $6^{1/3}$ ✓

(c) $\frac{AB}{PQ} = \frac{AC}{PR}$ ✓

(d) 3 ✓

(e)(i) $\sqrt{75}$
$= \sqrt{25}\sqrt{3}$
$= 5\sqrt{3}$ ✓ No working required

(ii) $\sqrt{7}$ ✓

(iii) $\sqrt{2}(3\sqrt{2}+1)$
$= 3(2) + \sqrt{2}$
$= 6 + \sqrt{2}$ ✓

(f)(i) $3200,000$
$= 3.2 \times 10^{6}$ ✓

(ii) 0.000005
$= 5 \times 10^{-6}$ ✓

(g)(i) $x^2 - 7^2$
$= x^2 - 49$ ✓

(ii) $2x + 3 - 2(x-4)$
$= 2x + 3 - 2x + 8$ ✓
$= 11$ ✓

(g)(iii) $(3+x)^2$
$= 9 + 6x + x^2$ ✓

Question 3.

(a) (i) mean = 1·5 times ✓
S.D = 1·25 ✓

(b) (i) $(m-4)(m+4)$ ✓

(ii) $x^2 - 10x + 25$
$= (x-5)(x-5)$ ✓
$= (x-5)^2$

(c) (i) x^2 ✓

(ii) $\frac{1}{(2a)^3}$
$= \frac{1}{8a^3}$ ✓

(d) (i) $\frac{8a^6b^5}{4ab^3}$
$= 2a^5b^2$ ✓

(ii) $2x^0 + (4y)^0$
$= 2(1) + 1$
$= 3$ ✓

(iii) $\frac{(a^3b^2)^2}{(a^2b)^3}$
$= \frac{a^6b^4}{a^6b^3}$ ✓
$= b.$ ✓

(e) (i) $4x - 5 = 2x - 9$
$2x = -4$ ✓
$x = -2$ ✓

(ii) $-5.9 + 0.8x = -1.7$
$0.8x = 4.2$
$x = \frac{4.2}{0.8}$
$x = 5.25.$ ✓

Question 4.

(a) $\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{2(\sqrt{5}+1)}{5-1}$ ✓

$= \frac{\cancel{2}(\sqrt{5}+1)}{\cancel{4}\ 2}$

award 1 mark for solution only with no working!

$= \frac{\sqrt{5}+1}{2}$ ✓

(b) Let Sam's current age be 'x' years.

	Now	Next year
Sam	x	$x+1$
father	$x+25$	$x+25+1$

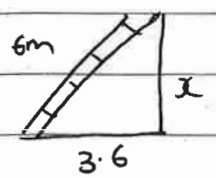
$2(x+1) = x+26$ ✓

$2x+2 = x+26$

$x = 24$

Sam is 24 years old. ✓

(c)



$6^2 = x^2 + 3 \cdot 6^2$ ✓

$36 = x^2 + 12 \cdot 96$

$26 \cdot 04 = x^2$

$x = \sqrt{26 \cdot 04}$

$x = 4 \cdot 8m$. ✓

(d)(i) 125cm ✓

(ii) 25%. ✓

(iii) $Q1 = 115$ $Q3 = 140$

$IQR = Q3 - Q1$

$= 140 - 115$

$= 25cm$ ✓

(e) $b = 3ac^2 + 4$

$-4 \qquad -4$

$(b-4) = 3ac^2$

$\frac{(b-4)}{3c^2} = \frac{3ac^2}{3c^2}$

$\frac{b-4}{3c^2} = a$

$a = \frac{b-4}{3c^2}$ ✓

(f)(i) $x^2 - 9x + 20$

$= (x-5)(x-4)$ ✓

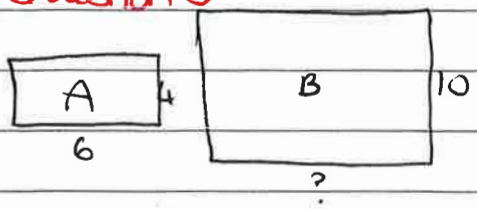
(ii) $2x^2 - 4x - 30$

$= 2(x^2 - 2x - 15)$ ✓

$= 2(x+3)(x-5)$ ✓

Question 5

(a)



$\frac{10}{4} = 2.5$

$6 \times 2.5 = 15\text{ cm}$ (accept $\frac{20}{3}$ cm)

Do not penalise units.

