SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2022 Half-Yearly Examination

Form III Mathematics

Monday 30th May 2022

1:15pm

General Instructions

- Working time — 1 hour 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators may NOT be used.

Total Marks: 104

Questions 1 – 8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Place everything inside this question booklet.

3A: OMD	3B: PJE	3C: JAZ/LYL	3D: YH
3E: CWG/RR	3F: AEA/WJM	3G: FMW	3H: NJL
3I: LRP/GMC	3J: BR		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 199 pupils

Writer: NJL

QUESTION ONE (13 marks) Start a new page.

(a) Fully simplify:

(i) $2\sqrt{3} + 5\sqrt{3}$

(ii) $\sqrt{50}$

(iii) $\dfrac{\sqrt{66}}{\sqrt{33}}$

(b) Evaluate:

(i) 2^4

(ii) $27^{\frac{1}{3}}$

(c) Simplify $x^8 \div x^2$.

(d) Factorise fully:

(i) $8x - 12$

(ii) $9ab + 3b$

(iii) $x^2 - 10x$

(iv) $a^2 - b^2$

(e) Find the value of P if $P = \dfrac{RT}{V}$, given $R = 6$, $T = 3$ and $V = 9$.

(f) Write x^{-2} without a negative index.

(g) Write 9137 correct to three significant figures.

QUESTION TWO (13 marks) Start a new page.

(a) Nine people were asked to complete a puzzle. Their completion times, in minutes, are presented in the stem-and-leaf plot below.

0	4
1	0 4 9
2	1 2 7 7
3	2

2 | 1 means 21 minutes

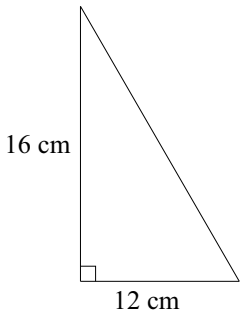
For the time taken to complete the puzzle, find:

(i) the mode

(ii) the median

(iii) the range

(b)



The diagram above shows a right-angled triangle of base length 12 cm and height 16 cm. Find the length of the hypotenuse.

(c) Solve the following equations

(i) $4 + x = 1$

(ii) $-10x + 4 = -46$

(iii) $6(x - 4) = 18$

(iv) $\dfrac{2x + 1}{5} = x - 4$

(d) Expand and simplify $\left(3\sqrt{2} - 2\right)\left(2\sqrt{2} - 1\right)$.

QUESTION THREE (13 marks) Start a new page.

(a) Find 14% of 150.

(b) Express $\dfrac{5}{\sqrt{3}}$ with a rational denominator.

(c) Simplify $\dfrac{2p^3}{9} \times \dfrac{6}{p^2}$.

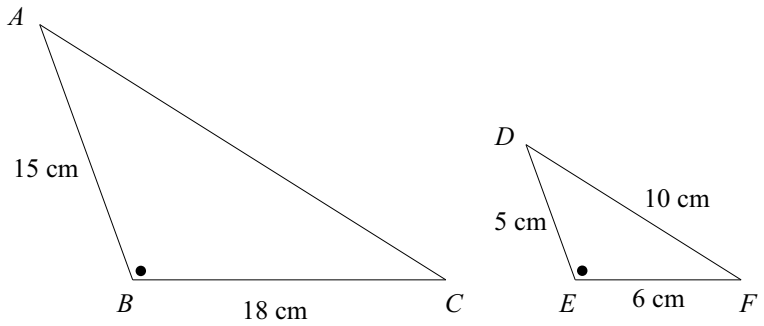
(d) Factorise fully:

(i) $x^2 + 6x + 9$

(ii) $x^2 + 4x - 21$

(iii) $3x^2 - 15x + 12$

(e)



The diagram above shows $\triangle ABC$ and $\triangle DEF$ where $\angle ABC = \angle DEF$, $AB = 15\,\text{cm}$, $BC = 18\,\text{cm}$, $DE = 5\,\text{cm}$, $EF = 6\,\text{cm}$ and $FD = 10\,\text{cm}$.

(i) Which similarity test can be used to show that $\triangle ABC$ is similar to $\triangle DEF$?

