SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2023 Half-Yearly Examination

Form III Mathematics

Wednesday 17th May 2023

8:42am

General Instructions

- Working time — 1 hour 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators may NOT be used.

Total Marks: 104

Questions 1 – 8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Place everything inside this question booklet.

3A: BLR	3B: NJL	3C: PKS	3D: PC
3E: AEA/BR	3F: LRP/GMC	3G: OMD	3H: NWS
3I: LYL	3J: YH		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 202 pupils

Writer: AEA

QUESTION ONE (13 marks) Start a new page.

(a) Simplify:

(i) $3x + 4x$

(ii) $7s^2 + s^2 - 11$

(b) Evaluate $6m - 4n$ when $m = 2$ and $n = 1$.

(c) Simplify $\sqrt{32}$.

(d) Factorise:

(i) $2x + 4$

(ii) $3pq + 12p$

(iii) $x^2 - 5x$

(e) Solve the following equations for x:

(i) $x + 5 = 7$

(ii) $3 + 4x = -2x + 9$

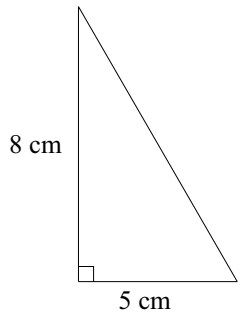
(f) Simplify:

(i) $e^6 \times e^8$

(ii) $4a^7 \times 2a^{-3}$

QUESTION TWO (13 marks) Start a new page.

(a)



The diagram above shows a right-angled triangle of base length 5 cm and height 8 cm. Find the length of the hypotenuse, leaving your answer in exact form.

(b) Rewrite $\frac{3}{x}$ with a negative index.

(c) Write 0·004683 in scientific notation correct to two significant figures.

(d) Ten pupils ran a 10 km race. Their completion times, in minutes, are below:

49 53 58 59 60 61 67 67 71 72

Find:

(i) the mode

(ii) the median

(iii) the range

(e) Expand and simplify:

(i) $2(x+3)$

(ii) $5a(a+2)-6a$

(iii) $(x+2)(x+5)$

(iv) $(x+6)^2$

(v) $(b+4)(b-4)$

QUESTION THREE (13 marks) Start a new page.

(a) Simplify:

(i) $\sqrt{27} + \sqrt{45} - 2\sqrt{5} + \sqrt{3}$

(ii) $\sqrt{8} \times \sqrt{3}$

(b) Evaluate $(3b^2)^0 \times 4b^0$.

(c) Simplify $\dfrac{3x^4}{8} \times \dfrac{4}{x^2}$.

(d) Factorise fully:

(i) $x^2 + 7x + 12$

(ii) $x^2 - 11x + 24$

(iii) $5x^2 - 50x - 120$

(e) Make x the subject of the formula $y = mx + c$.

(f) Express $\dfrac{3}{\sqrt{5}}$ with a rational denominator in simplest form.

QUESTION FOUR (13 marks) Start a new page.

(a) Evaluate:

(i) $25^{\frac{3}{2}}$

(ii) $\left(\frac{27}{64}\right)^{-\frac{1}{3}}$

(b) Write down all irrational numbers from the following list: π, $0\cdot\dot{7}$, $\frac{1}{2}$, -1, $\sqrt{2}$.

(c) Factorise fully:

(i) $x^2 - 14x + 49$

(ii) $7x + 18 - x^2$

(iii) $4x^2 - 9$

(d) Solve the inequality $6 - 2x < 14$, and graph the solution on a number line.

QUESTION FIVE (13 marks) Start a new page.

(a) Solve the following equations for x:

(i) $\dfrac{2x}{7} - 6 = 2$

(ii) $\dfrac{x+1}{7} = \dfrac{2x-2}{3}$

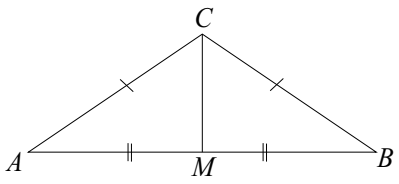
(iii) $\dfrac{3x+2}{5} + 2 = \dfrac{x}{3}$

(b) Michael's father is six times as old as Michael. In twenty years time Michael's father will be twice as old as Michael. Let m be Michael's current age. By forming and solving an equation, find Michael's current age.