(b)(i) $\frac{x+2}{3} \geq \frac{5x-1}{4}$

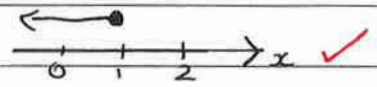
$4(x+2) \geq 3(5x-1)$

$4x + 8 \geq 15x - 3$ ✓

$11 \geq 11x$

$x \leq 1$ ✓

(ii)



✓

(c) $\frac{5.972 \times 10^{24}}{1.673 \times 10^{-27}}$

$= 3.5696 \times 10^{51}$

$= 3.570 \times 10^{51}$ ✓ penalise incorrect rounding.

(d) In $\triangle ADE$ and $\triangle ABC$

$\angle ADE = \angle ABC$ (corresponding angles, DE || BC) ✓

$\angle AED = \angle ACB$ (corresponding angles, DE || BC) ✓

$\therefore \triangle ADE$ is similar to $\triangle ABC$ (AAA) ✓

(e) $A = \frac{1}{2}(a+b)h$

$2A = (a+b)h$

$\frac{2A}{h} = a + b$ ✓

Also accept $b = \frac{2A - ah}{h}$

$\frac{2A}{h} - a = b$

$b = \frac{2A}{h} - a$ ✓

Do not accept $b = \frac{A}{\frac{1}{2}h} - a$

(f) $AC^2 = 5^2 + 7^2$

$AC^2 = 74$

$AC = \sqrt{74}\text{ cm}$ ✓

$AD^2 = (\sqrt{74})^2 + 5^2$

$AD = \sqrt{74 + 25}$

$AD = \sqrt{99}\text{ cm}$ ✓

$x^2 = (\sqrt{99})^2 + 5^2$

$x = \sqrt{124}$

$= \sqrt{4}\sqrt{31}$

$x = 2\sqrt{31}\text{ cm}$ ✓ (Accept $\sqrt{124}$ as 3 marks)

or

$AC^2 = 5^2 + 7^2$

$= 74$

$AD^2 = 5^2 + 74$

$= 99$

$x^2 = AD^2 + 5^2$

$= 124$

$x = 2\sqrt{31}\text{ cm}$.

Question 6.

(a) (i) $3(2x-4)+5 = 2(x+7)+1$

$6x - 12 + 5 = 2x + 14 + 1$

$6x - 7 = 2x + 15$ ✓

$4x = 22$

$x = \frac{22}{4}$

$x = \frac{11}{2}$ ✓

(ii) $\frac{(2x+7)}{4} - \frac{(3x-1)}{5} = 1$ (×20)

$5(2x+7) - 4(3x-1) = 20$ ✓

$10x + 35 - 12x + 4 = 20$

$-2x + 39 = 20$ ✓

$-2x = -19$

$x = \frac{19}{2}$ ✓

(b) $\left(\frac{81x^{-4}y^{1/2}}{9x^2y^{-5/2}}\right)^{1/2}$

$= \left(9x^{-6}y^{3}\right)^{1/2}$ ✓

$= 3x^{-3}y^{3/2}$ ✓ or $\frac{3y^{3/2}}{x^3}$

$= \frac{3\sqrt{y^3}}{x^3}$ or $\frac{3y\sqrt{y}}{x^3}$

(c) $\frac{1}{\sqrt{3}-1} + \frac{1}{(\sqrt{5}+2)^2}$

$= \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} + \frac{1}{5+4\sqrt{5}+4}$ ✓

$= \frac{\sqrt{3}+1}{3-1} + \frac{1}{9+4\sqrt{5}}$

6(c) continued

$= \frac{\sqrt{3}+1}{2} + \frac{1}{9+4\sqrt{5}} \times \frac{9-4\sqrt{5}}{9-4\sqrt{5}}$