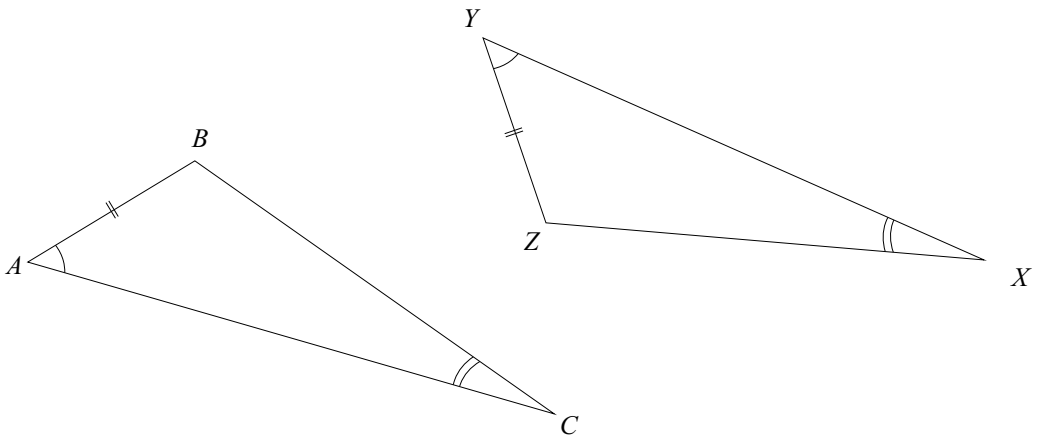
(ii) Find the length of CA, giving a geometric reason.

(f) (i) Write 185 000 000 in scientific notation.

(ii) Write $5{\cdot}02 \times 10^{-3}$ in decimal form.

QUESTION FOUR (13 marks) Start a new page.

(a)



Prove that the triangles above are congruent.

(b) Express $\dfrac{7}{3-\sqrt{5}}$ with a rational denominator in simplest form.

(c) Evaluate:

(i) $\left(3\sqrt{2}\right)^2$

(ii) $\left(\dfrac{4}{9}\right)^{-\frac{1}{2}}$

(d) Solve the following equations:

(i) $3(4-x)-7(2x+8)=0$

(ii) $\dfrac{3p}{14}-1=\dfrac{6}{7}$

(iii) $\dfrac{3(6-x)}{2}=-\dfrac{2(x+5)}{5}$

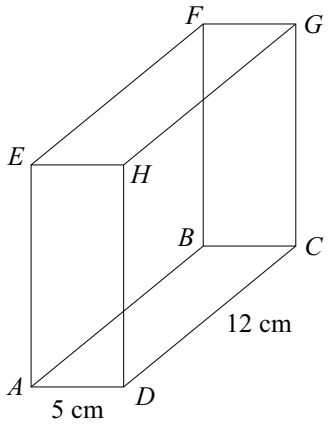
QUESTION FIVE (13 marks) Start a new page.

(a) Last year Nigel's cake shop made a profit of \$70 000. The profit was 40% of his costs. Calculate:

(i) his costs

(ii) his sales

(b)



The diagram above shows a rectangular prism where $AD = 5\,\text{cm}$ and $DC = 12\,\text{cm}$.

(i) Find the length of AC.

(ii) The length of the space diagonal AG is 20 cm. Find the exact length of CG.

(c) Rearrange the following formulae to make a the subject.

(i) $w = H + Ca$

(ii) $S = \dfrac{n}{2}(a + l)$

(d) Simplify and express with positive indices:

(i) $3x^2y^{-4} \times 2x^3y^7$

(ii) $\left(2a^2b\right)^3 \div a^{-1}b^4$

(e) In a colony of 2300 penguins, 920 of them are parents. What percentage of the penguins in the colony are parents?

QUESTION SIX (13 marks) Start a new page.

(a) On a school picnic, pupils chose either a chocolate muffin or a banana muffin. Ten fewer pupils chose a banana muffin. Sixty-eight pupils attended the picnic and each of them chose one muffin. Let b be the number of banana muffins chosen.

(i) Write a linear equation in terms of b representing the information above.