(c) Simplify $(7 \times 10^4) \times (3 \times 10^{-6})$. Give your answer in scientific notation.

(d) Simplify and express with positive indices: $5a^6b^{-7} \times 6a^2b^5$.

(e)



In the diagram above, $\triangle ABC$ is isosceles with $AC = BC$. Point M is the midpoint of the base AB. Prove that $\triangle AMC \equiv \triangle BMC$, giving clear geometric reasons.

QUESTION SIX (13 marks) Start a new page.

(a) Rationalise the denominator of $\dfrac{\sqrt{5}}{5-\sqrt{3}}$, giving your answer in simplest form.

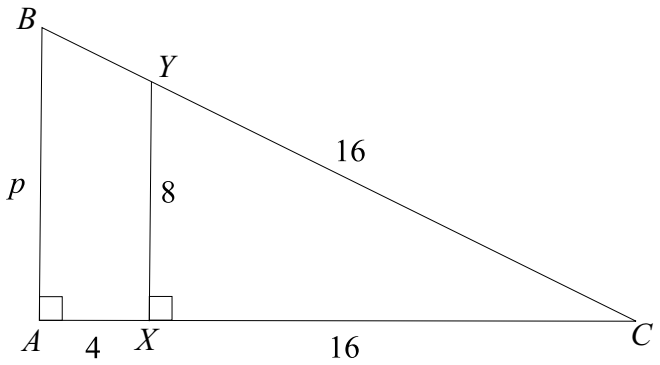
(b) Andrew has a certain number of 20-cent coins and twice as many 10-cent coins. The total value of the coins is \$12. Let n be the number of 20-cent coins.

(i) Write an equation in n for the value of the coins.

(ii) Solve your equation to find the number of 10-cent and 20-cent coins.

(c) Find the size of an interior angle in a regular nine-sided polygon.

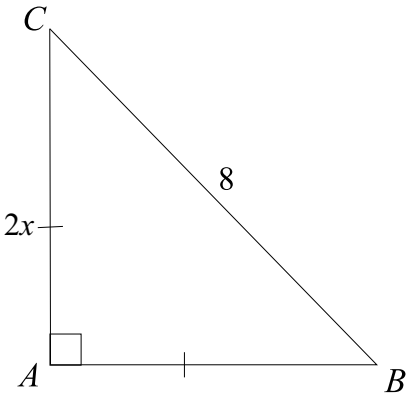
(d)



(i) Prove that $\triangle ABC$ is similar to $\triangle XYC$.

(ii) Hence find the value of p, giving reasons.

(e)



Find the value of x, leaving your answer in simplified exact form.

QUESTION SEVEN (13 marks) Start a new page.

(a) Consider the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$.

(i) Find the value of A if $a = 2$, $b = 3$, $c = 4$, $s = 6$.

(ii) Rearrange the formula to make c the subject.

(iii) Find the value of c if $A = 84$, $s = 21$, $a = 14$ and $b = 13$.

(b) Tom drove at a constant speed, x km/h, for 4 hours. He then increased his speed by 20 km/h and drove at this speed for $2\frac{1}{2}$ hours. In total, he travelled a distance of 427 km.

(i) Form an equation to represent the information above.

(ii) By solving your equation from (i), find Tom's original speed.

(c) Write $\sqrt[3]{\left(\frac{8}{27}\right)^{-4}}$ as a mixed numeral.

(d) Simplify $2x + 6 - \dfrac{12}{3 - \dfrac{3x}{4+x}}$.

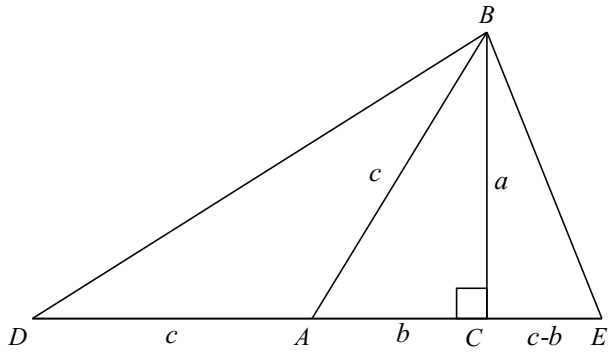
QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $\dfrac{\left(3x^4y\right)^2}{x^{-2}y^2} \times \dfrac{2xy^{-2}}{x^5y}$.

