NAME MASTER

SYDNEY GRAMMAR SCHOOL



2019 Half-Yearly Examination

FORM III

MATHEMATICS

Wednesday 29th May 2019

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Calculators are not to be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: SO	3B: NJL	3C: LYL/RR	3D: REJ/PKH
3E: WJM/RCF	3F: DWH-GMC	3G: LRP	3H: RDP
3I: DNW	3J: RPL		

Checklist

- Writing paper — approx. 16 sheets
- Candidature — 197 boys

Examiner

RPL

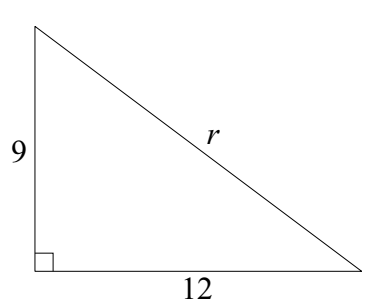
QUESTION ONE (13 marks) Start a new page.

(a) Evaluate 5^3.

(b) Simplify $5x^2 \times 2x^5$.

(c) Write 8085 correct to 3 significant figures.

(d)



Find the value of r in the diagram above.

(e) If $a = -2$ and $b = -3$, evaluate:

(i) $a^2 + b$

(ii) $3b - 2a$

(iii) $(a + b)^2$

(f) Expand and simplify where appropriate:

(i) $5(x + 2)$

(ii) $(x + 3)(x - 4)$

(g) Simplify $8b^2 - 5b - 7b - 9b^2$.

(h) Factorise:

(i) $15xy + 10x$

(ii) $p^2 - 5p$

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Evaluate $27^{\frac{1}{3}}$.

(b) Simplify:

(i) $4\sqrt{2} - \sqrt{2}$

(ii) $\sqrt{5} \times \sqrt{2}$

(iii) $\sqrt{72}$

(c) Solve the following equations:

(i) $5 - 2x = 1$

(ii) $4(x - 6) = 2(3 - x)$

(d) (i) Solve the inequation $8 - 4x \geq -16$.

(ii) Represent your solution to (d)(i) on a number line.

(e) For the following questions, use the data set below:

12 8 12 31 9 4 1

(i) Find the mean.

(ii) Find the median.

(iii) Find the mode.

(iv) Find the range.

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

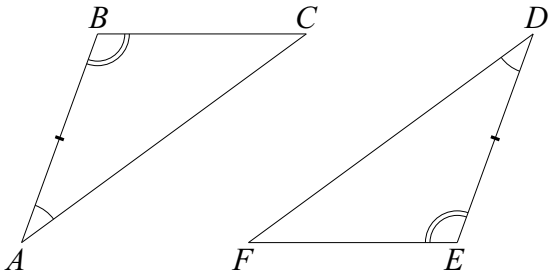
(a) Express $\dfrac{6}{\sqrt{3}}$ with a rational denominator.

(b) Expand:

(i) $(2b-3c)^2$

(ii) $(d+4e)(d-4e)$

(c)



Which congruency test would be used to prove that $\triangle ABC \equiv \triangle DEF$?

(d) Write the following numbers in scientific notation correct to 3 significant figures:

(i) 934 578

(ii) 0·007 284

(e) E.T. phoned home on a regular basis over a year. The length of each phone call was recorded in the table below to the nearest minute.

Length of Call (mins)	**Frequency**
$0-4$	4
$5-9$	7
$10-14$	12
$15-19$	18
$20-24$	15
$25-30$	9

(i) Draw a frequency histogram to correctly represent this data.

(ii) How many phone calls were less than 15 minutes long?

(f) Factorise:

(i) x^2+5x+6

(ii) $x^2-8x+16$

(iii) x^2-9

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) Simplify:

(i) $\dfrac{2a}{3} \times \dfrac{6}{a}$

(ii) $\dfrac{6b}{4} \div \dfrac{3b}{2a}$

(iii) $\dfrac{2a}{5} + \dfrac{b}{10}$

(b) Evaluate $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$.

(c) Simplify:

(i) $\left(3\sqrt{2}\right)^2$

(ii) $\sqrt{3}\left(4\sqrt{3} - 2\sqrt{6}\right)$

(d) Write $0\cdot\dot{1}\dot{2}$ as a fraction in its lowest terms.

