NAME MASTER

SYDNEY GRAMMAR SCHOOL



2018 Half-Yearly Examination

FORM III

MATHEMATICS

Friday 25th May 2018

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Calculators are not to be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: LRP	3B: LL	3C: OMD	3D: RPL/NJL
3E: RDP	3F: SO/BR	3G: FMW/LYL	3H: RCF
3I: BDD	3J: WJM		

Checklist

- Writing paper — approx. 16 sheets
- Candidature — 194 boys

Examiner

BR

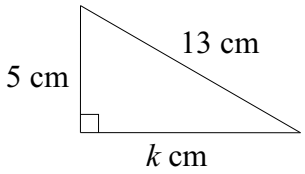
QUESTION ONE (13 marks) Start a new page.

(a) Evaluate 3^4.

(b) Simplify $4x^3 \times 6x^7$.

(c) Expand $7(x - 6)$.

(d)



Use the diagram above to find the value of k.

(e) Simplify:

(i) $\sqrt{50}$

(ii) $2\sqrt{5} + 3\sqrt{5}$

(iii) $\sqrt{3} \times \sqrt{5}$

(f) Simplify:

(i) $15a + 17b - 19b$

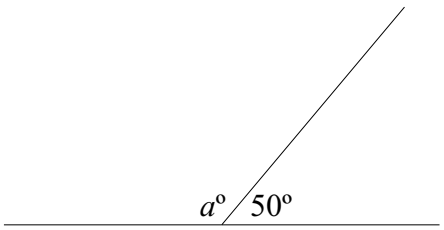
(ii) $63y \div 9$

(g) Factorise:

(i) $a^2 - 4a$

(ii) $12uv + 8u$

(h)



Find the value of a. You must give a geometrical reason.

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Evaluate $16^{\frac{1}{2}}$.

(b) Write x^{-1} without a negative index.

(c) Write in scientific notation correct to 3 significant figures:

(i) 85 446

(ii) 0·064 853

(d) For the following questions, use the data set below:

7 2 5 7 7 3 4 1 9

(i) Find the mean.

(ii) Find the median.

(iii) Find the mode.

(iv) Find the range.

(e) Express $\dfrac{5}{\sqrt{2}}$ with a rational denominator.

(f) Expand and simplify:

(i) $(a+2)(a+4)$

(ii) $(2x-6)^2$

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

(a) Simplify $\dfrac{21p^2}{8} \times \dfrac{4}{p}$.

(b) Factorise:

(i) $x^2 + 7x + 12$

(ii) $x^2 - 5x - 14$

(c) A survey was conducted in Northmead. Each household was asked to indicate the number of cars registered to that address. The results are summarised in the table below.

Number of Cars	**Frequency**
0	401
1	1854
2	2105
3	582
4	38
Total	4980

(i) Write down the mode of the number of cars in a household.

(ii) How many households have less than 3 cars registered to that address?

(d) Solve the following equations:

(i) $2m + 9 = 25$

(ii) $2x + 12 = 6x + 7$

(iii) $5(p - 3) = 6(p + 2)$

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) The following ordered stem-and-leaf plot shows the ages of eighteen people presenting for treatment at a physiotherapist.

Stem	Leaf
3	0 7 7 9
4	1 4 5 7 7 8
5	0 2 3 4 4 5 9
6	2

5 | 2 represents 52 years old

(i) Calculate the range of this data.

(ii) Find the median of this data.

(b) Expand and simplify:

(i) $\left(2\sqrt{3}\right)^2$

(ii) $\sqrt{6}\left(2\sqrt{6}-3\sqrt{2}\right)$

(c) Rationalise the denominator of $\dfrac{1}{\sqrt{7}-2}$.

(d) Fully factorise:

(i) b^2-100

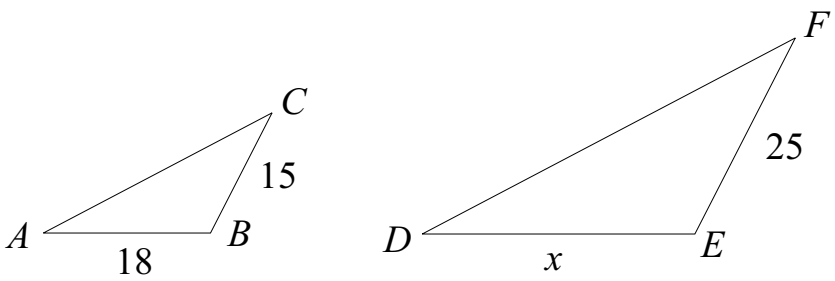
(ii) $5a^2-60a+180$

(e) Evaluate:

(i) $8^{\frac{2}{3}}$

(ii) $\left(\dfrac{1}{49}\right)^{-\frac{1}{2}}$

(f)



It is known that $\triangle ABC$ is similar to $\triangle DEF$ (you do not need to prove this). Find the value of x.