(ii) Solve your equation to determine the number of each type of muffin chosen.

(b) Solve the inequality $8 - 4x > 4$ and graph the solution on a number line.

(c) The number of cookies eaten each day by the Cookie Monster was recorded for 20 days. The results were recorded in the table below.

Number of Cookies	**Frequency**
8	4
9	10
10	3
11	2
12	1

(i) For the number of cookies eaten per day find:

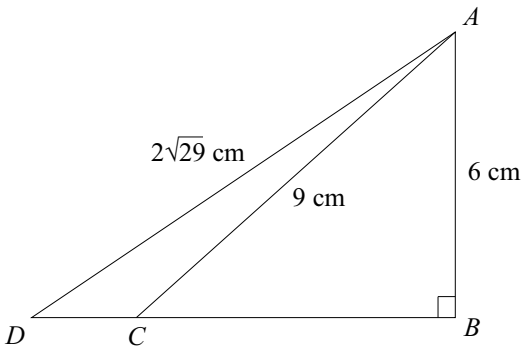
(α) the mode

(β) the median

(γ) the mean

(ii) State whether this distribution is symmetrical, positively skewed or negatively skewed.

(d)



The diagram above shows two right-angled triangles ABC and ABD, where $AB = 6\,\text{cm}$, $AC = 9\,\text{cm}$ and $AD = 2\sqrt{29}\,\text{cm}$.

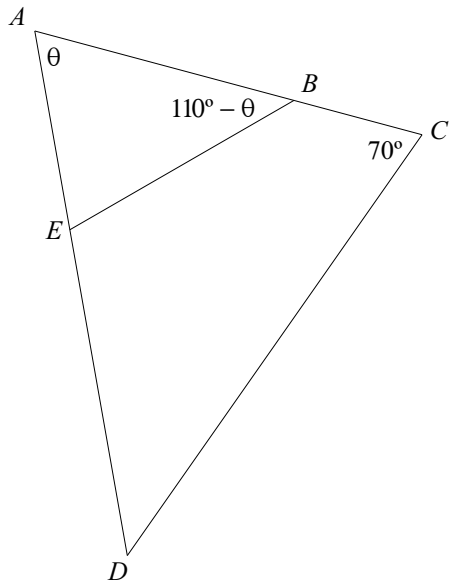
Find the exact length of DC.

QUESTION SEVEN (13 marks) Start a new page.

(a) A scale model of a proposed building is in the ratio of 1 : 400. Calculate the width of the lobby entrance on the real building if it is 12 mm wide on the model. Give your answer in metres.

(b) Express $x^2 + \frac{4}{3}x + \frac{4}{9}$ in the form $(x + a)^2$.

(c)



The diagram above shows $\triangle ABE$ and $\triangle ACD$ where $\angle BAE = \theta$, $\angle ABE = 110^\circ - \theta$ and $\angle ACD = 70^\circ$.

(i) Prove that $\triangle ABE \,|||\, \triangle ADC$.

(ii) Find the length of AE given that $AD = 7\,\text{cm}$, $AB = 3\,\text{cm}$ and $AC = 5\,\text{cm}$.

(d) Make c the subject of the formula $\frac{a}{b} = \sqrt{1 - \frac{c^2}{d^2}}$.

(e) A rectangle has a width which is 4 cm longer than its length. A new rectangle is made by increasing both the width and length of the original rectangle by 3 cm each. The area of the new rectangle is $63\,\text{cm}^2$ greater than the original. Let the length of the original rectangle be x.

Write and solve an equation in x to find the length of the original rectangle.

(f) Express $\left(2{\cdot}7 \times 10^4\right)^3 \div \left(3^2 \times 10^3\right)^2$ as a single number in decimal form.

QUESTION EIGHT (13 marks) Start a new page.

(a) Mable pays tax on her income at the rates given in the table below.

Pre-Tax Income	**Tax Rate**
first \$40 000	no tax
from \$40 001 to \$80 000	20c for each dollar over \$40 000
From \$80 001 to \$120 000	35c for each dollar over \$80 000
from \$120 001	48c for each dollar over \$120 000

If Mable pays \$60 400 in tax, calculate her pre-tax income.