(e) Five honey pots are added to Winnie the Pooh's initial collection of honey pots. He shares them between himself, Eeyore and Piglet and each of them gets exactly four honey pots each.

(i) If x is the number of honey pots in the initial collection, form an equation to represent the situation.

(ii) Hence calculate how many honey pots there were in the initial collection.

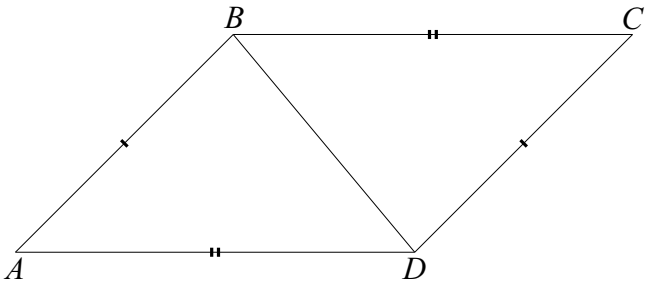
(f) Express $\dfrac{5^{-1}a^{-3}}{y^{-3}}$ without negative indices.

Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Calculate $\left(8 \times 10^{14}\right) \times \left(9 \times 10^{9}\right)$, expressing your answer in scientific notation.

(b)



The diagram above shows a quadrilateral $ABCD$ where $AB = DC$ and $AD = BC$.

(i) Prove $\triangle ABD \equiv \triangle CDB$.

(ii) Hence or otherwise, prove that $AD \parallel BC$.

(c) Michael is flying a kite with a string of length 30 m from an anchor point. The kite's horizontal distance from its anchor point is 18 m. What is the kite's vertical distance from its anchor point?

(d) Rationalise the denominator of $\dfrac{2}{\sqrt{3}-4}$.

(e) Simplify $\dfrac{5x^8y^{-12}}{x^{10}} \times \dfrac{\left(x^2y^5\right)^2}{10}$, expressing your answer without negative indices.

(f) Factorise, and hence simplify $\dfrac{x^2-64}{x^2+12x+32} \times \dfrac{x^2+8x+16}{x^2-16}$.

Examination continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a) Dash Incredible and Frozone ran some 100 m races. The times in seconds were recorded as follows:

Dash : 3.7 3.2 5.7 4.5 4.7 5.2 4.2 3.5 5.6 5.2

Frozone : 4.0 5.9 5.5 5.8 6.2 6.5 4.8 5.6 4.6 5.2

(i) Express these results as a back-to-back stem-and-leaf plot.

(ii) Find Dash's median time.

(iii) Compare the number of times Dash and Frozone ran 100 m in less than 5 seconds.

(b) Solve the equation $\dfrac{x+5}{2} - \dfrac{2x+3}{3} = \dfrac{x+7}{5}$.

(c) (i) Make r the subject of $V = \dfrac{4}{3}\pi r^3$.

(ii) Make t the subject of $m = \dfrac{1+at}{1-at}$.

(d) Jim Hawkins sails due north for 2 km and then north-east for 3 km. How far north is he from his starting point? Give your answer in exact form.

Examination continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

(a) If $p = \dfrac{4}{5}$ and $q = \dfrac{2}{7}$, evaluate $2pq + \sqrt{pq^2}$, giving your answer as a single fraction with a rational denominator.

(b) Factorise fully:

(i) $(m+1)^2 - n^2$

(ii) $p^2 + 2p + 1 - q^2$

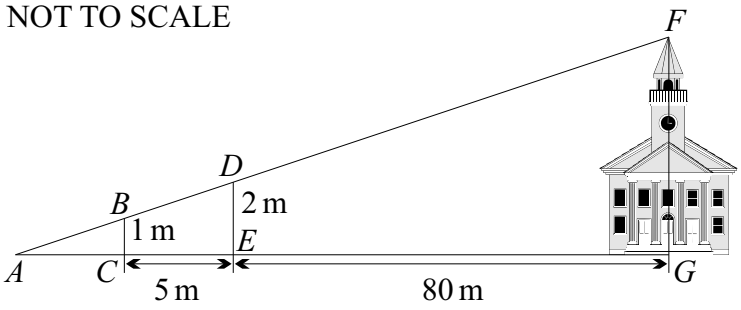
(c) Diophantus was a mathematician who lived in Alexandria during the 3rd century BC. His epitaph is a poetic version of the following:

> *Here lie the remains of Diophantus.*
> *He was a child for* $\frac{1}{6}$ *of his life.*
> *After* $\frac{1}{12}$ *more, he became a man.*
> *After* $\frac{1}{7}$ *more, he married, and* 5 *years later his son was born.*
> *His son lived* $\frac{1}{2}$ *as long as his father, and died* 4 *years before his father.*

Let x be how long Diophantus was alive. Form an equation in x and solve it to determine how long he lived.

