SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2021 Half-Yearly Examination

Form III Mathematics

Thursday 13th May 2021

8:40am

General Instructions

- Working time — 1 hour 30 minutes
- Attempt all questions.
- Write using black pen.
- Calculators may NOT be used.

Total Marks: 104

Questions 1–8

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.

Collection

- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Write your name and master on this page and on each page of writing paper.
- Place everything inside this question booklet.

3A: BR	3B: BLR	3C: RR/FMW	3D: WJM/RCF
3E: LRP/GMC	3F: AEA/SDP	3G: JAZ	3H: YH
3I: LL	3J: BDD		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 201 pupils

Writer: BLR

QUESTION ONE (13 marks)

(a) Fully simplify:

(i) $6x - 4x$

(ii) $2x \times 3y$

(iii) $15x \div 3$

(iv) $x^{12} \div x^{7}$

(b) Expand and simplify:

(i) $2\,(5x - 3)$

(ii) $(x + 9)\,(x + 1)$

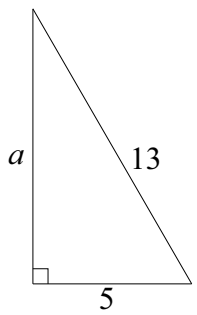
(c) Solve the equations:

(i) $-7x + 2 = 23$

(ii) $2\,(x - 1) = 14$

(iii) $2x = 3x - 6$

(d)



Find the value of a in the diagram above.

(e) Fully simplify:

(i) $\sqrt{7} \times \sqrt{7}$

(ii) $\sqrt{3} \times \sqrt{10}$

(iii) $\dfrac{\sqrt{22}}{\sqrt{11}}$

QUESTION TWO (13 marks)

(a) Fully simplify:

(i) $\sqrt{12}$

(ii) $8\sqrt{2} + \sqrt{5} + 10\sqrt{2} - 7\sqrt{5}$

(b) (i) Solve $5 - 9x > 32$.

(ii) Graph your solution from part (i) on a number line.

(c) Expand and simplify:

(i) $(a - b)^2$

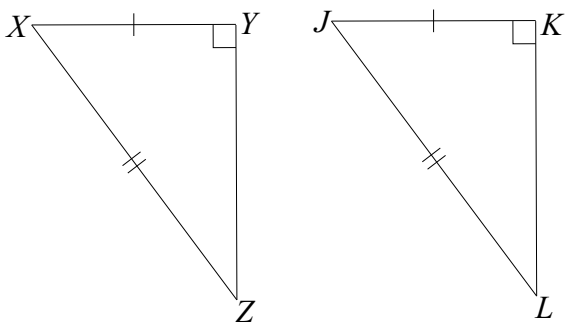
(ii) $\left(5 + \sqrt{3}\right)\left(5 - \sqrt{3}\right)$

(d) Express 154 000 in scientific notation.

(e) Round 13·486 to 3 significant figures.

(f) Evaluate 2^{-1}.

(g)



State the test that can be used to prove $\triangle XYZ \equiv \triangle JKL$ in the diagram above. **Do not prove this result.**

(h) The formula to convert a temperature from the number of degrees Fahrenheit F to the number of degrees Celsius C is

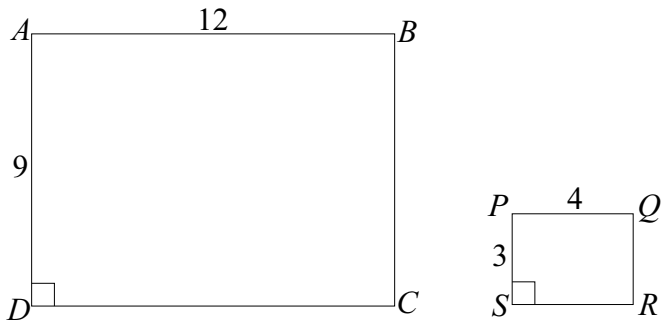
$$C = \frac{5}{9}(F - 32).$$

Use this formula to convert a temperature of 86 degrees Fahrenheit to degrees Celsius.