(b) Two pans of melted chocolate and milk are poured into a jug already containing a litre of warm milk. Once poured, it is mixed together thoroughly and the resulting drink is 20% chocolate.Prior to mixing, Pan A contained half a litre of liquid which was 80% chocolate. Pan B contained 5 litres of liquid but the percentage of chocolate was unknown.

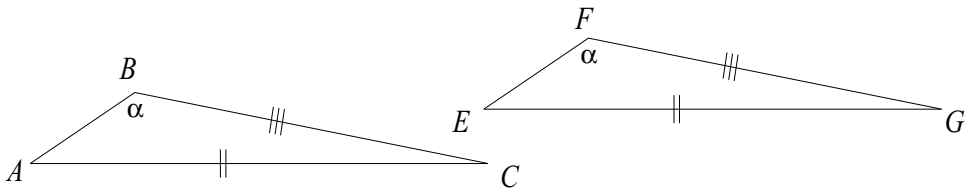
What percentage of the liquid in pan B was chocolate?

(c) Peter has some money in a jar. Every time he uses a rude word Briana triples the amount of money in the jar and then subtracts \$27. By the time Peter uses a rude word for the third time his jar is empty.

What is the maximum amount of money that could originally have been in the jar?

(d) Express $\left(\dfrac{c^2+b^2-a^2}{2bc}-1\right)\div\left(\dfrac{c^2-b^2-a^2}{2ab}-1\right)$ as a single fraction in simplest form.

(e)



The diagram above shows $\triangle ABC$ and $\triangle EFG$, where $\angle ABC = \angle EFG = \alpha$ and α is obtuse. It is known that $AC = EG$ and $BC = FG$.

Using a construction prove that $\triangle ABC \equiv \triangle EFG$.

(f) Solve $\dfrac{2}{x-6+\sqrt{x}}+\dfrac{3}{\sqrt{x}-2}=\dfrac{4}{\sqrt{x}+3}$.

END OF PAPER

B L A N K P A G E

B L A N K P A G E

B L A N K P A G E

Form III Half-Yearly 2022
Solutions.

1. a) i) $2\sqrt{3} + 5\sqrt{3}$
$= 7\sqrt{3}$ ✓

ii) $\sqrt{50}$
$= \sqrt{25 \times 2}$
$= 5\sqrt{2}$ ✓

iii) $\frac{\sqrt{66}}{\sqrt{33}}$
$= \sqrt{\frac{66}{33}}$
$\sqrt{2}$ ✓

b) i) $2^4 = 16$ ✓

ii) $27^{1/3} = \sqrt[3]{27}$
$= 3$ ✓

c) $x^8 \div x^2 = x^6$ ✓

d) i) $8x - 12 = 4(2x - 3)$ ✓

ii) $9ab + 3b = 3b(3a + 1)$ ✓

iii) $x^2 - 10x = x(x - 10)$ ✓

iv) $a^2 - b^2 = (a - b)(a + b)$ ✓

e) $P = \frac{6 \times 3}{9}$
$= 2$ ✓

f) $x^{-2} = \frac{1}{x^2}$ ✓

g) 9/40 ✓

Q2) a) i) mode = 27 minutes ✓

ii) 9 people, look for 5th person

median = 21 minutes ✓

iii) range = 32 − 4

= 28 minutes ✓

b) $h^2 = a^2 + b^2$

$= 12^2 + 16^2$ ✓

$= 144 + 256$

$= 400$

$h = \sqrt{400}$

$h = 20$ cm ✓ (units required)