(d) Find the value of $\dfrac{99}{100} \times \dfrac{80}{81} \times \dfrac{63}{64} \times \dfrac{48}{49} \times \dfrac{35}{36} \times \dfrac{24}{25} \times \dfrac{15}{16} \times \dfrac{8}{9} \times \dfrac{3}{4}$. Write your answer in the form $\dfrac{a}{b}$, where a and b are positive integers with no common factors other than 1.

(e)



Vertical poles BC and DE with lengths 1 m and 2 m respectively are placed 5 m apart. The longer pole is 80 m from the base of a vertical clock tower, and a line of sight is taken from A, as shown in the diagram above.

(i) Prove $\triangle ABC \,|||\, \triangle ADE \,|||\, \triangle AFG$.

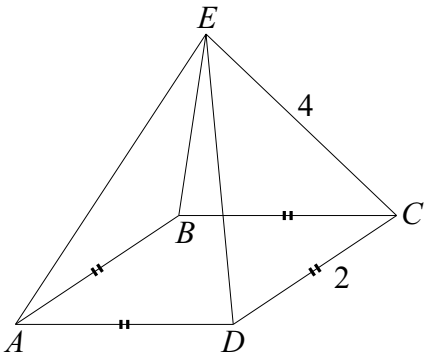
(ii) Hence calculate the height of the tower.

Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Find the value of $\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2\right)\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2-2^2\right)$.

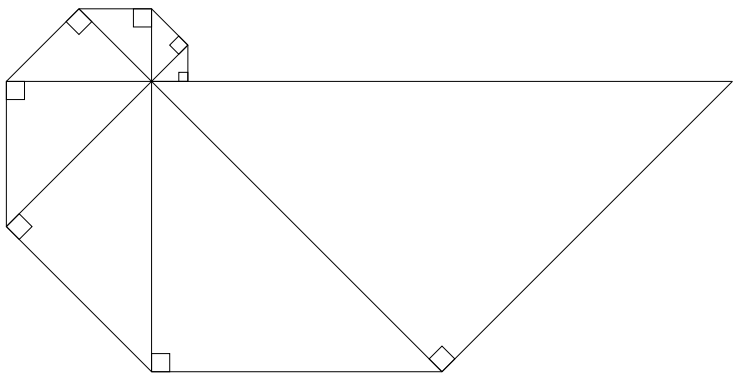
(b)



For the right square pyramid illustrated above, find the perpendicular height of the pyramid as a simplified surd.

(c) Simplify $\dfrac{12^n-18^n}{3^n-2^n}$.

(d)



Eight isosceles right-angled triangles, whose sides coincide as shown, form a shape whose perimeter is an irregular nonagon.
The shortest side of the nonagon has length 1 cm.
Calculate the perimeter of the nonagon, giving your answer in the form $a\left(b+\sqrt{b}\right)$.

(e) In a Wonderland jousting tournament, each of the several competing knights receives 17 points for every bout they enter. The winner of each bout receives an extra 3 points. At the end of the tournament, the Black Knight has exactly one more point than the White Knight. What is the smallest number of bouts that the Black Knight could have entered? You must justify your answer.

END OF EXAMINATION

FORM III HALF YEARLY SOLUTIONS

Question One

(a) 125 ✓

(b) $10x^7$ ✓

(c) 8090 ✓

(d) $9^2 + 12^2 = 81 + 144 = 225$

$r = \sqrt{225} = 15$ ✓

(e) (i) $(-2)^2 + (-3) = 4 - 3 = 1$ ✓

(ii) $3(-3) - 2(-2) = -9 + 4 = -5$ ✓

(iii) $(-2 + (-3))^2 = (-2-3)^2 = (-5)^2 = 25$ ✓

(f) (i) $5x + 10$ ✓

(ii) $x^2 + 3x - 4x - 12$ ✓

$= x^2 - x - 12$ ✓

(g) $-b^2 - 12b$ ✓

(h) (i) $5x(3y + 2)$ ✓

(ii) $p(p - 5)$ ✓

Question Two

(a) $\sqrt[3]{27} = 3$ ✓

(b) (i) $4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$ ✓

(ii) $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ ✓

(iii) $\sqrt{72} = \sqrt{9}\sqrt{8}$
$= 3\sqrt{8}$
$= 3\sqrt{4}\sqrt{2}$
$= 6\sqrt{2}$ ✓

