NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL



2017 Half-Yearly Examination

FORM III

MATHEMATICS

Wednesday 17th May 2017

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Calculators are not to be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: GMC	3B: SDP	3C: FMW	3D: LJF
3E: REJ/LRP	3F: LYL	3G: WJM	3H: CMDB
3I: NJL	3J: LL		

Checklist

- Writing paper required.
- Candidature — 188 boys

Examiner

CMDB

QUESTION ONE (13 marks) Start a new page.

(a) Simplify:

(i) $\sqrt{50}$

(ii) $4\sqrt{2} \times 3\sqrt{5}$

(b) Simplify:

(i) $2x - 3y + 4x$

(ii) $18p^2q \div 6pq$

(c) Evaluate $2p - 3q$, given that $p = 5$ and $q = 4$.

(d) Solve $3x + 5 = 17$.

(e) Write the following in scientific notation:

(i) 26 391

(ii) 0·004 17

(f) (i) Solve $x - 4 \geq 3$.

(ii) Graph your solution on a number line.

(g) Factorise $5p + 10$.

(h) Find the mode of $14, 15, 15, 17, 19, 20$.

(i) Expand and simplify $(x + 2)(x - 2)$.

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Write an expression for twice the sum of x and 3.

(b) Write 3·4812 correct to three significant figures.

(c) Simplify $6y^0$

(d) Evaluate:

(i) $8^{\frac{1}{3}}$

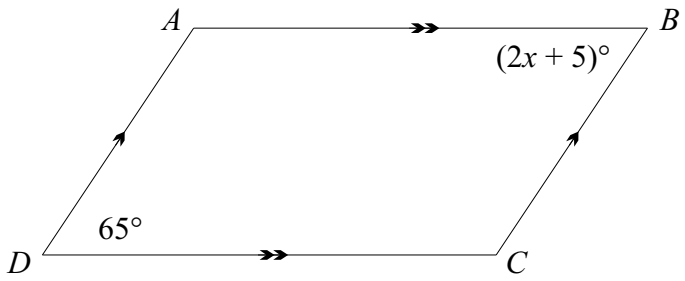
(ii) $\left(2\sqrt{5}\right)^2$

(e) Expand and simplify:

(i) $(x+5)(2x+1)$

(ii) $(x+6)^2$

(f)



Find the value of x in the above diagram, giving a geometric reason.

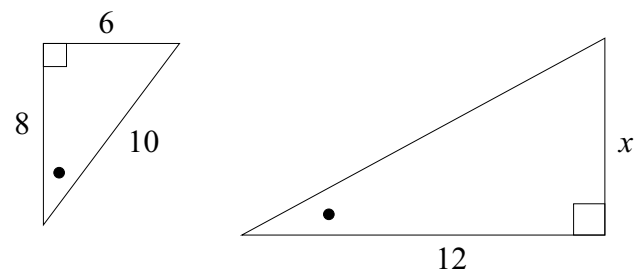
(g) Jim and Kevin are selling raffle tickets. They sell a total of 104 tickets, with Jim selling 32 tickets more than Kevin.

Let x be the number of tickets that Kevin sells.

(i) Write an equation to represent the information above.

(ii) Hence find the number of tickets sold by Kevin.

(h)



The diagram above shows two similar triangles.

Find the value of x, giving a geometric reason.

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

(a) Factorise fully:

(i) $14x^2 - 7x$

(ii) $x^2 - 36$

(iii) $x^2 + 9x - 10$

(b) Simplify $3\sqrt{50} - 4\sqrt{32} + 2\sqrt{27}$.

(c) Simplify:

(i) $p^{12} \div p^3$

(ii) $\left(3p^5\right)^2$

(iii) $4x^5y^{-2} \times 2xy^6$

(d) Consider the formula $y = wx + z$.

(i) Make x the subject of the formula.

(ii) Hence find the value of x when $y = 20$, $w = 8$ and $z = -12$.