$= \frac{\sqrt{3}+1}{2} + \frac{9-4\sqrt{5}}{81-80}$

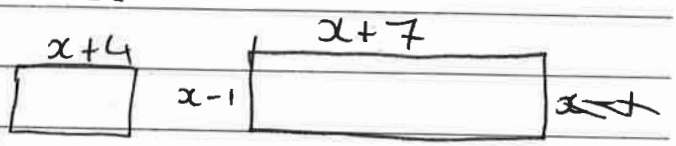
$= \frac{\sqrt{3}+1}{2} + \frac{9-4\sqrt{5}}{1}$ ✓

$= \frac{\sqrt{3}+1}{2} + \frac{2(9-4\sqrt{5})}{2}$

$= \frac{\sqrt{3}+1+18-8\sqrt{5}}{2}$

$= \frac{19+\sqrt{3}-8\sqrt{5}}{2}$ ✓

(d)



original new area

(i) $x(x+4)$ ✓

$= x^2 + 4x$

(ii) $x^2 + 4x = (x-1)(x+7)$ ✓

$x^2 + 4x = x^2 - x + 7x - 7$

$-2x = -7$

$x = 3.5\text{m}$.

∴ new dimensions are:

Width: 2.5m

Length: 10.5m ✓

Question 7.

(a) $A = \sqrt{b^2 - 2c}$ $(A=4, b=6$

$4 = \sqrt{(6)^2 - 2c}$

$4 = \sqrt{36 - 2c}$ (square both sides)

$16 = 36 - 2c$ ✓

$-20 = -2c$

$c = 10$ ✓

(b)(i) $112a - 28a^3$

$= 28a(4 - a^2)$ ✓

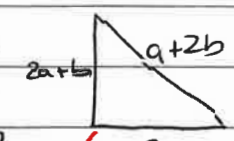
$= 28a(2-a)(2+a)$ ✓

(ii) $a^2 - 2ab + b^2 - 1$

$= (a-b)^2 - 1$ ✓ difference of two squares

$= (a-b-1)(a-b+1)$ ✓

(c)(i) $a+2b$ must be longest side as $0 < a < b$.



$\therefore (a)^2 + (2a+b)^2 = (a+2b)^2$ ✓

$a^2 + 4a^2 + 4ab + b^2 = a^2 + 4ab + 4b^2$

$4a^2 + b^2 = 4b^2$

Recognising longest side is $a+2b$

$4a^2 = 3b^2$

$a^2 = \frac{3b^2}{4}$

$a = \frac{\sqrt{3b^2}}{\sqrt{4}}$ ✓

$a = \frac{\sqrt{3}b}{2}$

(ii) $P = 2a + b + a + 2b + a$

$= 4a + 3b$

$= 4^2\left(\frac{\sqrt{3}b}{2}\right) + 3b$ (from (i))

$= 2\sqrt{3}b + 3b$

(c)(ii) Continued

$P = 2\sqrt{3}b + 3b$ but also,

$P = 2 + \sqrt{3}$

equating both:

$2\sqrt{3}b + 3b = 2 + \sqrt{3}$

$b(2\sqrt{3}+3) = 2+\sqrt{3}$

$b = \frac{2+\sqrt{3}}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3}$

$= \frac{4\sqrt{3} - 6 + 2(3) - 3\sqrt{3}}{12 - 9}$

$b = \frac{\sqrt{3}}{3}$

$\therefore a = \frac{\sqrt{3}b}{2}$ from (i)

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{3}\right)$

$= \frac{3}{6}$

$a = \frac{1}{2}$ ✓

base of $\Delta = \frac{1}{2}$ height $= 1 + \frac{\sqrt{3}}{3}$

$\therefore$ area $= \frac{1}{2}\left(\frac{1}{2}\right) \times \left(1 + \frac{\sqrt{3}}{3}\right)$

$= \frac{1}{4}\left(1 + \frac{\sqrt{3}}{3}\right)$

$= \left(\frac{1}{4} + \frac{\sqrt{3}}{12}\right) m^2$ ✓

$= \frac{3+\sqrt{3}}{12} m^2$

Question 7 Continued...