or $\sqrt{72} = \sqrt{36}\sqrt{2}$
$= 6\sqrt{2}$

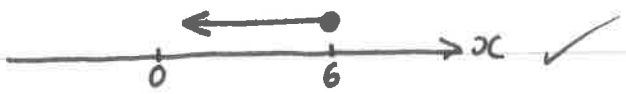
(c) (i) $4 = 2x$
$x = 2$ ✓

(ii) $4x - 24 = 6 - 2x$ ✓
$6x - 24 = 6$
$6x = 30$
$x = 5$ ✓

or $2(x-6) = 3 - x$
$2x - 12 = 3 - x$ ✓
$3x = 15$
$x = 5$ ✓

(d) (i) $8 \geq -16 + 4x$
$24 \geq 4x$
$6 \geq x$
$x \leq 6$ ✓

(ii)



✓

(e) (i) $12+8+12+31+9+4+1 = 77$ ∴ mean $= 77 \div 7 = 11$ ✓

(ii) 1, 4, 8, 9, 12, 12, 31 ∴ median $= 9$ ✓

(iii) mode $= 12$ ✓

(iv) range $= 31 - 1 = 30$ ✓

Question Three

(a) $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ ✓ ✓ ✓

(b) (i) $4b^2 - 12bc + 9c^2$ ✓

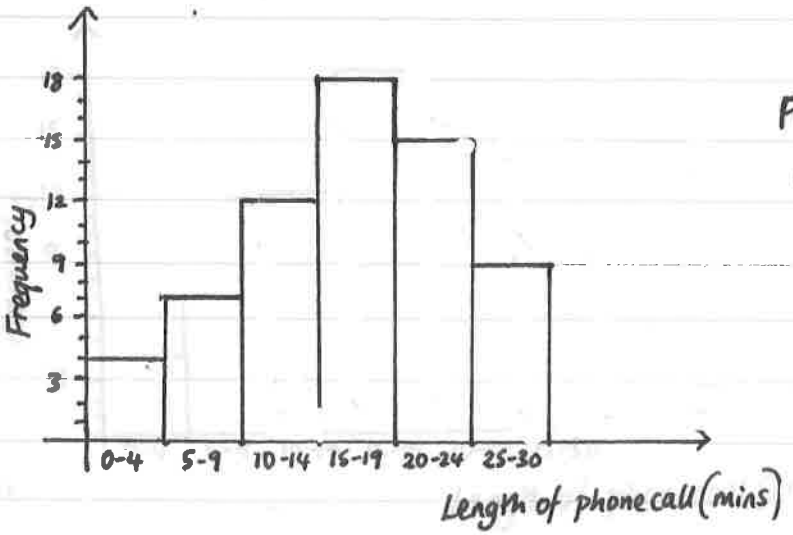
(ii) $d^2 - 16e^2$ ✓

(c) AAS ✓

(d) (i) 9.35×10^5 ✓
(ii) 7.28×10^{-3} ✓

} only penalise once for incorrect rounding if standard form is correct.

(e) (i)



Frequency labelled & bars correct height ✓

bars labelled correctly ✓

(no gaps between columns, and widths should be equal.)