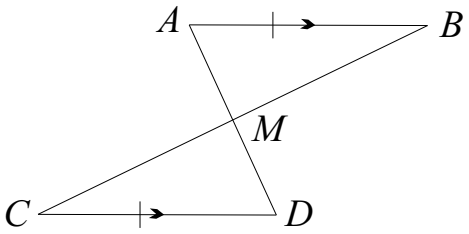
Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Simplify $\left(\dfrac{m^2 n^{\frac{1}{2}}}{n^{\frac{3}{2}}}\right)^2$.

(b) I think of a number, double it, then add 5. The result is 17. Write an equation and solve it to find the number.

(c)



In the diagram above, AB is equal in length and parallel to CD. Write a formal proof to show that $\triangle ABM$ is congruent to $\triangle DCM$.

(d) (i) Solve $3 - 4m < 31$.

(ii) Graph your solution on a number line.

(e) Make x the subject of the equation $y = 2x + 3$.

(f) Find the value of s if:

$$v^2 = u^2 + 2as,$$

given $v = 5, u = 4$ and $a = 10$.

Examination continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a) The number of downloads per day of a music track for two bands *Plastic Cutlery* and *Explosion* were analysed over a two week period. This data is displayed below:

Plastic Cutlery: 2, 4, 7, 11, 13, 15, 17, 18, 19, 20, 24, 26, 26, 32
Explosion: 8, 12, 15, 15, 22, 24, 25, 26, 27, 28, 29, 30, 35, 38

(i) Draw a back-to-back stem-and-leaf plot to represent the data above.

(ii) Which band is more popular? Justify your answer.

(b) Expand and simplify $5(4w - 8) - 4(8w - 6)$.

(c) If $a = \dfrac{3}{2}$ and $b = \dfrac{4}{5}$, evaluate $a^2 - b^{-2}$.

(d) Make x the subject of the equation $y = \dfrac{2x + 1}{x - 1}$.

(e) A bank teller notices that he has 50 coins all of which are 5 cent or 10 cent coins. He finds that the value of the coins is \$4.20. Let t be the number of 10 cent coins. Write an equation and solve it to find the number of 10 cent coins.

Examination continues overleaf ...

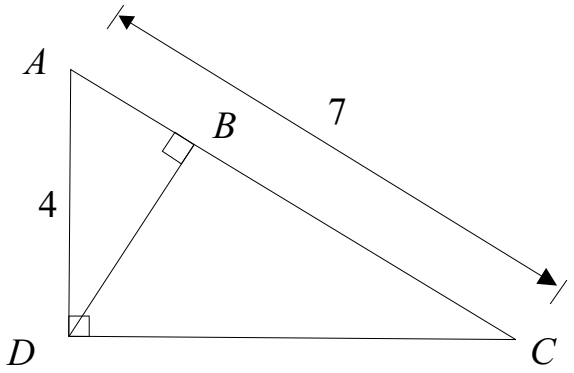
QUESTION SEVEN (13 marks) Start a new page.

(a) Expand and simplify:

(i) $8x^7\left(x^3 - 7x^8\right)$

(ii) $\left(x^9 - x^4\right)^2$

(b)



In the diagram above, $AD = 4$ units and $AC = 7$ units.

(i) Prove that $\triangle ABD \;|||\; \triangle ADC$.

(ii) Find the length of AB, giving a geometrical reason.

(c) Expand and simplify $\left(\dfrac{3x}{4} - 5\right)^2 + \left(\dfrac{x}{3} + 4\right)^2$.

(d) A car travels between Town A and Town B at an average speed of 60 km/h. If the car increased its average speed to 100 km/h it would take 10 minutes less to make the trip. By forming an equation and solving it, determine how far it is between the towns.

Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $\sqrt{4{\cdot}9 \times 10^{-7}}$. Give your answer in scientific notation.

(b) Simplify $\dfrac{1}{\sqrt{5}-\sqrt{3}} - \dfrac{1}{\sqrt{7}+\sqrt{5}}$. Give your answer with a rational denominator.

(c) Solve $\dfrac{2(3x+2)}{5} - \dfrac{6(1-x)}{7} = x$.