(b) Factorise fully $a\,(b+c)^4 - a\,(b-c)^4$.

(c) If $\dfrac{a}{b} = k$, express $\dfrac{4a-5b}{\sqrt{18a^2-4b^2}}$ in terms of k.

(d)



The following question is designed to prove Pythagoras Theorem. Do NOT assume this result in answering the question.
Triangle ABC is right angled at C. Side CA is produced to D such that $AD = c$ and side AC is produced to E such that $CE = c - b$.

(i) Prove that $\triangle BCD$ is similar to $\triangle BCE$.

(ii) Prove that $\dfrac{a}{b+c} = \dfrac{c-b}{a}$.

(iii) Hence prove the theorem of Pythagoras.

(e) A group of n students sat a test and the mean was calculated.

(i) The result of a student, Pierre, was excluded and the mean increased by 1. Find a simplified expression in terms of n for the difference between Pierre's score and the mean without his score.

(ii) If instead, Pierre's mark was halved, the mean would still decrease by 1. Find a simplified expression for the mean of the test (including Pierre's score) in terms of n.

END OF PAPER

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Form 3 HY 2023

Question 1

a) i) $7x$ (1)

ii) $8s^2 - 11$ (1)

b) $6m - 4n$

$6(2) - 4(1) = 8$ (1)

c) $\sqrt{32} = 4\sqrt{2}$ (1)

d) i) $2(x+2)$ (1)

ii) $3p(q+4)$ (1)

iii) $x(x-5)$ (1)

e) i) $x = 2$ (1)

ii) $3 + 4x = -2x + 9$

$3 + 6x = 9$ (1) collecting x's

$6x = 6$

$x = 1$ (1) answer

f) i) e^{14} (1)

ii) $8a^4$ (1) 8

(1) a^4

Question 2

a) $c^2 = 8^2 + 5^2$ (1) pythag

$c^2 = 64 + 25$

$c^2 = 89$

$c = \sqrt{89}$ units (1) answer in exact form

b) $3x^{-1}$ (1) or $\left(\frac{x}{3}\right)^{-1}$

c) 4.7×10^{-3} (1) Significant figures (1) Scientific notation

d) i) 67 (1)

ii) 60.5 (1)

iii) 23 (1)

e) i) $2x + 6$ (1)

ii) $5a^2 + 10a - 6a$

$= 5a^2 + 4a$ (1)

iii) $x^2 + 7x + 10$ (1)

iv) $x^2 + 12x + 36$ (1)

v) $b^2 - 16$ (1)

Question 3

a) i) $3\sqrt{3} + 3\sqrt{5} - 2\sqrt{5} + \sqrt{3}$ (1)

$= 4\sqrt{3} + \sqrt{5}$ (1)

ii) $\sqrt{24} = 2\sqrt{6}$ (1) multiplying (1) simplifying

b) $1 \times 4(1) = 4$ (1)

c) $\frac{3x^{2}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{x^{2}}} = \frac{3x^{2}}{2}$ (1)

d) i) $(x+3)(x+4)$ (1)

ii) $(x-8)(x-3)$ (1)

iii) $5(x^{2} - 10x - 24)$ (1) factoring out 5

$= 5(x-12)(x+2)$ (1)

e) $y = mx + c$

$mx = y - c$ (1) 1 correct step

$x = \frac{y-c}{m}$ (1) answer.

f) $\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$ (1)

Question 4

a) i) $\left(25^{\frac{1}{2}}\right)^3 = 5^3$ ① evaluating to 5^3 or $25^{\frac{1}{2}}$

$= 125$ ①

ii) $\left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$ ① flipping fraction for neg index

① answer

b) $\pi, \sqrt{2}$ ① both

c) i) $(x-7)^2$ ①

ii) $-(x^2-7x-18)$ ① factoring out negative

$= -(x-9)(x+2)$ ① answer or ②$(9-x)(2+x)$

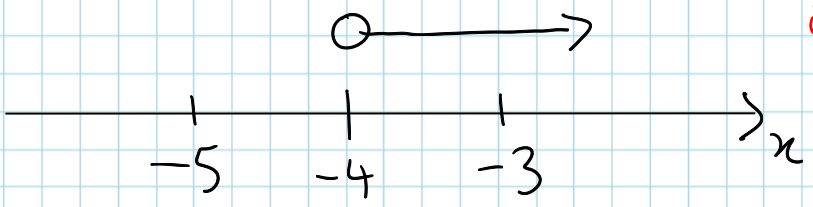
iii) $(2x-3)(2x+3)$ ① Some form of difference of two squares

① Correct answer

d) $6-2x < 14$

$-2x < 8$ ① 1 step correct

$x > -4$ ① correct answer



① numberline correct to their solution.