QUESTION THREE (13 marks)

(a) Evaluate $100^{\frac{1}{2}}$.

(b)



The diagram above shows two similar rectangles. Write down the similarity ratio.

(c) Fully factorise:

(i) $21x^2 + 15xy$

(ii) $x^2 - 121$

(iii) $x^2 + 6x - 40$

(iv) $x^2 + 8x + 16$

(d) Express $3\sqrt{2}$ as the square root of a whole number.

(e) Simplify:

(i) $8a\,(3 - a) - 5a\,(a + 4)$

(ii) $\dfrac{13a^2}{4} \times \dfrac{8b}{5a}$

(iii) $\dfrac{3x}{4} + \dfrac{x}{6}$

QUESTION FOUR (13 marks)

(a) Simplify:

(i) $14 \times (3x)^0$

(ii) $\left(2x^2\right)^3$

(b) Express $\sqrt[4]{x^3}$ in index form.

(c) Simplify $\sqrt{3} \times \sqrt{18} - \sqrt{24}$.

(d) Express each fraction in simplest form with a rational denominator:

(i) $\dfrac{1}{\sqrt{7}}$

(ii) $\dfrac{5}{\sqrt{3}-1}$

(e) Make x the subject of the formula $A = \dfrac{x+y}{2}$.

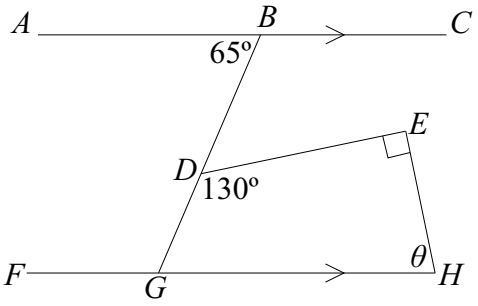
(f) Pythagoras owns n books. Hippasus owns 13 more books than Pythagoras.

(i) Write an expression in terms of n for the number of books Hippasus owns.

(ii) Pythagoras and Hippasus own a total of 27 books. Write and solve an equation to determine the number of books each man owns.

QUESTION FIVE (13 marks)

(a)



In the diagram above, $AC \parallel FH$, $\angle ABG = 65°$, $\angle EDG = 130°$, $\angle DEH = 90°$, and $\angle EHG = \theta$. Find the value of θ, giving clear geometric reasons for your answer.

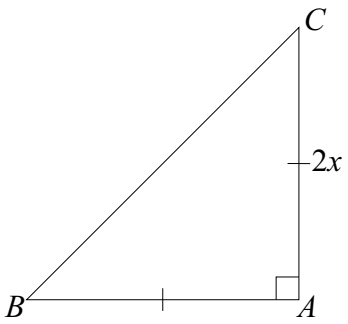
(b) Simplify $9x^8 \times x^{-1}$.

(c) Express with positive indices:

(i) $3a^{-2}$

(ii) $\left(\frac{5}{2}\right)^{-1}$

(d)



The diagram above shows a right-angled isosceles triangle with short sides of length $2x$. Find a simplified expression in terms of x for the length of BC.

(e) Expand and simplify $\left(2\sqrt{5} + 4\sqrt{2}\right)\left(3\sqrt{2} - 5\sqrt{10}\right)$.

(f) Fully factorise:

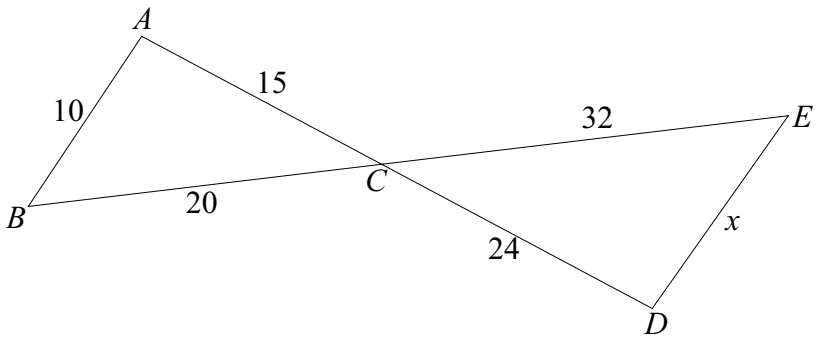
(i) $4x^2 + 24x - 64$

(ii) $x^2 + 7x + \frac{49}{4}$

QUESTION SIX (13 marks)

(a) Simplify $\left(8x^3y^{-9}\right)^{-\frac{1}{3}}$, leaving your answers in terms of positive indices.