Let 'S' be speed, and 'd' be distance

(d) from town X to town Y,

$\text{Time} = \frac{\text{Distance}}{\text{speed}}$

$1.5 = \frac{\text{distance}}{S}$

$1.5s = \text{distance}$ (1) ✓ two finding equations for distance

from town Y to town X,

$\text{Time} = \frac{\text{distance}}{\text{speed}}$

$2 = \frac{\text{distance}}{s-4}$

$2(s-4) = \text{distance}$ (2)

equating (1) and (2)

$2(s-4) = 1.5s$ ✓

$2s - 8 = 1.5s$

$0.5s = 8$

$s = 16 \text{ km/hr}.$

Subbing 16 km/hr into (1)

$\text{Distance} = 1.5(16)$

$= 24 \text{ km}.$ ✓

Alterative solution:

Let d km be distance between town X and Y

$\text{Speed} = \frac{\text{dist}}{\text{time}}$

Speed for $X \to Y = \frac{d}{3/2} = \frac{2d}{3}$ ✓

Speed for $Y \to X = \frac{d}{2}$

$\therefore \frac{2d}{3} - 4 = \frac{d}{2}$ $(\times 6)$ $(\times 6)$ ✓

$4d - 24 = 3d$

$d = 24 \text{ km}.$

Distance between towns is 24km ✓

Question 8

(a) If $2^{72} > 3^{48}$ (HCF of 72 and 48 is 24)

Then, $(2^3)^{24} > (3^2)^{24}$

✓ correct justification

So $8^{24} > 9^{24}$

But $8 < 9$, so $8^{24} < 9^{24}$, therefore it is false. $(8^{24} \not> 9^{24})$.

(b) $\dfrac{p}{q+\sqrt{r}} + \dfrac{t}{\sqrt{r}}$

$= \dfrac{p}{q+\sqrt{r}} \times \dfrac{q-\sqrt{r}}{q-\sqrt{r}} + \dfrac{t\sqrt{r}}{\sqrt{r}\sqrt{r}}$ (rationalising)

1 mark for rationalising and adding fractions.

$= \dfrac{p(q-\sqrt{r})}{q^2-(\sqrt{r})^2} + \dfrac{t\sqrt{r}}{r}$

$= \dfrac{rp(q-\sqrt{r}) + t\sqrt{r}(q^2-r)}{r(q^2-r)}$ expanding

$= \dfrac{pqr - pr\sqrt{r} + tq^2\sqrt{r} - tr\sqrt{r}}{r(q^2-r)}$ factorise out $\sqrt{r}$.

$= \dfrac{\sqrt{r}[tq^2 - pr - tr] + pqr}{r(q^2-r)}$ ✓

Since the original expression is rational, and $\sqrt{r}$ must be irrational (as r is not a square num.)

Then, $tq^2 - pr - tr = 0$ ✓ with justification

$q^2t = pr + tr$

$q^2t = r(p+t)$

(c) Let x be Lily's swimming speed in km/hr.

	Swim	Paddle	Run
speed (km/hr)	x	$\frac{6x}{5}$	$\frac{12x}{5}$
Dist. (km)	$3/2$	2	$7/2$
time hrs	$\frac{3/2}{x} = \frac{3}{2x}$	$\frac{2}{6x/5} = \frac{5}{3x}$	$7/2 \div \frac{12x}{5} = \frac{35}{24x}$

$\text{Time} = \dfrac{\text{Dist}}{\text{Speed}}$. ✓ progress

$\text{Total time} = \dfrac{3}{2x} + \dfrac{5}{3x} + \dfrac{35}{24x}$

$= \dfrac{36}{24x} + \dfrac{40}{24x} + \dfrac{35}{24x}$

$\text{Total time} = \dfrac{111}{24x}$

Total time is also $1\text{hr } 51\text{min} = \frac{111}{60}$ hr

$\therefore \dfrac{111}{60} = \dfrac{111}{24x}$

$24x = 60$

$x = 5/2$ km/hr. ✓

Time for swim $= \dfrac{3}{2(5/2)}$

$= 3/5$

$= 36$min

Time for paddle $= \frac{5}{3}(5/2)$

$= 4/3$

$= 40$min

Time for Run $= \frac{35}{24}(5/2)$

$= 35$min ✓ all times

Question 8 continued

(d) (i) $\frac{1}{4} - \frac{1}{5} = \frac{5}{20} - \frac{4}{20} = \frac{1}{20}$

∴ We want to remove $\frac{1}{20}$ of pure cordial from the drink.