Question 5

a) i) $\frac{2x}{7} - 6 = 2$

$\frac{2x}{7} = 8$ (1) one correct step

$2x = 56$

$x = 28$ (1) correct answer

ii) $\frac{x+1}{7} = \frac{2x-2}{3}$ (1) common denom or cross mult.

$3(x+1) = 7(2x-2)$

$3x + 3 = 14x - 14$

$17 = 11x$

$x = \frac{17}{11}$ (1) correct answer

iii) $\frac{3x+2}{5} + \frac{2}{1} = \frac{x}{3}$ (1) A common denom. or $\times 15$

$\times 15$

$3(3x+2) + 30 = 5x$

$9x + 6 + 30 = 5x$

$4x = -36$

$x = -9$ (1) answer

b) Michael is m

Michael father $= 6m$

In 20 years

michaels father $+ 20 = 2(m + 20)$

$6m + 20 = 2(m+20)$ (1)

$6m + 20 = 2m + 40$

$4m = 20$

$m = 5$

$\therefore$ Michael is 5 (1)

c) $7 \times 3 \times 10^{4} \times 10^{-6}$

$= 21 \times 10^{-2}$ (1)

$= 2.1 \times 10^{-1}$ (1)

d) $\frac{5a^{6}}{b^{7}} \times 6a^{2}b^{5}$ $\quad 7 - 2$

$= \frac{30a^{8}}{b^{2}}$ (1)

e) In $\triangle AMC$ and $\triangle BMC$

CM is common

$AC = BC$ (equal sides of isos. $\triangle ABC$)

$AM = BM$ (M is midpoint of AB)

$\therefore \triangle AMC \equiv \triangle BMC$ (SSS)

(1) one correct statement with reason

(1) full proof.

OR other test with full proof.

Question 6

a) $\frac{\sqrt{5}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{\sqrt{5}(5+\sqrt{3})}{25-3}$ ① conjugate

$= \frac{\sqrt{5}(5+\sqrt{3})}{22}$ } ① either

$= \frac{5\sqrt{5}+\sqrt{15}}{22}$

b) If amount of 20c coins is n

$\therefore$ $2n$ 10c coins

① progress towards eqn

i) $2n \times 10 + n \times 20 = 1200$

ii) $20n + 20n = 1200$ ① correct eqn

$40n = 1200$

$n = 30$ ① answering question.

$\therefore$ 30 20c coins and 60 10c coins.

c) $S = 180(n-2)$

$= 180(9-2)$

$= 1260^\circ$ ①

OR $360 \div 9$ ①

$= 40$

then $180 - 40$

$= 140$ ①

Interior angle $= 1260 \div 9$

$= 140^\circ$ ①

d) i) In $\triangle ABC$ and $\triangle XYC$

$\angle C$ is common

$\angle BAC = \angle YXC = 90^\circ$ (given)

$\angle ABC = \angle XYC$ (corresponding angles, $AB \parallel XY$)

$\therefore \triangle ABC \parallel\parallel \triangle XYC$ (AAA)

(1) One angle with reason

(1) Two (or three) angles w/ reason/s and stating proof used.

ii) Since $\triangle ABC \parallel\parallel \triangle XYC$, the sides will be in the same ratio

$\frac{p}{8} = \frac{20}{16}$ (1) ratio statement

$p = \frac{160}{16}$

$p = 10$ (1) answer

e) $(2x)^2 + (2x)^2 = 8^2$ (1) using pythag to set up equation

$4x^2 + 4x^2 = 64$

$8x^2 = 64$

$x^2 = 8$

$x = \sqrt{8}$ $(x>0)$

$x = 2\sqrt{2}$ (1) correct, simplified answer.