c) i) $4 + x = 1$

$x = -3$ ✓

ii) $-10x + 4 = -46$

$-10x = -50$

$x = 5$ ✓

iii) $6(x - 4) = 18$

$x - 4 = 3$ ✓

$x = 7$ ✓

iv) $\frac{2x+1}{5} = x - 4$

$2x + 1 = 5x - 20$ ✓

$1 = 3x - 20$

$21 = 3x$

$x = 7$ ✓

d) $(3\sqrt{2}-2)(2\sqrt{2}-1)$

$= 12 - 3\sqrt{2} - 4\sqrt{2} + 2$ ✓

$= 14 - 7\sqrt{2}$ ✓

Q3.) a) i) 10% of $150 = 15$ ✓

1% of $150 = 1.5$

4% of $150 = 6$

14% of $150 = 21$ ✓

b) $\frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ ✓

c) $\frac{2p^3}{9} \times \frac{6}{p^2}$

$= \frac{4p}{3}$ ✓

d) i) $x^2 + 6x + 9$

$= (x+3)(x+3)$ ✓

$= (x+3)^2$

ii) $x^2 + 4x - 21$

$= (x-3)(x+7)$ ✓

iii) $3x^2 - 15x + 12$

$= 3(x^2 - 5x + 4)$ ✓

$= 3(x-4)(x-1)$ ✓

e) i) SAS ✓

ii) $\frac{ED}{BA} = \frac{FD}{CA} = \frac{1}{3}$ (Matching sides of similar triangles.) ✓

$FD = \frac{CA}{3}$

$10 = \frac{CA}{3}$

$CA = 30\text{ cm}$ ✓

f) i) 1.85×10^{8} ✓

ii) 0.00502 ✓

Q4) a) $AB = YZ$ (Given)
$\angle BCA = \angle ZXY$ (Given)
$\angle CAB = \angle XYZ$ (Given) } ✓
$\therefore \Delta ABC \equiv \Delta YZX$
(AAS) ✓

b) $\frac{7}{3-\sqrt{5}}$

$= \frac{7(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$ ✓

$= \frac{7(3+\sqrt{5})}{9-5}$

$= \frac{7(3+\sqrt{5})}{4}$ ✓

c) i) $(3\sqrt{2})^2 = 3\sqrt{2} \times 3\sqrt{2}$
$= 9 \times 2$
$= 18$ ✓

ii) $\left(\frac{4}{9}\right)^{-1/2}$

$= \left(\frac{9}{4}\right)^{1/2}$ ✓

$= \frac{\sqrt{9}}{\sqrt{4}}$

$= \frac{3}{2}$ ✓

d) i) $3(4-x)-7(2x+8)=0$

$12-3x-14x-56=0$ ✓

$-17x-44=0$

$-17x=44$

$x=-\frac{44}{17}$ ✓

$x=-2\frac{10}{17}$

ii) $\frac{3p}{14}-1=\frac{6}{7}$

$\frac{3p}{14}=1+\frac{6}{7}$

$\frac{3p}{14}=\frac{13}{7}$ ✓

$3p=13\times 2$

$p=\frac{26}{3}$ ✓

$p=8^{2/3}$

iii) $\frac{3(6-x)}{2}=-\frac{2(x+5)}{5}$

$15(6-x)=-4(x+5)$

$90-15x=-4x-20$ ✓

$90=11x-20$

$110=11x$

$x=10$ ✓

Q5.) a) i) $\frac{\text{Profit}}{\text{costs}} \times 100 = 40$

$\frac{70,000}{\text{costs}} \times 100 = 40$

$\text{costs} = \frac{7000000}{4}$

$\text{costs} = 175,000$

Nigel's costs were $175,000. ✓

ii) Profit = sales − costs.

$70\,000 = \text{sales} - 175000$

$\text{Sales} = 70000 + 175000$

$= 245,000$ ✓

Nigel's sales were $245,000.

b) i) $AC^2 = 5^2 + 12^2$

$AC = \sqrt{169}$

$AC = 13$ cm ✓

ii) $AQ^2 = AC^2 + CQ^2$

$20^2 = 169 + CQ^2$ ✓

$CQ^2 = 400 - 169$

$CQ^2 = \sqrt{231}$ cm ✓

c) i) $w = H + Ca$

$w - H = Ca$

$a = \frac{w-H}{C}$ ✓

ii) $S = \frac{n}{2}(a + l)$

$2S = n(a + l)$ ✓

$\frac{2S}{n} = a + l$

$a = \frac{2S}{n} - l$ ✓

d) i) $3x^{2-4} \times 2x^{3}y^{7}$

$= 6x^{-2}y^{7}$ ✓

ii) $(2a^{2}b)^{3} \div a^{-1}b^{4}$

$= 8a^{6}b^{3} \div a^{-1}b^{4}$ ✓

$= 8a^{7}b^{-1}$

$= \frac{8a^{7}}{b}$ ✓

e) $\frac{920}{2300} \times 100$ ✓

$= \frac{92}{230} \times 100$

$= \frac{4}{10} \times 100$

$= 40\%$ ✓

$\therefore$ 40% of the penguins are parents.