(d) Mandi went shopping with all of her birthday money. First she bought three tops for the same price each and then spent a third of what she had left on a pair of jeans. Then she bought a pair of shorts for \$20. The last item she bought was a jacket that cost three-quarters of what she had left. After buying the jacket she went home with \$12. If each top costs half the price of the jacket, find:

(i) the price of the jacket.

(ii) how much birthday money Mandi received.

(e) If $x^a = y^b = (xy)^c$, prove that $ab = c(a+b)$ for $x, y > 1$.

END OF EXAMINATION

Form III HY Soln

Question 1

a) 81 ✓

b) $24x^{10}$ ✓

c) $7x - 42$ ✓

d) $x^2 = 13^2 - 5^2$
$x^2 = 169 - 25$
$x^2 = 144$
$x = 12$ ✓

e) i) $\sqrt{50} = 5\sqrt{2}$ ✓

ii) $5\sqrt{5}$ ✓

iii) $\sqrt{15}$ ✓

f) i) $15a - 2b$ ✓

ii) $7y$ ✓

g) i) $a^2 - 4a = a(a-4)$ ✓

ii) $12uv + 8u = 4u(3v + 2)$ ✓

h) $a = 130$ [straight angle]
✓ ✓

Question 2

a) $16^{\frac{1}{2}} = \sqrt{16}$
$= 4$ ✓

b) $x^{-1} = \frac{1}{x}$ ✓

c) i) 8.54×10^4 ✓

ii) 6.49×10^{-2} ✓

d) i) $\frac{7+2+5+7+7+3+4+1+9}{9}$

$= \frac{4\times 7 + 7 + 10}{9}$

$= \frac{45}{9}$

$= 5$ ✓

ii) ~~1 2~~ 1 2 3 4 5 7 7 7 9

Median is 5 ✓

iii) 7 ✓

iv) 8 ✓

e) $\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ ✓

f) i) $(a+2)(a+4) = a^2 + 4a + 2a + 8$ ✓
$= a^2 + 6a + 8$ ✓

ii) $(2x-6)^2 = (2x-6)(2x-6)$
$= 4x^2 - 12x - 12x + 36$ ✓
$= 4x^2 - 24x + 36$ ✓

Question 3

a) $\frac{21p^8}{82} \times \frac{\cancel{4}1}{\cancel{p}1} = \frac{21p}{2}$ ✓ coefficient ✓ pronumeral

b) i) $x^2 + 7x + 12 = (x+3)(x+4)$ ✓

ii) $x^2 - 5x - 14 = (x+2)(x-7)$ ✓

c) i) 2 cars ✓

ii)
$$\begin{array}{r} 401 \\ 1854 \\ 12105 \\ \hline 4360 \end{array} \quad +$$ ✓ ✓

households

4360 have less than 3 cars

d) i) $2m + 9 = 25$

$2m = 16$ ✓

$\therefore m = 8$ ✓

ii) $2x + 12 = 6x + 7$

$6x + 7 = 2x + 12$

$4x + 7 = 12$ ✓

$4x = 5$

$\therefore x = \frac{5}{4}$ ✓

iii) $5(p-3) = 6(p+2)$ ✓

$5p - 15 = 6p + 12$

$6p + 12 = 5p - 15$

$p + 12 = -15$

$\therefore p = -27$ ✓

Question 4

a) i) $62 - 30 = 32$ yrs ✓

ii) $\frac{47+48}{2} = 47\frac{1}{2}$ yrs ✓

b) i) $(2\sqrt{3})^2 = 4 \times 3$
$= 12$ ✓

ii) $\sqrt{6}(2\sqrt{6} - 3\sqrt{2})$
$= 12 - 3\sqrt{12}$ ✓
$= 12 - 6\sqrt{3}$ ✓

c) $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4}$
$= \frac{\sqrt{7}+2}{3}$ ✓

d) i) $b^2 - 100 = (b+10)(b-10)$ ✓

ii) $5a^2 - 60a + 180 = 5(a^2 - 12a + 36)$ ✓
$= 5(a-6)(a-6)$
$= 5(a-6)^2$ ✓

e) i) $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2$
$= (2)^2$
$= 4$ ✓

ii) $\left(\frac{1}{49}\right)^{-\frac{1}{2}} = (49)^{\frac{1}{2}}$

$= \sqrt{49}$

$= 7$ ✓

f) $\frac{x}{18} = \frac{25}{15}$ ✓ {matching sides of similar triangles}

$\frac{x}{18} = \frac{5}{3}$

$x = \frac{5 \times 18^{6}}{\cancel{3}_{1}}$

$x = 30$ ✓

Question 5

a) $\left(\frac{m^2 n^{1/2}}{n^{3/2}}\right)^2 = \frac{m^4 n}{n^3}$ ✓

$= \frac{m^4}{n^2}$ ✓

b) Let the number be n ✓

$2n + 5 = 17$

$2n = 12$ ✓

$\therefore n = 6$ ✓

$\therefore$ the number is 6.