(b)



In the diagram above, AD intersects BE at C, $AB = 10$, $AC = 15$, $BC = 20$, $DC = 24$, $EC = 32$, and $ED = x$.

(i) Find $\dfrac{AC}{DC}$.

(ii) Prove that $\triangle ABC \,|||\, \triangle DEC$.

(iii) Hence, find the value of x. Give a clear geometric reason for your answer.

(c) Solve $\dfrac{x+1}{9} - 1 = \dfrac{x-7}{4}$.

(d) Simplify $\dfrac{8a}{3} - \dfrac{2-ab}{5b}$. Give your answer as a single simplified fraction.

(e) The length of all the DNA in a single human cell is $2{\cdot}2 \times 10^9$ nanometres.

(i) Given that there are approximately $3{\cdot}7 \times 10^{13}$ cells in the human body, calculate the total length of DNA contained in the human body in nanometres. Give your answer in scientific notation.

(ii) The DNA in each cell is composed of a series of base pairs, each 0·37 nanometres long. Use your answer from part (i) to determine the number of base pairs in the human body. Give your answer in scientific notation.

QUESTION SEVEN (13 marks)

(a) Fully factorise:

(i) $72x - 32x^3$

(ii) $8 + 7x - x^2$

(b) Make x the subject of $ax - ab = x + c$.

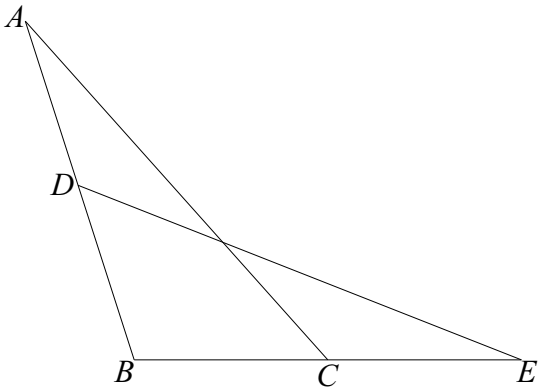
(c) Ismail first looks at his watch and notices that it reads x minutes after 6:00 am. He looks at his watch fifteen minutes later and notices that the number of minutes after 7:00 am is one sixth the number of minutes after the hour when he checked his watch the first time.

(i) Write an equation to represent the situation.

(ii) Solve your equation from part (i) for x.

(iii) Hence, determine what time it was when he looked at his watch the second time.

(d)



In the diagram above, $\angle BAC = \angle BED$ and $AB = EB$.

(i) Prove that $\triangle ABC \equiv \triangle EBD$.

(ii) Prove that $AD = EC$.

QUESTION EIGHT (13 marks)

(a) Determine whether $5^{\frac{1}{3}}$ or $3^{\frac{1}{2}}$ is greater by considering an appropriate power of each number. Support your answer with appropriate working.

(b) (i) Express $\left(x^{\frac{1}{2}} + (x+1)^{\frac{1}{2}}\right)^{-1}$ without negative or fractional indices.

(ii) Write your answer from part (i) as a simplified expression with a rational denominator.

(c) Achilles and Hercules competed in a 4 km race. Achilles ran at a constant speed, and Hercules ran the first 2 km at a speed 1 km/h faster than Achilles and the second 2 km at a speed 1 km/h slower than Achilles.