Q6) a) b is number of banana muffins.

i) $68 = b + b + 10$ ✓

ii) $68 = 2b + 10$

$58 = 2b$

$b = 29$ ✓

$b + 10 = 39$

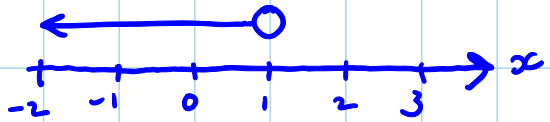
29 pupils chose banana muffins and 39 chose chocolate. } ✓

(Worded response required.)

b) $8 - 4x > 4$

$-4x > -4$ ✓

$x < 1$ ✓



✓

c) α) 9 cookies per day. ✓

β) median: 10.5th

∴ median = 9 cookies per day ✓

γ) mean $= \frac{4\times8 + 10\times9 + 3\times10 + 2\times11 + 1\times12}{20}$

$= \frac{186}{20}$

$= \frac{93}{10}$

$= 9.3$ ✓

cookies per day.

ii) Positively skewed ✓

d) $(2\sqrt{29})^2 = 6^2 + DB^2$

$4 \times 29 - 36 = DB^2$

$DB^2 = 116 - 36$

$DB^2 = 80$

$DB = \sqrt{80}$ cm ✓

$BC^2 = 9^2 - 6^2$

$BC^2 = 45$

$BC = \sqrt{45}$ cm ✓

$DC = \sqrt{80} - \sqrt{45}$

$= 4\sqrt{5} - 3\sqrt{5}$

$= \sqrt{5}$ cm ✓

Q7) a) $\frac{12 \times 400}{1000}$ ✓

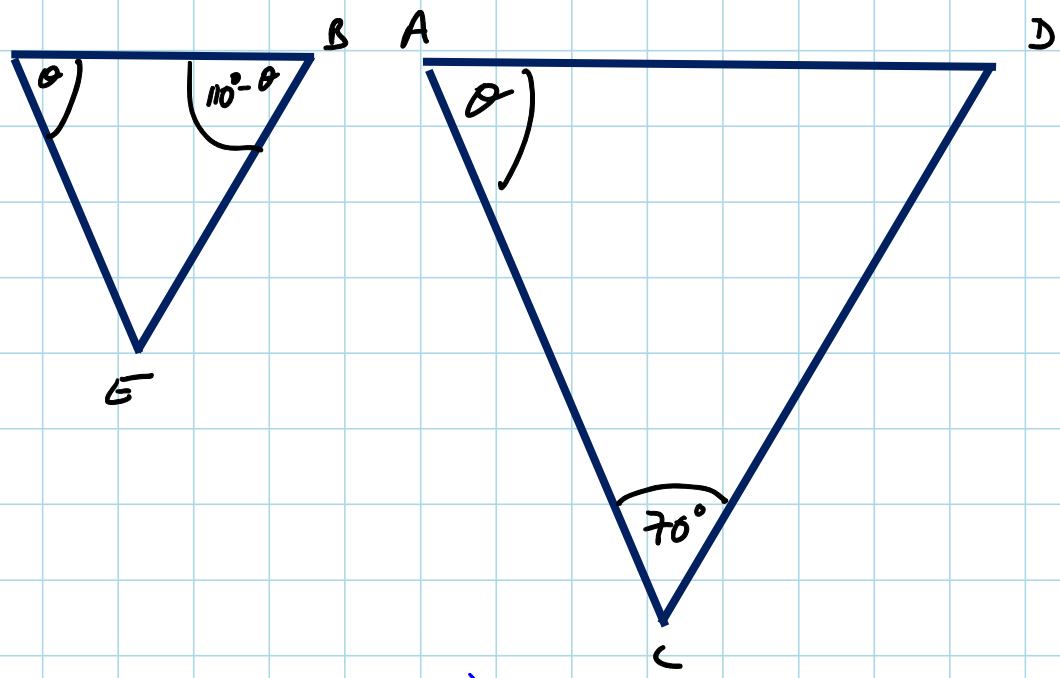
$= \frac{12 \times 4}{10}$

$= 4.8\text{m}$ wide ✓

b) $x^2 + \frac{4}{3}x + \frac{4}{9}$

$= (x + \frac{2}{3})^2$ ✓

c) i)