c) In $\triangle$'s ABM, DCM

AB = DC [given] ✓

$\angle MAB = \angle MDC$ [alternate angles, AB || CD]

$\angle AMB = \angle DMC$ [vertically opposite angles] ✓

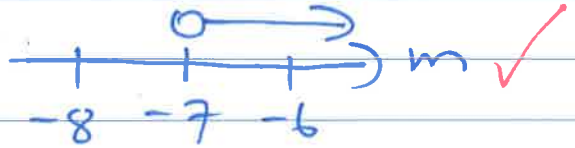
$\therefore \triangle ABM \equiv \triangle DCM$ [AAS] ✓

d) i) $3 - 4m < 31$

$-4m < 28$ ✓

$m > -7$ ✓

ii)



✓

e) $y = 2x + 3$

$2x + 3 = y$

$2x = y - 3$

$x = \frac{y - 3}{2}$ ✓

f) $v = 5, u = 4, a = 10$

$$v^2 = u^2 + 2as$$
$$(5)^2 = (4)^2 + 2(10)s \quad \checkmark$$
$$25 = 16 + 20s$$
$$20s + 16 = 25$$
$$20s = 9$$
$$s = \frac{9}{20} \quad \checkmark$$

Question 6

a) i)

Plastic Cutlery		Explosion
7 4 2	0	8
9 8 7 5 3 1	1	2 5 5
6 6 4 0	2	2 4 5 6 7 8 9
2	3	0 5 8

✓ LHS
✓ RHS ✓ ordered

1|2 = 12

ii) Explosion is more popular as it has a greater ✓ average (mean or median). ✓

b) $5(4w-8)-4(8w-6)$
$= 20w - 40 - 32w + 24$ ✓
$= -12w - 16$ ✓

c) $a = \frac{3}{2}$, $b = \frac{4}{5}$

$a^2 - b^{-2} = \left(\frac{3}{2}\right)^2 - \left(\frac{4}{5}\right)^{-2}$

$= \frac{9}{4} - \left(\frac{5}{4}\right)^2$

$= \frac{9}{4} - \frac{25}{16}$ ✓

$= \frac{36-25}{16}$

$= \frac{11}{16}$ ✓

d) $y = \frac{2x+1}{x-1}$

$xy - y = 2x + 1$ ✓

$xy - 2x = y + 1$

$x(y-2) = y + 1$

$x = \frac{y+1}{y-2}$ ✓

e) $10t + 5(50-t) = 420$ ✓

$10t + 250 - 5t = 420$

$5t + 250 = 420$ ✓

$5t = 170$

$\therefore t = 34$

$\therefore$ The teller has 34 ten cent coins.

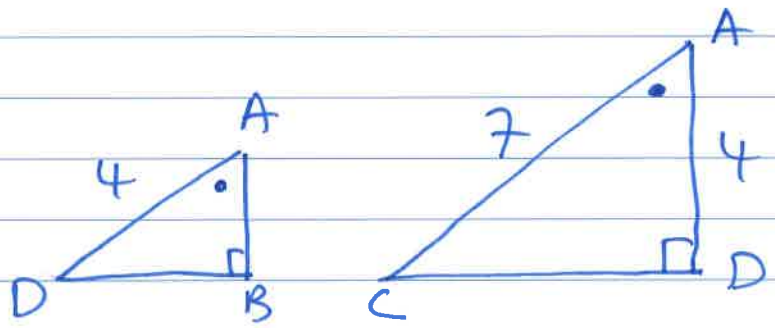
✓

Question 7

a) i) $8x^7(x^3 - 7x^8) = 8x^{10} - 56x^{15}$ ✓

ii) $(x^9 - x^4)^2 = (x^9 - x^4)(x^9 - x^4)$
$= x^{18} - x^{13} - x^{13} + x^8$ ✓
$= x^{18} - 2x^{13} + x^8$ ✓

b)



i) In $\triangle$'s ABD, ADC

$\angle ABD = \angle ADC = 90°$ [given]
$\angle DAB = \angle CAD$ [common] ✓

$\therefore \triangle ABD \,|||\, \triangle ADC$ [AA] ✓

ii) $\frac{AB}{4} = \frac{4}{7}$ [matching sides of similar triangles are in the same ratio] ✓ {reason}