(ii) $12 + 7 + 4 = 23$ ✓

(f) (i) $(x+2)(x+3)$ ✓
(ii) $(x-4)^2$ ✓
(iii) $(x+3)(x-3)$ ✓

also accept $(x-4)(x-4)$

Question Four

(a) (i) $\frac{12a}{3a} = 4$ ✓

(ii) $\frac{6b}{4} \times \frac{2a}{3b} = \frac{12ab}{12b} = a$ ✓

(iii) $\frac{2a}{5} + \frac{b}{10} = \frac{4a}{10} + \frac{b}{10}$ ✓

$= \frac{4a+b}{10}$ ✓

(b) $\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ ✓

(c) (i) $3\sqrt{2} \times 3\sqrt{2} = 9 \times 2 = 18$ ✓

(ii) $4\sqrt{3}\sqrt{3} - 2\sqrt{3}\sqrt{6} = 12 - 2\sqrt{18}$ ✓

$= 12 - 2\sqrt{9}\sqrt{2}$

$= 12 - 6\sqrt{2}$ ✓

(d) Let $n = 0.\dot{1}\dot{2}$

$100n = 12.\dot{1}\dot{2}$ ✓

$99n = 12$

$n = \frac{12}{99} = \frac{4}{33}$ ✓

(e) (i) $\frac{x+5}{3} = 4$ ✓

(ii) $x + 5 = 12$

$x = 7$ ∴ there were 7 honey pots in the initial collection ✓

(f) $\frac{5^{-1}a^{-3}}{y^{-3}} = \frac{y^3}{5a^3}$ ✓

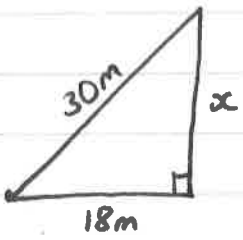
Question Five

(a) $72 \times 10^{23} = 7.2 \times 10^{24}$ ✓ ✓

(b) (i) BD common to both
$AB = CD$ (given)
$AD = CB$ (given) ✓
$\therefore \triangle ABD \equiv \triangle CDB$ (SSS) ✓

(ii) $\angle ADB = \angle CBD$ (matching angles in congruent $\triangle$s)
$\therefore AD \parallel BC$ ✓ (alternate angles are equal)

(c)



$x^2 = 30^2 - 18^2$
$= 576$ ✓
$\therefore x = 24$ m Kite is flying at 24m. ✓

(d) $\frac{2}{\sqrt{3}-4} \times \frac{\sqrt{3}+4}{\sqrt{3}+4}$ ✓ $= \frac{2(\sqrt{3}+4)}{3+\sqrt{3}4-\sqrt{3}4-16} = \frac{2\sqrt{3}+8}{-13}$ ✓ $\left(\text{or } \frac{-2\sqrt{3}-8}{13}\right)$

(e) $\frac{5x^8y^{-12}}{x^{10}} \times \frac{x^4y^{10}}{10} = \frac{5x^{12}y^{-2}}{10x^{10}}$ ✓

$= \frac{x^2y^{-2}}{2}$

$= \frac{x^2}{2y^2}$ ✓

(f) $\frac{(x-8)(x+8)}{(x+4)(x+8)} \times \frac{(x+4)(x+4)}{(x+4)(x-4)} = \frac{x-8}{x-4}$ ✓

✓ at least 2 factorised correctly

Question Six

(a) (i)

Dash		Frozone
7 5 2	3	
7 5 2	4	0 6 8
7 6 2 2	5	2 5 6 8 9
	6	2 5

✓ stem and leaves separate
✓ incorrect numerical order

4|0 represents 4.0 seconds

(ii) $\frac{4.5+4.7}{2} = 4.6$ seconds ✓

(iii) Dash ran 100m in less than 5 seconds 6 times out of 10.
Frozone ran 100m in less than 5 seconds 3 times out of 10. ✓

Accept
Dash = 6
Frozone = 3

(b) $\frac{\overset{15}{\cancel{30}}(x+5)}{\cancel{2}} - \frac{\overset{10}{\cancel{30}}(2x+3)}{\cancel{3}} = \frac{\overset{6}{\cancel{30}}(x+7)}{\cancel{5}}$ ✓ determining common denominator

$15x + 75 - 20x - 30 = 6x + 42$ ✓

$45 = 11x + 42$

$11x = 3$

$x = \frac{3}{11}$ ✓

(c) (i) $4\pi r^3 = 3V$

$r^3 = \frac{3V}{4\pi}$ ✓

$r = \sqrt[3]{\frac{3V}{4\pi}}$ ✓

(ii) $m(1-at) = 1+at$

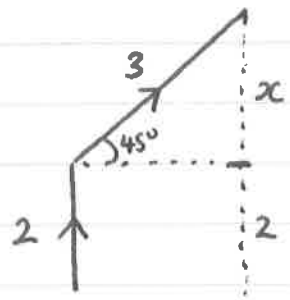
$m - mat = 1 + at$

$m - 1 = mat + at$ ✓

$m - 1 = t(ma + a)$

$\therefore t = \frac{m-1}{ma+a}$ ✓

(d)