$\angle DAC = \angle BAE = \theta$ (Common)

$\angle AEB = 180 - \theta - (110 - \theta)$

$= 180 - \theta - 110 + \theta$

$= 70°$ (Interior angle sum of $\triangle AEB$) ✓

$= \angle ACD$

$\therefore \triangle ABE \parallel\parallel \triangle ADC$ (AA) ✓

ii) $5 \times \frac{3}{7} = \frac{15}{7} = 2\frac{1}{7}$ ✓

d) $\frac{a}{b} = \sqrt{1 - \frac{c^2}{d^2}}$

$\frac{a^2}{b^2} = 1 - \frac{c^2}{d^2}$

$\frac{c^2}{d^2} = 1 - \frac{a^2}{b^2}$ ✓

$c^2 = d^2\left(1 - \frac{a^2}{b^2}\right)$

$c = d\sqrt{1 - \frac{a^2}{b^2}}$ ✓

e) $(x+3)(x+7) = x(x+4) + 63$ ✓

$x^2 + 10x + 21 = x^2 + 4x + 63$ ✓

$10x + 21 = 4x + 63$

$6x = 42$

$x = 7$ ✓

The original length was 7cm.

f) $(2.7 \times 10^4)^3 \div (3^2 \times 10^3)^2$

$= (27 \times 10^3)^3 \div (3^2 \times 10^3)^2$

$= (3^3 \times 10^3)^3 \div (3^4 \times 10^6)$

$= \frac{3^9 \times 10^9}{3^4 \times 10^6}$

$= 3^5 \times 10^3$ ✓

$= 3^2 \times 3^2 \times 3 \times 10^3$

$= 81 \times 3 \times 1000$

$= 243000$ ✓

8) a) • 1st $40,000 pays no tax.

• 2nd $40,000 pays 20% of $40,000
= $8000 tax

• 3rd $40,000 pays 35% of $40,000
= $14,000 tax.

$$\begin{array}{r} 60,400 \\ -\ 14,000 \\ -\ 8,000 \\ \hline \$\ 38,400 \end{array}$$ ✓

let x be amount Mable earns over $120,000.

$$\frac{48}{100} \times x = 38\,400$$

$$x = \frac{38400 \times 100}{48}$$

$$x = \frac{3200 \times 100}{4}$$

$$x = \$80,000$$

$$\begin{array}{r} 80,000 \\ +\ 40,000 \\ +\ 40,000 \\ +\ 40,000 \\ \hline \$\ 200,000 \end{array}$$

Mable earns $200,000 ✓

b) Let x be % chocolate in pan B.

$$0.5 \times \frac{80}{100} + 6 \times \frac{x}{100} = 6.5 \times \frac{20}{100} \quad \checkmark$$

$$40 + 5x = 130$$

$$5x = 90$$

$$x = 18\% \quad \checkmark$$

The liquid in pan B was 18% chocolate.

c) Let x be number of dollars in jar.

N° Rude Words	Amount in jar.
0	x
1	$3x - 27$
2	$3(3x - 27) - 27$
3	$3(3(3x - 27) - 27) - 27$

$$3(3(3x-27)-27)-27 = 0 \quad \checkmark$$

$$3(3(3x-27)-27) = 27$$

$$3(3x-27)-27 = 9$$

$$3(3x-27) = 36$$

$$3x - 27 = 12$$

$$3x = 39$$

$$x = 13 \quad \checkmark$$

The most Peter could have had in the jar was \$13.

d) $\left(\frac{c^2+b^2-a^2}{2bc}-1\right) \div \left(\frac{c^2-b^2-a^2}{2ab}-1\right)$