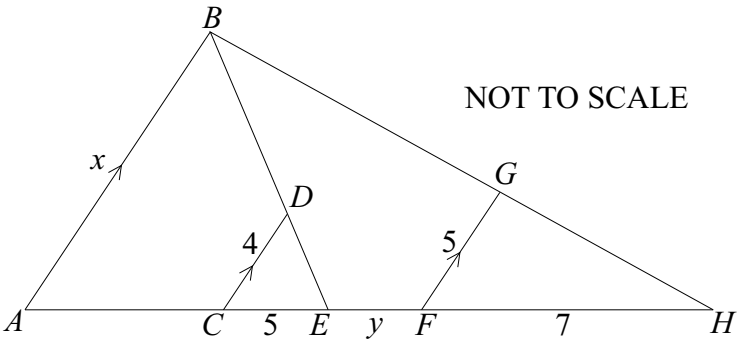
(i) By letting x represent Achilles' speed in km/h, write expressions in terms of x to represent Hercules' speed for each of the two parts of the race.

(ii) Write an expression that represents the time it took Achilles to complete the race.

(iii) Find an expression that represents the time it took Hercules to complete the race. Give your answer as a single simplified fraction.

(iv) Hence, determine who won the race. Justify your answer.

(d)



In the diagram above, $CD = 4$, $CE = 5$, $FG = 5$, $FH = 7$, $AB = x$, and $EF = y$. The intervals AB, CD, and FG are all parallel to each other.

(i) Copy and complete the statements below, and write down the similarity tests that can be used to prove each pair of triangles are similar. You do **not** need to prove these results.

$\triangle ABE \,|||\, \square$ and $\triangle FGH \,|||\, \square$

(ii) Hence, find x in terms of y.

END OF PAPER

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Form III Half-Yearly Examination

Question 1

a) i) $6x - 4x$

$= 2x$ ✓

ii) $2x \times 3y$

$= 6xy$ ✓

iii) $15x \div 3$

$= 5x$ ✓

iv) $x^{12} \div x^{7}$

$= x^{5}$ ✓

b) i) $2(5x - 3)$

$= 10x - 6$ ✓

ii) $(x + 9)(x + 1)$

$= x^{2} + 9x + x + 9$

$= x^{2} + 10x + 9$ ✓

c) i) $-7x + 2 = 23$

$-7x = 21$

$x = -3$ ✓

ii) $2(x-1)=14$

$x-1=7$

$x=8$ ✓

iii) $2x=3x-6$

$-x=-6$

$x=6$ ✓

d) $a^2+5^2=13^2$

$a^2+25=169$

$a^2=144$

$a=12$ ✓

e) i) $\sqrt{7}\times\sqrt{7}$

$=7$ ✓

ii) $\sqrt{3}\times\sqrt{10}$

$=\sqrt{30}$ ✓

iii) $\frac{\sqrt{22}}{\sqrt{11}}$

$=\sqrt{2}$ ✓

Question 2

a) i) $\sqrt{12}$

$= \sqrt{4} \times \sqrt{3}$

$= 2\sqrt{3}$ ✓

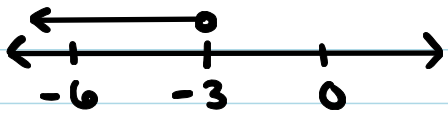
ii) $8\sqrt{2} + \sqrt{5} + 10\sqrt{2} - 7\sqrt{5}$

$= 18\sqrt{2} - 6\sqrt{5}$ ✓

b) i) $5 - 9x > 32$

$-9x > 27$ ✓ subtracting 5 from both sides

$x < -3$ ✓ dividing both sides by -9 and changing direction of inequality sign

ii) 