Question 7

a) $A = \sqrt{s(s-a)(s-b)(s-c)}$

i) $A = \sqrt{6(6-2)(6-3)(6-4)}$

$= \sqrt{6(4)(3)(2)}$

$= \sqrt{144}$

$= 12$ (1)

ii) $A^2 = s(s-a)(s-b)(s-c)$ (1) Squaring

$\frac{A^2}{s(s-a)(s-b)} = s-c$

$c = s - \frac{A^2}{s(s-a)(s-b)}$ (1)

iii) $c = 21 - \frac{84^2}{21(21-14)(21-13)}$

$= 21 - \frac{7056}{21(7)(8)}$ (1) Substitution and progress towards answer

$= 21 - 6$

$= 15$ (1)

b) 

$d = 4x$

$d = 2.5(x+20)$ } 427km

i) $4x + 2.5(x+20) = 427$ (1) or similar

ii) $4x + 2.5x + 50 = 427$ } (1) one correct step

$6.5x = 377$

$x = 58$ (1)

$\therefore$ Tom's OG speed is 58 km/h

c) $\sqrt[3]{\left(\frac{8}{27}\right)^{-4}} = \left(\frac{2}{3}\right)^{-4}$

$= \left(\frac{3}{2}\right)^{4}$ } (1) for one correct step: flipping / $\sqrt[3]{}$ or ^{4}

$= \frac{81}{16}$

$= 5\frac{1}{16}$ } (1) either form of correct answer

d) $2x+6-\dfrac{12}{3-\dfrac{3x}{4+x}}$

$= 2x+6-\dfrac{12}{\dfrac{3(4+x)-3x}{4+x}}$ (1) common denom.

$= 2x+6-\dfrac{12(4+x)}{12+3x-3x}$ (1) 4+x in numerator and correct expansion

$= 2x+6-\dfrac{\cancel{12}(4+x)}{\cancel{12}}$

$= 2x+6-4-x$

$= x+2$ (1)

Question 8

a) $\dfrac{(3x^4y)^2}{x^{-2}y^2} \times \dfrac{2xy^{-2}}{x^5y}$

$= \dfrac{9x^8y^2x^2}{y^2} \times \dfrac{2x}{x^5y^3}$ (1) bracket or negatives dealt with correctly.

$= \dfrac{18x^6}{y^3}$ (1)

b) $a(b+c)^4 - a(b-c)^4$

$= a[(b+c)^4 - (b-c)^4]$

$= a\{[(b+c)^2+(b-c)^2][(b+c)^2-(b-c)^2]\}$ (1) diff of 2 □'s. or expansion

$= a\{[b^2+2bc+c^2+b^2-2bc+c^2][b^2+2bc+c^2-b^2+2bc-c^2]\}$

$= a\{(2b^2+2c^2)(4bc)\}$

$= 8abc(b^2+c^2)$ (1)

c) $\dfrac{4a-5b}{\sqrt{18a^2-4b^2}} \quad \begin{matrix} \times \frac{1}{b} \\ \\ \times \frac{1}{b} \end{matrix} = \dfrac{\frac{4a-5b}{b}}{\sqrt{\frac{18a^2-4b^2}{b^2}}}$ (1)

$= \dfrac{\frac{4a}{b}-5}{\sqrt{18\frac{a^2}{b^2}-4}}$

$= \dfrac{4k-5}{\sqrt{18k^2-4}}$ (1)

d) i) $AE = b + (c-b)$ (given)

$= c$

$AD = AB = AE = c$ (given)

Let $\angle AOB = \alpha$ and let $\angle CEB = \beta$

$\angle ABE = \beta$ (base angles of isosceles $\triangle ABE$)

$\angle OBA = \alpha$ (base angles of isosceles $\triangle ABD$)

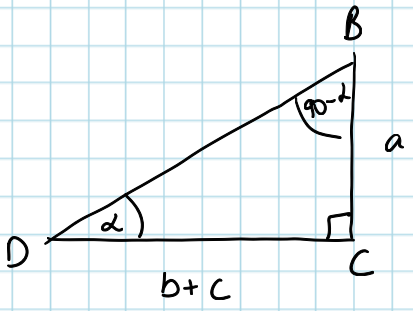
$\alpha + \alpha + \beta + \beta = 180$

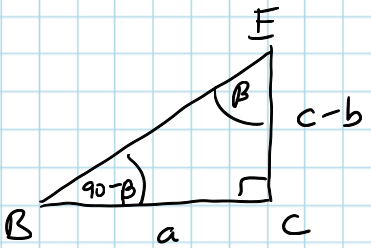
$2\alpha + 2\beta = 180$

$\alpha + \beta = 90$

$\therefore \triangle OBE$ is right-angled, $\angle OBE = 90°$

(1)