(e) Express $\dfrac{5}{\sqrt{6}}$ with a rational denominator.

(f) Solve $\dfrac{7 - 2x}{3} < -5$.

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) Expand and simplify $\left(2\sqrt{3}+5\right)^2$.

(b) Express the following using positive indices:

(i) $x^{-4}y^{-1}z^2$

(ii) $\left(\dfrac{3a}{b^{-5}}\right)^{-2}$

(c) Find the median of the following numbers:

21 24 17 29 28 24 32 28 23 27

(d) Find the interior angle sum of an octagon.

(e) Simplify:

(i) $\dfrac{2x^2}{5} \times \dfrac{10}{xy}$

(ii) $\dfrac{4x+12}{x^2-9}$

(f) Write $\dfrac{4}{3x} + \dfrac{2}{5}$ as a single fraction.

(g) Simplify $\left(4{\cdot}2 \times 10^6\right) \times \left(3 \times 10^5\right)$, giving your answer in scientific notation.

Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

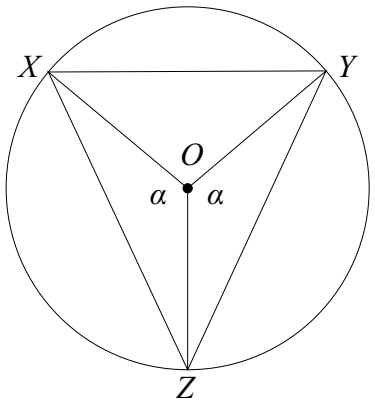
(a) Solve:

(i) $4(2x - 3) = 6x + 20$

(ii) $\dfrac{x+2}{3} + \dfrac{x-1}{4} = 5$

(b) Four parcels are weighed. The weights have a mode of 8 kg, median of 9 kg and mean of 10 kg. Find the range.

(c)



The diagram above has a circle with centre O. There are three points, X, Y and Z, on the circle. Let $\angle XOZ = \angle YOZ = \alpha$.

(i) Prove that $\triangle XOZ$ is congruent to $\triangle YOZ$.

(ii) Hence prove that $\triangle XYZ$ is isosceles.

(d) Simplify $\dfrac{x^2 - 9}{8x + 24} \div \dfrac{x^2 + x - 12}{2x^2 + 8x}$.

Examination continues next page ...

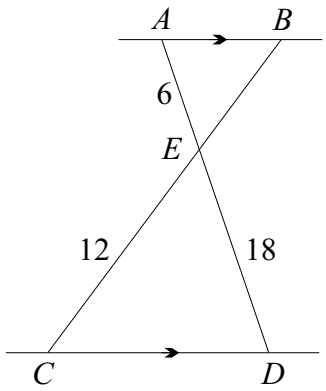
QUESTION SIX (13 marks) Start a new page.

(a) Express $\dfrac{\sqrt{3}}{\sqrt{6}+2}$ with a rational denominator.

(b) Factorise fully $2x^2 + 10x - 48$.

(c) Find the values of a and b if $\left(5\sqrt{2}-3\right)\left(\sqrt{2}-1\right) = a - b\sqrt{2}$, where a and b are integers.

(d)



In the diagram above, $AB \parallel CD$, $AE = 6$, $EC = 12$ and $ED = 18$.

(i) Prove that the triangles above are similar, giving all geometric reasons.

(ii) Find the value of BE.

(e) Make m the subject of $mp = \dfrac{mn}{p} + 2k$.

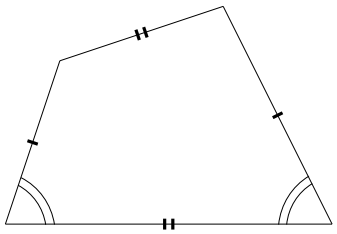
(f) Write $0{\cdot}4\dot{8}\dot{3}$ as a fraction in simplest form.