✓ using solution from (i)

c) i) $(a - b)^2$

$= a^2 - 2ab + b^2$ ✓

ii) $(5 + \sqrt{3})(5 - \sqrt{3})$

$= 5^2 - (\sqrt{3})^2$

$= 25 - 3$

$= 22$ ✓

d) 154 000

$= 1.54 \times 10^5$ ✓

e) 13.486

÷13.5 ✓

f) 2^{-1}

$= \frac{1}{2}$ ✓ award mark for $\frac{1}{2^1}$

g) RHS ✓ award mark for SSS if stated that Pythagoras' theorem could be used to find lengths of unknown sides

h) $C = \frac{5}{9}(F - 32)$

sub $F = 86$

$C = \frac{5}{9}(86 - 32)$ ✓ correct substitution into formula

$= \frac{5}{9} \times 54$

$= 30$ ✓ evaluating C award mark even if no conclusion or units given

$\therefore$ 30 degrees Celsius

Question 3

a) $100^{\frac{1}{2}}$

$= \sqrt{100}$

$= 10$ ✓

b) similarity ratio is 1:3 ✓ award mark even if unsimplified

also accepted:

$\frac{1}{3}$, 3:1, 3

c) i) $21x^2 + 15xy$

$= 3x(7x + 5y)$ ✓

ii) $x^2 - 121$

$= (x + 11)(x - 11)$ ✓

iii) $x^2 + 6x - 40$

$= (x + 10)(x - 4)$ ✓

iv) $x^2 + 8x + 16$

$= (x + 4)(x + 4)$

$= (x + 4)^2$ } ✓ either expression

d) $3\sqrt{2} = \sqrt{9} \times \sqrt{2}$

$= \sqrt{18}$ ✓

e) i) $8a(3-a) - 5a(a+4)$

$= 24a - 8a^2 - 5a^2 - 20a$ ✓ correct expansion of one set of brackets

✓ correct expansion of other set of brackets and collecting like terms

$= -13a^2 + 4a$

ii) $\frac{13a^{1}}{4_{1}} \times \frac{2\cancel{8}b}{5\cancel{a}}$ ✓ cancelling constants to get $\frac{26}{5}$

$= \frac{26ab}{5}$ ✓ cancelling pronumerals to get ab

iii) $\frac{3}{3} \times \frac{3x}{4} + \frac{x}{6} \times \frac{2}{2}$

$= \frac{9x}{12} + \frac{2x}{12}$ ✓ common denominator

$= \frac{11x}{12}$ ✓ simplified fraction

Question 4

a) i) $14 \times (3x)^0$

$= 14 \times 1$

$= 14$ ✓

ii) $(2x^2)^3$

$= 2^3 x^6$

$= 8x^6$ } ✓ either expression

b) $\sqrt[4]{x^3}$

$= x^{\frac{3}{4}}$ ✓

c) $\sqrt{3} \times \sqrt{18} - \sqrt{24}$

$= \sqrt{54} - \sqrt{24}$

$= \sqrt{9} \times \sqrt{6} - \sqrt{4} \times \sqrt{6}$ ✓ taking square factor out of one term

$= 3\sqrt{6} - 2\sqrt{6}$

$= \sqrt{6}$ ✓ taking square factor out of other term and collecting like surds

d) i) $\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$ ✓

ii) $\frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ ✓ showing multiplication by the conjugate of the denominator

$= \frac{5(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{5\sqrt{3}+5}{3-1}$

$= \frac{5\sqrt{3}+5}{2}$ ✓ simplified fraction

e) $A = \frac{x+y}{2}$

$2A = x + y$

$x = 2A - y$ ✓

f) i) Hippasus owns $n+13$ books ✓

ii) $n + (n+13) = 27$ ✓ equation, using answer from (i)

$2n + 13 = 27$

$2n = 14$

$n = 7$ ✓ solving for n

∴ Pythagoras owns 7 books

and Hippasus owns 20 books } ✓ answering the question

Question 5

a) $\angle DGH = 65°$ (alternate angles, AC || FH) ✓ ∠DGH, with reason

$\theta + 65° + 90° + 130° = 360°$ (angle sum of $\triangle EHG$)