Since $\alpha + \beta = 90$

$90 - \beta = \alpha$ $\quad \therefore \angle EBC = \angle BDC = \alpha$

and $\beta = 90 - \alpha$ $\quad \therefore \angle DBC = \angle BEC = \beta$

$\angle DCB = \angle BCE = 90^\circ$ (given)

$\therefore \Delta DCB \,|||\, \Delta BCE$ (AAA) (1)

ii) $\dfrac{a}{b+c} = \dfrac{c-b}{a}$ (1) (matching sides in similar triangles are in same ratio)

iii) $a^2 = (c-b)(c+b)$

$a^2 = c^2 - b^2$ (1)

$\therefore c^2 = a^2 + b^2$

e) Let x be the total number of marks scored by the students, excluding Pierre. Let Pierre's mark be a.

Then, $$\frac{x+a}{n} = \frac{x}{n-1} - 1$$

Solving for a:

$$(n-1)(x+a) = xn - n(n-1)$$
$$nx + na - x - a = nx - n(n-1)$$
$$a(n-1) - x = -n(n-1)$$
$$a(n-1) = x - n(n-1)$$
$$a = \frac{x}{n-1} - n$$

This is n marks less than $\frac{x}{n-1}$ (the mean without Pierre). (1)

Alternative solution

To decrease a mean score of n students by 1, he would have to score n less than the mean since this would decrease the mean by $\frac{n}{n} = 1$.

ii) Pierre's mark is now $\frac{a}{2}$.

$$\frac{x+\frac{a}{2}}{n} = \frac{x+a}{n} - 1$$

$$x + \frac{a}{2} = x + a - n$$

$$\frac{a}{2} = n$$

$$a = 2n \quad (1)$$

from i) $a = \frac{x}{n-1} - n$

$$\therefore \quad 2n = \frac{x}{n-1} - n$$

$$3n = \frac{x}{n-1}$$

$$3n(n-1) = x$$

The mean was $\frac{x+a}{n}$

$$\therefore \quad \frac{3n(n-1) + 2n}{n} \text{ is the mean in terms of } n$$

$$= 3n - 3 + 2$$

$$= 3n - 1 \quad (1)$$

Alternative Solution.

e) i) let p be Pierre's mark, $\bar{x}$ be the mean

$$\bar{x} + 1 = \frac{n\bar{x} - p}{n-1}, \text{ find } \frac{n\bar{x} - p}{n-1} - p$$

$$(\bar{x}+1)(n-1) = n\bar{x} - p$$

$$n\bar{x} - \bar{x} + n - 1 = n\bar{x} - p$$

$$-\bar{x} + n - 1 = -p$$

$$\bar{x} = n + p - 1$$

finding difference:

$$\frac{n\bar{x} - p}{n-1} - p$$

$$= \frac{n(n+p-1) - p}{n-1} - p$$

$$= \frac{n(n+p-1) - p - p(n-1)}{n-1}$$

$$= \frac{n^2 + np - n - np + p - p}{n-1}$$

$$= \frac{n^2 - n}{n-1}$$

$$= \frac{n(n-1)}{n-1}$$

$$= n$$

ii) mean = mean without Pierre − 1 = mean with $\frac{1}{2}$ Pierre + 1

$$\frac{n\bar{x} - p}{n-1} - 1 = \frac{n\bar{x} - \frac{1}{2}p}{n} + 1$$

$$\frac{n\bar{x} - (\bar{x} + 1 - n)}{n-1} = \frac{2n\bar{x} - (\bar{x} + 1 - n)}{2n} + 2$$

$$2n(n\bar{x} - \bar{x} - 1 + n) = (n-1)(2n\bar{x} - \bar{x} - 1 + n) + 2(n-1)(2n)$$

$$2n^2\bar{x} - 2n(\bar{x} + 1 - n) = 2n^2\bar{x} - n(\bar{x} + 1 - n) - 2n\bar{x} + (\bar{x} + 1 - n) + 4n(n-1)$$