Examination continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

(a) Expand and simplify $2(x-3)(x+3)-(4x+3)^2$.

(b)



NOT TO SCALE

The diagram above shows one of the special quadrilaterals. Use the properties of the special quadrilaterals to identify which quadrilateral it must be.

(c) Express $3^7+3^7+3^7$ in the form a^b, where a is a prime number.

(d) Solve $\dfrac{3}{4}(x+3)-\dfrac{5}{6}(x+7)=\dfrac{2}{3}$.

(e) After 8 tests, Joel has an average mark of 23·5. What mark does he need to get in his next test to lift his average to 25?

(f) If $a(c+d)+b(c+d)=72$ and $c+d=6$, what is the value of $a+b+c+d$?

(g) Michael is four times as old as Angela. Seven years ago, the sum of their ages was 56.

(i) Write this information as an equation.

(ii) Hence find Angela's age.

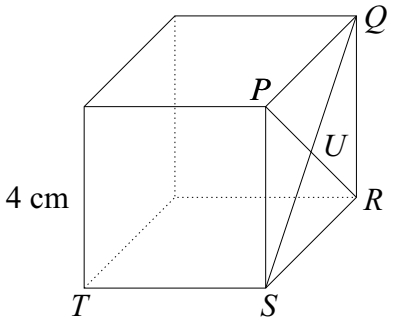
Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $9^m \times 3^{m-2}$.

(b) Simplify $\sqrt[4]{p \times \sqrt[3]{p^{-1}}}$, giving your answer in surd form.

(c)



The diagram above shows a cube with side length 4 cm. The diagonals of $PQRS$ intersect at U.

(i) Find the length of SU, leaving your answer as a simplified surd.

(ii) Hence, find the length of TU, leaving your answer as a simplified surd.

(d) Write $\dfrac{x^{-1}+y}{x+y^{-1}}$ in simplest form without negative indices.

(e) The diagonals of a rhombus with side length 40 cm are in the ratio 1 : 2. Find the lengths of its diagonals. State any properties of special quadrilaterals that you use.

(f) A train which is 15 metres long travels at a constant speed of 18 kilometres per hour. Adam and Bob are running parallel to the train tracks. Adam is running in the same direction as the train, while Bob is running towards the train. They are both running at a constant speed. The train takes 6 seconds to pass Adam. It meets Bob 32 seconds later, taking 2 seconds to pass him. How long after the train passes Bob will the runners meet?

END OF EXAMINATION

Form III Half-Yearly 2017 Solutions

Question One

a) (i) $\sqrt{50} = 5\sqrt{2}$ ✓

(ii) $4\sqrt{2} \times 3\sqrt{5} = 12\sqrt{10}$ ✓

b) (i) $2x - 3y + 4x = 6x - 3y$ ✓

(ii) $18p^2q \div 6pq = 3p$ ✓

c) $2 \times 5 - 3 \times 4 = 10 - 12$

$= -2$ ✓

d) $3x + 5 = 17$

$3x = 12$

$x = 4$ ✓

e) (i) 2.6391×10^4 ✓

(ii) 4.17×10^{-3} ✓

f) (i) $x - 4 \geq 3$

$x \geq 7$ ✓ ✓

(ii)



g) $5p + 10 = 5(p + 2)$ ✓

h) mode = 15 ✓

i) $(x + 2)(x - 2) = x^2 - 4$ ✓

Question Two

a) $2(x+3)$ or $2x+6$ ✓

b) $3 \cdot 48$ ✓ ✓

c) $6y^0 = 6$ ✓

d) (i) $8^{\frac{1}{3}} = \sqrt[3]{8}$
$= 2$ ✓

(ii) $(2\sqrt{5})^2 = 4 \times 5$
$= 20$ ✓

e) (i) $(x+5)(2x+1) = 2x^2 + x + 10x + 5$
$= 2x^2 + 11x + 5$ ✓

(ii) $(x+6)^2 = x^2 + 12x + 36$ ✓

f) $2x + 5 = 65$ (opposite $\angle$'s of parallelogram are equal) ✓ reason
$2x = 60$
$x = 30^\circ$ ✓