$x^2 + x^2 = 9$ ✓

$2x^2 = 9$

$x = \sqrt{\frac{9}{2}}$

$x = \frac{3}{\sqrt{2}}$

$\therefore 2 + \frac{3\sqrt{2}}{2}$ km north ✓

(or $2 + \frac{3}{\sqrt{2}}$ or $\frac{4+3\sqrt{2}}{2}$
don't penalise for not rationalising the denominator)

Question Seven

(a) $\sqrt{\frac{4}{5} \times \frac{4}{49}} + 2 \times \frac{4}{5} \times \frac{2}{7} = \frac{4}{7\sqrt{5}} + \frac{16}{35}$ ✓

$= \frac{20}{35\sqrt{5}} + \frac{16\sqrt{5}}{35\sqrt{5}}$ ✓ common denominator

$= \frac{20 + 16\sqrt{5}}{35\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{4\not{20}\sqrt{5} + 16 \times \not{5}}{35 \times \not{5}}$

$= \frac{4\sqrt{5} + 16}{35}$ ✓

(or: $= \frac{4\sqrt{5}}{35} + \frac{16}{35}$ ✓

$= \frac{4\sqrt{5} + 16}{35}$ ✓)

(b) (i) $((m+1)-n)((m+1)+n) = (m+1-n)(m+1+n)$ ✓

(ii) $p^2 + 2p + 1 - q^2 = (p+1)^2 - q^2 = (p+1-q)(p+1+q)$ ✓

(c) Number of years until son born $= \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5$

(Note x was given in the question)

$= \frac{14}{84}x + \frac{7}{84}x + \frac{12}{84}x + 5$

$= \frac{33}{84}x + 5$ ✓

son died when father was $x-4$ years old } ✓

son lived $\frac{x}{2}$ years.

$\frac{33}{84}x + 5 + \frac{x}{2} = x - 4$

$33x + 420 + 42x = 84 - 336$

$420 + 75x = 84x - 336$

$756 = 9x$

$x = 84$ ✓

∴ Diophantus was alive for 84 years.

(d) $\frac{9.11}{10.10} \times \frac{8.10}{9.9} \times \frac{7.9}{8.8} \times \frac{6.8}{7.7} \times \frac{5.7}{6.6} \times \frac{4.6}{5.5} \times \frac{3.5}{4.4} \times \frac{2.4}{3.3} \times \frac{1.3}{2.2}$

then can cancel through on right diagonals:

$\frac{9.11}{10.10} \times \frac{8.10}{9.9} \times \frac{7.9}{8.8} \times \frac{6.8}{7.7} \times \frac{8.7}{6.6} \times \frac{4.6}{8.5} \times \frac{8.5}{4.4} \times \frac{2.4}{3.3} \times \frac{1.3}{2.2}$

and left diagonals:

$\frac{9.11}{10.10} \times \frac{8.10}{9.9} \times \frac{7.9}{8.8} \times \frac{6.8}{7.7} \times \frac{8.7}{6.6} \times \frac{4.6}{8.8} \times \frac{8.8}{4.4} \times \frac{2.4}{3.3} \times \frac{1.3}{2.2}$

leaving $\frac{11 \times 1}{10 \times 2} = \frac{11}{20}$ ✓

✓ for indication of factoring or cancelling correctly in ≥4 instances.

(e) (i) $\angle BAC = \angle DAE = \angle FAG$ (common to all)

$\angle ACB = \angle AED = \angle AGF$ (corresponding angles are equal, $BC \parallel DE \parallel FG$)

$\therefore \triangle ABC \parallel\parallel \triangle ADE \parallel\parallel \triangle AFG$ (AAA) ✓

(*) If angles are noted to be equal to 90° as vertical, instead of corresponding angles.