$\theta + 285° = 360°$

$\theta = 75°$ ✓ finding θ, with reason

b) $9x^8 \times x^{-1}$

$= 9x^7$ ✓

c) i) $3a^{-2}$

$= \frac{3}{a^2}$ ✓

ii) $\left(\frac{5}{2}\right)^{-1}$

$= \frac{2}{5}$ ✓

d) $BC^2 = (2x)^2 + (2x)^2$ ✓ showing correct use of Pythagoras' theorem

$BC^2 = 4x^2 + 4x^2$

$BC^2 = 8x^2$

$BC = \sqrt{8x^2}$

$BC = 2\sqrt{2}\,x$ ✓ simplified expression for BC

e) $(2\sqrt{5} + 4\sqrt{2})(3\sqrt{2} - 5\sqrt{10})$

$= 6\sqrt{10} - 10\sqrt{50} + 12\sqrt{4} - 20\sqrt{20}$ ✓ expanding

$= 6\sqrt{10} - 50\sqrt{2} + 24 - 40\sqrt{5}$ ✓ ✓ simplifying any term, simplifying remaining two terms

f) i) $4x^2 + 24x - 64$

$= 4(x^2 + 6x - 16)$ ✓ taking out HCF

$= 4(x + 8)(x - 2)$ ✓ factorising trinomial

ii) $x^2 + 7x + \frac{49}{4}$

$= \left(x + \frac{7}{2}\right)^2$ ✓ also accept $\left(x + \frac{7}{2}\right)\left(x + \frac{7}{2}\right)$

Question 6

a) $(8x^3y^{-9})^{-\frac{1}{3}}$ or $= \left(\frac{8x^3}{y^9}\right)^{-\frac{1}{3}}$

$= \frac{y^3}{2x}$

$= \left(\frac{y^9}{8x^3}\right)^{\frac{1}{3}}$ ✓ reciprocal

$= \frac{y^3}{2x}$ ✓ cube root

b) i) $\frac{AC}{DC} = \frac{15}{24}$

$= \frac{5}{8}$ ✓ either value

ii) in ΔABC and ΔDEC,

$\frac{AC}{DC} = \frac{15}{24}$

$= \frac{5}{8}$ ✓ any two correct steps, with any necessary reasons

$\frac{BC}{EC} = \frac{20}{32}$

$= \frac{5}{8}$ correct third step with any necessary reason, correct similarity statement, and similarity test

$\angle ACB = \angle DCE$ (vertically opposite angles)

$\therefore \Delta ABC \,|||\, \Delta DEC$ (SAS) ✓

iii) $\frac{10}{x} = \frac{5}{8}$ (matching sides of similar triangles)

$5x = 80$

$x = 16$ ✓ correct value of x and geometric reason

c) $36 \times \frac{x+1}{9} - 1 \times 36 = \frac{x-7}{4} \times 36$

$4(x+1) - 36 = 9(x-7)$ ✓ cancelling denominators or forming a common denominator

$4x + 4 - 36 = 9x - 63$

$4x - 32 = 9x - 63$

$5x = 31$

$x = \frac{31}{5}$ ✓

d) $\frac{5b}{5b} \times \frac{8a}{3} - \frac{(2-ab)}{5b} \times \frac{3}{3}$

$= \frac{40ab}{15b} - \frac{6-3ab}{15b}$ ✓ common denominator

$= \frac{40ab - 6 + 3ab}{15b}$

$= \frac{43ab - 6}{15b}$ ✓ simplified fraction

e) i) $(2.2 \times 10^{9}) \times (3.7 \times 10^{13})$

$= 8.14 \times 10^{22}$ nanometres ✓ award if no units given

ii) $8.14 \times 10^{22} \div 0.37$ ✓ showing division by 0.37

$= 22 \times 10^{22}$

$= 2.2 \times 10^{23}$ base pairs ✓ evaluating and expressing in scientific notation

Question 7

a) i) $72x - 32x^3$

$= 8x(9 - 4x^2)$ ✓ taking out HCF

$= 8x(3 - 2x)(3 + 2x)$ ✓ factorising difference of two squares

ii) $8 + 7x - x^2$

$= -(x^2 - 7x - 8)$ ✓ taking out factor of -1

$= -(x - 8)(x + 1)$ ✓ factorising trinomial
full marks for $(8 - x)(x + 1)$ or similar

b) $ax - ab = x + c$

$ax - x = ab + c$ ✓ all terms in x on one side

$x(a - 1) = ab + c$

$x = \frac{ab + c}{a - 1}$ ✓ correctly solving for x

c) i) $\frac{x}{6} + 60 = x + 15$ ✓ a correct equation

ii) $x + 360 = 6x + 90$

$5x = 270$

$x = 54$ ✓ solving equation

iii) it was 7:09 am ✓ using solution from (ii) to find the time ie $7:\frac{x}{6}$ am
award if "am" left off

d) i) in ΔABC and ΔEBD,

$\angle BAC = \angle BED$ (given) ✓ any two correct steps

$AB = EB$ (given)