alternative:
$\angle DCB = 180 - 65$ (co-interior $\angle$'s, $AD \parallel BC$)
or $\angle DAB$ $= 115^\circ$ $AB \parallel DC$ if finding $\angle DAB$
$2x + 5 = 180 - 115$ (co-interior $\angle$'s, $AB \parallel DC$)
$= 65$
$2x = 60$
$x = 30^\circ$

Both reasons required, specifying parallel lines, if using this method.

g) (i) $x + x + 32 = 104$ } ✓ either line
$2x + 32 = 104$

(ii) $2x = 72$
$x = 36$ ✓

∴ Kevin sold 36 tickets.

h) $\frac{x}{6} = \frac{12}{8}$ (matching sides of similar Δ's are in the same ratio) ✓ reason

$x = 6 \times \frac{12}{8}$
$= 9$ ✓

Question Three

a). (i). $14x^2 - 7x = 7x(2x-1)$ ✓

(ii). $x^2 - 36 = (x-6)(x+6)$ ✓

(iii). $x^2 + 9x - 10 = (x+10)(x-1)$ ✓

b). $3\sqrt{50} - 4\sqrt{32} + 2\sqrt{27} = 3 \times 5\sqrt{2} - 4 \times 4\sqrt{2} + 2 \times 3\sqrt{3}$ ✓

$= 15\sqrt{2} - 16\sqrt{2} + 6\sqrt{3}$

$= 6\sqrt{3} - \sqrt{2}$ ✓

c). (i). p^9 ✓

(ii). $9p^{10}$ ✓

(iii). $8x^6y^4$ ✓

d). (i). $y = wx + z$

$y - z = wx$

$x = \frac{y-z}{w}$ ✓

OR $y - wx = z$

$-wx = z - y$

$x = \frac{z-y}{-w}$

(ii). $x = \frac{20 - -12}{8}$

$= \frac{32}{8}$

$= 4$ ✓

e). $\frac{5}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$ ✓

f). $7 - 2x < -15$ ✓

$-2x < -22$ ✓

$x > 11$

Question Four

a). $(2\sqrt{3}+5)^2 = 4\times 3 + 2\times 10\sqrt{3} + 25$

$= 12 + 20\sqrt{3} + 25$ ✓

$= 20\sqrt{3} + 37$ ✓

b) (i). $x^{-4}y^{-1}z^2 = \frac{z^2}{x^4 y}$ ✓

(ii). $\left(\frac{3a}{b^{-5}}\right)^{-2} = \left(\frac{b^{-5}}{3a}\right)^2$

$= \frac{b^{-10}}{9a^2}$ ✓ for $9a^2$

$= \frac{1}{9a^2 b^{10}}$ ✓ for b^{10}

c). ~~17~~ ~~21~~ ~~23~~ 24 (24 27) ~~28~~ ~~28~~ ~~29~~ 32

median $= \frac{24+27}{2}$

$= 25.5$ ✓ fractions ok

d). $S = 180(8-2)$

$= 180\times 6$ ✓ don't have to use rule, any method accepted.