$$(n - 1 - 2n)(\bar{x} + 1 - n) = -2n\bar{x} + 4n(n-1)$$

$$-(n+1)(\bar{x} + 1 - n) = -2n\bar{x} + 4n(n-1)$$

$$-\bar{x}(n+1) - (n+1)(1-n) = -2n\bar{x} + 4n(n-1)$$

$$-\bar{x}(n+1) + 2n\bar{x} = 4n(n-1) + (n+1)(1-n)$$

$$\bar{x}(2n - (n+1)) = 4n(n-1) - (n+1)(n-1)$$

$$\bar{x}(n-1) = (n-1)(4n - (n+1))$$

$$\bar{x} = \frac{(n-1)(3n-1)}{n-1}$$

$$\bar{x} = 3n - 1$$

Alternative Methods

Question 8

a) $\dfrac{(3x^4y)^2}{x^{-2}y^2} \times \dfrac{2xy^{-2}}{x^5y}$

$= \dfrac{9x^8y^2}{x^{-2}y^2} \times \dfrac{2}{x^4y^3}$

$= \dfrac{18x^6}{y^3}$

b) $a(b+c)^4 - a(b-c)^4$

$= a[(b+c)^4 - (b-c)^4]$

$= a[(b+c)^2 + (b-c)^2][(b+c)^2 - (b-c)^2]$

$= a(b^2+2bc+c^2+b^2-2bc+c^2)[(b+c)+(b-c)][(b+c)-(b-c)]$

$= a(2b^2+2c^2)(2b)(2c)$

$= 8abc(b^2+c^2)$

c) $\dfrac{(4a-5b)\times\frac{1}{b}}{(\sqrt{18a^2-4b^2})\times\frac{1}{b}}$

$= \dfrac{4\times\frac{a}{b}-5}{\sqrt{18\times\frac{a^2}{b^2}-4}}$

$= \dfrac{4k-5}{\sqrt{18k^2-4}}$

e) i) let p be Pierre's mark, $\bar{x}$ be the mean

$$\bar{x} + 1 = \frac{n\bar{x} - p}{n-1} \quad , \text{ find } \quad \frac{n\bar{x} - p}{n-1} - p$$

$$(\bar{x}+1)(n-1) = n\bar{x} - p$$

$$n\bar{x} - \bar{x} + n - 1 = n\bar{x} - p$$

$$-\bar{x} + n - 1 = -p$$

$$\bar{x} = n + p - 1$$

finding difference:

$$\frac{n\bar{x} - p}{n-1} - p$$

$$= \frac{n(n+p-1) - p}{n-1} - p$$

$$= \frac{n(n+p-1) - p - p(n-1)}{n-1}$$

$$= \frac{n^2 + np - n - np + p - p}{n-1}$$

$$= \frac{n^2 - n}{n-1}$$

$$= \frac{n(n-1)}{n-1}$$

$$= n$$

ii) mean = mean without Pierre − 1 = mean with $\frac{1}{2}$ Pierre + 1

$$\frac{n\bar{x} - p}{n-1} - 1 = \frac{n\bar{x} - \frac{1}{2}p}{n} + 1$$

$$\frac{n\bar{x} - (\bar{x} + 1 - n)}{n-1} = \frac{2n\bar{x} - (\bar{x} + 1 - n)}{2n} + 2$$

$$2n(n\bar{x} - \bar{x} - 1 + n) = (n-1)(2n\bar{x} - \bar{x} - 1 + n) + 2(n-1)(2n)$$

$$2n^2\bar{x} - 2n(\bar{x} + 1 - n) = 2n^2\bar{x} - n(\bar{x} + 1 - n) - 2n\bar{x} + (\bar{x} + 1 - n) + 4n(n-1)$$

$$(n - 1 - 2n)(\bar{x} + 1 - n) = -2n\bar{x} + 4n(n-1)$$

$$-(n+1)(\bar{x} + 1 - n) = -2n\bar{x} + 4n(n-1)$$

$$-\bar{x}(n+1) - (n+1)(1-n) = -2n\bar{x} + 4n(n-1)$$

$$-\bar{x}(n+1) + 2n\bar{x} = 4n(n-1) + (n+1)(1-n)$$

$$\bar{x}(2n - (n+1)) = 4n(n-1) - (n+1)(n-1)$$

$$\bar{x}(n-1) = (n-1)(4n - (n+1))$$

$$\bar{x} = \frac{(n-1)(3n-1)}{n-1}$$

$$\bar{x} = 3n - 1$$