$\angle B$ is common ✓ correct third step, congruence statement, and congruence test

$\therefore \Delta ABC \equiv \Delta EBD$ (AAS)

ii) $AB = AD + DB$, $EB = EC + CB$

$\because AB = EB$ (given)

$AD + DB = EC + CB$

$DB = CB$ (matching sides of $\Delta ABC \equiv \Delta EBD$)

$\therefore AD = EC$ ✓ establishing $DB = CB$, with correct geometric reason

✓ correct conclusion with sufficient working

Question 8

a) $(5^{\frac{1}{3}})^6$ $\qquad$ $(3^{\frac{1}{2}})^6$

$= 5^2$ $\qquad$ $= 3^3$

$= 25$ $\qquad$ $= 27$

$\because 27 > 25$

$\therefore 3^{\frac{1}{2}} > 5^{\frac{1}{3}}$

$\therefore 3^{\frac{1}{2}}$ is greater ✓ correct answer with sufficient supporting working

b) i) $\left(x^{\frac{1}{2}} + (x+1)^{\frac{1}{2}}\right)^{-1}$

$= \dfrac{1}{\sqrt{x} + \sqrt{x+1}}$ ✓

ii) $\dfrac{1}{\sqrt{x} + \sqrt{x+1}} \times \dfrac{\sqrt{x} - \sqrt{x+1}}{\sqrt{x} - \sqrt{x+1}}$ ✓ multiplying by conjugate of denominator

$= \dfrac{\sqrt{x} - \sqrt{x+1}}{x - (x+1)}$

$= \dfrac{\sqrt{x} - \sqrt{x+1}}{-1}$

$= \sqrt{x+1} - \sqrt{x}$ ✓ fully simplifying

c) i) first part: $x+1$ ✓ second part: $x-1$ ✓ must have both expressions

ii) $\frac{4}{x}$ ✓

iii) $\frac{2}{x+1} + \frac{2}{x-1}$ ✓ correct expression for time

$= \frac{2(x-1)+2(x+1)}{(x+1)(x-1)}$

$= \frac{2x-2+2x+2}{x^2-1}$

$= \frac{4x}{x^2-1}$ ✓ single, simplified fraction

iv) Achilles' time $= \frac{4}{x}$

$= \frac{4x}{x^2}$

Hercules' time $= \frac{4x}{x^2-1}$

$x^2-1 < x^2$

$\therefore \frac{4x}{x^2-1} > \frac{4x}{x^2}$

$\therefore$ Achilles had the shorter time, so he won the race ✓ correct answer with sufficient supporting working

d) i) $\Delta ABE \Vert\Vert \Delta CDE$ (AA) ✓ both similarity statements and tests correct

$\Delta FGH \Vert\Vert \Delta ABH$ (AA)

ii) $\frac{5}{AC+5} = \frac{4}{x}$

$5x = 4(AC+5)$

$AC+5 = \frac{5x}{4}$ ✓ an expresion for AC or AE

$\frac{7}{7+y+AC+5} = \frac{5}{x}$

$\because AC+5 = \frac{5x}{4}$

$\therefore \frac{7}{7+y+\frac{5x}{4}} = \frac{5}{x}$ ✓ a correct relationship between x and y

$7x = 5(7+y+\frac{5x}{4})$

$7x = 35+5y+\frac{25x}{4}$

$\frac{3x}{4} = 35+5y$

$3x = 140+20y$

$x = \frac{140+20y}{3}$ ✓ x in terms of y