$\therefore AB = \frac{16}{7}$ u ✓

c) $\left(\frac{3x}{4} - 5\right)^2 + \left(\frac{x}{3} + 4\right)^2$

$= \frac{9x^2}{16} - \frac{15x}{2} + 25 + \frac{x^2}{9} + \frac{8x}{3} + 16$ ✓

$= \left(\frac{9}{16} + \frac{1}{9}\right)x^2 + \left(\frac{8}{3} - \frac{15}{2}\right)x + 41$ ✓

$= \frac{81+16}{144}x^2 + \frac{16-45}{6}x + 41$

$= \frac{97x^2}{144} - \frac{29x}{6} + 41$ ✓

d) $s = \frac{d}{t}$

$t = \frac{d}{s}$

Let d be the distance between towns.

$\frac{d}{60} = \frac{d}{100} + \frac{10}{60}$ ✓

$d = \frac{3d}{5} + 10$

$5d = 3d + 50$ ✓

$2d = 50$

$d = 25$

∴ It is 25 km between towns.

✓

Question 8

a) $\sqrt{4.9 \times 10^{-7}} = (4.9 \times 10^{-7})^{1/2}$

$= (49 \times 10^{-8})^{1/2}$

$= 7 \times 10^{-4}$ ✓

b) $\frac{1}{\sqrt{5}-\sqrt{3}} - \frac{1}{\sqrt{7}+\sqrt{5}}$

$= \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \frac{\sqrt{5}+\sqrt{3}}{2} - \frac{\sqrt{7}-\sqrt{5}}{2}$ ✓

$= \frac{\sqrt{5}+\sqrt{3}-\sqrt{7}+\sqrt{5}}{2}$

$= \frac{2\sqrt{5}+\sqrt{3}-\sqrt{7}}{2}$ ✓

c) $\frac{2(3x+2)}{5} - \frac{6(1-x)}{7} = x$

$14(3x+2) - 30(1-x) = 35x$ ✓

$42x + 28 - 30 + 30x = 35x$

$72x - 2 = 35x$

$37x = 2$

$\therefore x = \frac{2}{37}$ ✓

d) Let M be birthday money
t be cost of top
j be cost of jeans
s be cost of shirts
k be cost of jacket

i) Now, $2t = k$

$$M - 3t - j - s - k = 12$$

$$\frac{M - 3t}{3} = j$$

$$s = 20$$

$$\frac{3}{4}(M - 3t - j - s) = k$$

So $\frac{3}{4}(M - 3t - j - s) - \frac{3}{4}k = \frac{3}{4}(12)$

$$k - \frac{3}{4}k = 9 \quad \checkmark$$

$$\frac{k}{4} = 9$$

$$k = \$36$$

$\therefore$ jacket costs \$36 $\checkmark$

ii) $k = 36$

$t = 18$

$s = 20$

$j = ?$

$$\frac{M - 3t}{3} = j$$

$$\frac{M - 54}{3} = j$$ ✓

$\therefore$ $M - 3t - j - s - k = 12$

$$M - 54 - \left(\frac{M-54}{3}\right) - 20 - 36 = 12$$ ✓

$$M - 110 - \left(\frac{M-54}{3}\right) = 12$$

$$M - \left(\frac{M-54}{3}\right) = 122$$

$$3M - M + 54 = 366$$

$$2M + 54 = 366$$

$$2M = 312$$

$$M = 156$$

$\therefore$ Mandi has $156 birthday money. ✓

ALTERNATIVE SOLUTION

Q7

d) i) left over = \$12 ← $\frac{1}{4}$ x 'what's left before jacket'

∴ jacket cost = 3 × \$12 ← $\frac{3}{4}$ x 'what's left before jacket'

= \$36

ii) Working backwards!

left over = \$12
jacket cost = \$36
shorts cost = \$20
] \$68 ← $\frac{2}{5}$ x 'what's left before jeans'

∴ jeans cost = \$68 ÷ 2 ← $\frac{1}{3}$ x 'what's left before jeans'

= \$34

3 tops cost = $3 \times \frac{\$36}{2}$

= \$54

∴ TOTAL = \$68 + \$34 + \$54

= \$156

e) $x^a = y^b = (xy)^c$

$x^a = (xy)^c$

$x^{ab} = (xy)^{cb}$ ✓

$x^{ab} = x^{cb} y^{cb}$

$x^{ab} = (x^b y^b)^c$

$x^{ab} = (x^b x^a)^c$ ✓

$x^{ab} = (x^{b+a})^c$

$x^{ab} = x^{c(a+b)}$

$\therefore ab = c(a+b)$. ✓