$= \frac{c^2+b^2-a^2-2bc}{2bc} \div \frac{c^2-b^2-a^2-2ab}{2ab}$

$= \left(\frac{c^2+b^2-a^2-2bc}{2bc}\right)\left(\frac{2ab}{c^2-b^2-a^2-2ab}\right)$ ✓

$= \left(\frac{c^2+b^2-a^2-2bc}{c}\right)\left(\frac{a}{c^2-b^2-a^2-2ab}\right)$

$= \left(\frac{(b-c)^2-a^2}{c}\right)\left(\frac{a}{-(b^2+a^2+2ab)+c^2}\right)$

$= -\left(\frac{(b-c)^2-a^2}{c}\right)\left(\frac{a}{(b+a)^2-c^2}\right)$ ✓

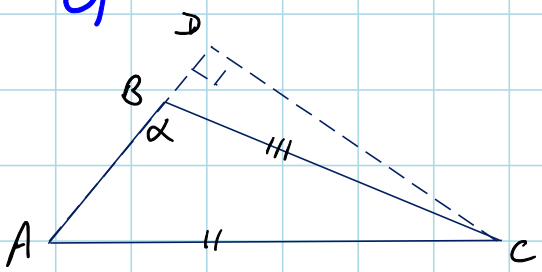
$= -\frac{a\left((b-c)^2-a^2\right)}{c\left((b+a)^2-c^2\right)}$

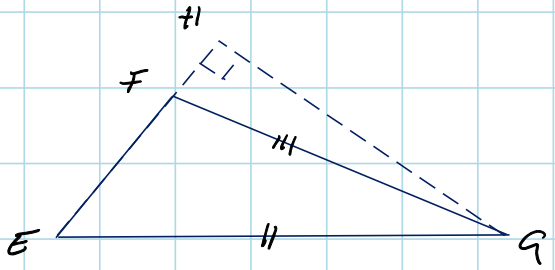
$= -\frac{a(b-c-a)(b-c+a)}{c(b+a-c)(b+a+c)}$

$= -\frac{a(b-c-a)}{c(b+a+c)}$

$= \frac{a(a-b+c)}{c(a+b+c)}$ ✓

e)





Extend AB to D and EF to H such that CD ⊥ AD and EH ⊥ GH.

$\angle BDC = \angle FHG = 90°$ (Given)

$BC = FG$ (Given)

$\angle DBC = \angle HFG = 180 - \alpha$ (Straight angle)

$\therefore \Delta BDC \equiv \Delta FHG$ (AAS)

$DC = HG$ (matching sides of congruent triangles $\Delta BDC \equiv \Delta FHG$)

$AC = EG$ (Given)

$\Delta ADC \equiv \Delta EHG$ (RHS) ✓ (Including above.)

$\therefore AD = EH$ (matching sides of congruent triangles $\Delta ADC \equiv \Delta EHG$)

$\therefore AD - AB = EH - EF$

$AB = EF$

$\therefore \Delta ABC \equiv \Delta EFG$ (SAS) ✓ (Including above.)

f) $\frac{2}{x-6+\sqrt{x}} + \frac{3}{\sqrt{x}-2} = \frac{4}{\sqrt{x}+3}$

Notice $(\sqrt{x}-2)(\sqrt{x}+3) = x-6+\sqrt{x}$ ✓

$\therefore \frac{2}{x-6+\sqrt{x}} = \frac{4}{\sqrt{x}+3} - \frac{3}{\sqrt{x}-2}$

$\frac{2}{x-6+\sqrt{x}} = \frac{4(\sqrt{x}-2)}{(\sqrt{x}+3)(\sqrt{x}-2)} - \frac{3(\sqrt{x}+3)}{(\sqrt{x}+3)(\sqrt{x}-2)}$

$\frac{2}{x-6+\sqrt{x}} = \frac{4(\sqrt{x}-2)-3(\sqrt{x}+3)}{x-6+\sqrt{x}}$

$2 = 4\sqrt{x} - 8 - 3\sqrt{x} - 9$

$2 = \sqrt{x} - 17$

$\sqrt{x} = 19$

$x = 361$ ✓