∴ $4 \times \frac{1}{20} = \frac{4}{20} = \boxed{\frac{1}{5}}$ of the total drink needs to be removed. ✓

(ii) When $x \leq 10$, nothing needs to be removed.

When $x > 10$, we start with x% and we need to remove a proportion. Let the proportion removed be y.

So $\frac{x}{100} - y\left(\frac{x}{100}\right) = \frac{1}{10}$ ✓ Any correct progress.

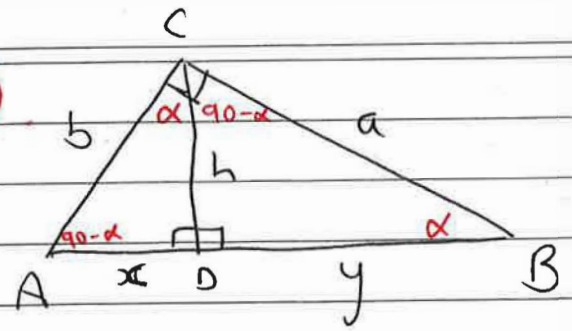
If we look only at the amount of pure cordial to remove, we start with x and we remove xy since we're removing y in total, of which x% is pure cordial.

Solving this equation:

$\frac{xy}{100} = \frac{x}{100} - \frac{1}{10} \quad \left(\times \frac{100}{x}\right)$

$\therefore y = 1 - \frac{10}{x}$ ✓

(e)



Let $\angle ACD$ be α, then $\angle BCD = 90-\alpha$ and $\angle CAD = 90-\alpha$ (Triangle sum ΔACD) and $\angle CBD = 90°$ (Angle sum of ΔCBD)

∴ ΔACD is similiar to ΔCDB is also similiar to ACB a all angles are the same (AAA) ✓ proving similarity

Since Δ's are similar, their sides are in proportion. So $\frac{h}{x} = \frac{y}{h}$ ∴ $h^2 = xy$ (1)

Also, $\frac{b}{x} = \frac{x+y}{b}$ and $\frac{a}{y} = \frac{x+y}{a}$

$b^2 = x^2 + xy$ and $a^2 = xy + y^2$

✓ find equations base on similar Δ's

But from (1), $h^2 = xy$.

So $b^2 = x^2 + h^2$ and $a^2 = h^2 + y^2$

Also, $\frac{b}{h} = \frac{x+y}{a}$

$\frac{ab}{h} = x+y$ (2)

$b^2 + a^2 = (x+y)^2$ (pythagoras' theorem, ΔABC)

$b^2 + a^2 = \left(\frac{ab}{h}\right)^2$ (from (2))

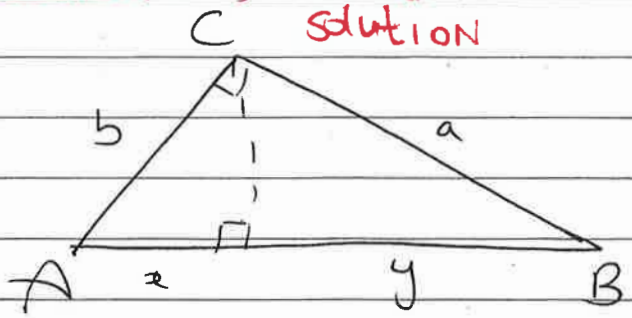
$b^2 + a^2 = \frac{a^2b^2}{h^2}$ (Divide each term by a^2)

$\frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2} = \frac{a^2b^2}{h^2} \div a^2b^2$

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$

✓ use of pytha and solutio

Question 8(c) alternative solution



$a^2+b^2=(x+y)^2$ by pythagoras' theorem.

and area $= \frac{1}{2}ab$

area is also $= \frac{1}{2}(x+y)h$

So $\frac{1}{2}ab = \frac{1}{2}(x+y)h$

$ab = (x+y)h$ ✓ equating areas

Then $\frac{1}{a^2}+\frac{1}{b^2} = \frac{b^2+a^2}{(ab)^2}$

$= \frac{(x+y)^2}{((x+y)h)^2}$ ✓

$\frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{h^2}$ ✓