(ii) Let h = height of tower. Let $AC = x$

$\therefore \frac{x}{x+5} = \frac{1}{2}$ (matching sides on similar Δs in ratio, $\Delta ABC \parallel\parallel \Delta ADE$)

$2x = x + 5$

$x = 5$ ✓

$\therefore \frac{h}{2} = \frac{90}{10}$ (matching sides on similar Δs in ratio, $\Delta AFG \parallel\parallel \Delta ADE$)

$h = \frac{180}{10} = 18\text{m}$ ✓

Question Eight

(a) $\left(2^{3/2}+2.2^{3/4}+1+2^{3/2}-2.2^{3/4}+1\right)\left(2^{3/2}+2.2^{3/4}+1+2^{3/2}-2.2^{3/4}+1-4\right)$

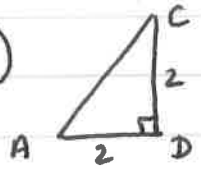
$=\left(2.2^{3/2}+2\right)\left(2.2^{3/2}-2\right)$ ✓

$=\left(2.2^{3/2}\right)^2-2^2$ ✓

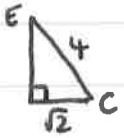
$=32-4$

$=28$ ✓

(b)



Base diagonal $=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$ ✓



$\therefore$ Height $=\sqrt{4^2-\left(\sqrt{2}\right)^2}=\sqrt{16-2}=\sqrt{14}$ ✓

(c) $\dfrac{12^n-18^n}{3^n-2^n}=\dfrac{\left(2^2.3\right)^n-\left(2.3^2\right)^n}{3^n-2^n}$ ✓ for prime reduction

$=\dfrac{2^{2n}3^n-2^n3^{2n}}{3^n-2^n}$

$=\dfrac{2^n3^n\left(2^n-3^n\right)}{3^n-2^n}$ ✓ for factorising correctly

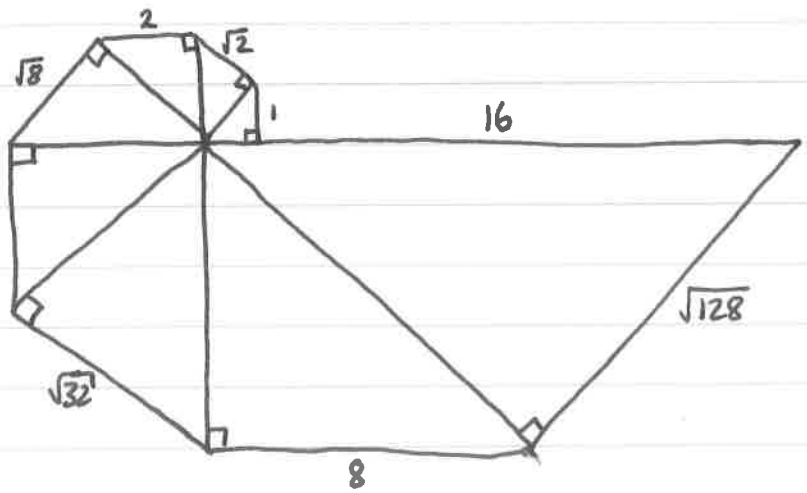
$=\dfrac{-2^n3^n\left(3^n-2^n\right)}{3^n-2^n}$

$=-2^n3^n$ ✓

[or: $\dfrac{6^n\left(2^n-3^n\right)}{3^n-2^n}$ ✓ for noticing negative needed.

$=-6^n$ ✓]

(d)



✓ at least 4 correct sides indicated

P = sum of all sides − 1

$=30+\sqrt{2}+2\sqrt{2}+4\sqrt{2}+8\sqrt{2}$

$=30+15\sqrt{2}$

$=15\left(2+\sqrt{2}\right)$ ✓

$\therefore a=15, b=2$

(e)

White Knight			Black Knight		
Win	Lose	Points	Win	Lose	Points
a	b	$20a+17b$	c	d	$20c+17d$

$\therefore 20a+17b+1 = 20c+17d$

$20(a-c)+17(b-d) = -1$ ✓ (or similar)

Need solutions to $20M+17N=-1$ where M and N are integers.

i.e. multiples of 20 and 17 that differ by 1. ✓ (or similar indication)

$\therefore$ A multiple of 17 must end in 1 or 9 → 17, 34, 51, 68, 85, 102, 119

↑ 50/52 not mult of 20.

↑ 120 is mult. of 20!

$\therefore N=7$, $M=-6$

$\therefore a-c=-6$, $b-d=7$ smallest non-negative integer solutions:

$a=0, c=6, b=7, d=0$

$\therefore$ Black Knight enters $c+d$ bouts

$\therefore$ 6 bouts. ✓