$= 1080°$

e). (i). $\frac{2x^2}{5} \times \frac{10^2}{xy} = \frac{4x}{y}$ ✓

(ii). $\frac{4x+12}{x^2-9} = \frac{4(x+3)}{(x-3)(x+3)}$ ✓

$= \frac{4}{x-3}$ ✓

(iii). $\frac{4}{3x} + \frac{2}{5} = \frac{5\times 4 + 2\times 3x}{15x}$

$= \frac{20+6x}{15x}$ ✓

f). $(4.2\times 10^6)\times(3\times 10^5) = 12.6\times 10^{11}$ ✓

$= 1.26\times 10^{12}$ ✓

Question Five

a) (i) $4(2x-3) = 6x+20$

$8x - 12 = 6x + 20$ ✓

$2x - 12 = 20$

$2x = 32$ ✓

$x = 16$

(ii) $\frac{x+2}{3} + \frac{x-1}{4} = 5$

$4(x+2) + 3(x-1) = 12 \times 5$ ✓

$4x + 8 + 3x - 3 = 60$

$7x + 5 = 60$

$7x = 55$

$x = \frac{55}{7}$ ✓

b) 8 8 10 14 ✓

range $= 14 - 8$ ✓

$= 6$

c) (i) In $\triangle XOZ$ and $\triangle YOZ$:

OZ is common.

$OX = OY$ (equal radii) ✓

$\angle XOZ = \angle YOZ = \alpha$ (given) ✓

$\therefore \triangle XOZ \equiv \triangle YOZ$ (SAS) ✓

2 marks for all three. Lose a mark for each incorrect.

(ii) $XZ = YZ$ (matching sides of congruent $\triangle$'s) ✓

$\therefore \triangle XYZ$ is isosceles (pair of equal sides)

d) $\frac{x^2-9}{8x+24} \div \frac{x^2+x-12}{2x^2+8x} = \frac{x^2-9}{8x+24} \times \frac{2x^2+8x}{x^2+x-12}$

$= \frac{(x-3)(x+3)}{8(x+3)} \times \frac{2x(x+4)}{(x+4)(x-3)}$ ✓ ✓

$= \frac{x}{4}$ ✓

Question Six

a). $\frac{\sqrt{3}}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2} = \frac{\sqrt{18}-2\sqrt{3}}{6-4}$ ✓

$= \frac{3\sqrt{2}-2\sqrt{3}}{2}$ ✓

b). $2x^2+10x-48 = 2(x^2+5x-24)$ ✓

$= 2(x+8)(x-3)$ ✓

c). $(5\sqrt{2}-3)(\sqrt{2}-1) = 10-5\sqrt{2}-3\sqrt{2}+3$

$= 13-8\sqrt{2}$ ✓

$\therefore a = 13, \quad b = 8$ ✓

d). (i). In $\triangle ABE$ and $\triangle DCE$:

$\angle AEB = \angle DEC$ (vertically opposite $\angle$'s) } ✓

$\angle ABE = \angle DCE$ (alternate $\angle$'s, $AB \parallel CD$)

$\therefore \triangle ABE \,|||\, \triangle DCE$ (AA) ✓

(ii). $\frac{BE}{12} = \frac{6}{18}$ (matching sides of similar $\triangle$'s)

$BE = 12 \times \frac{6}{18}$

$= 4$ ✓

e). $mp = \frac{mn}{p} + 2k$

$mp^2 = mn + 2kp$

$mp^2 - mn = 2kp$ ✓

$m(p^2-n) = 2kp$

$m = \frac{2kp}{p^2-n}$ ✓

f). Let $x = 0.4\dot{8}\dot{3}$

$10x = 4.\dot{8}\dot{3}$ } ✓

$1000x = 483.\dot{8}\dot{3}$

$990x = 479$

$x = \frac{479}{990}$ ✓

Question Seven

a). $2(x-3)(x+3) - (4x+3)^2$ ✓

$= 2(x^2 - 9) - (16x^2 + 24x + 9)$ ✓

$= 2x^2 - 18 - 16x^2 - 24x - 9$ ✓

$= -14x^2 - 24x - 27$

b). Opposite sides equal.
One pair of adjacent angles equal. ✓
Rectangle.

c). $3^7 + 3^7 + 3^7 = 3 \times 3^7$ ✓

$= 3^8$

d). $\frac{3}{4}(x+3) - \frac{5}{6}(x+7) = \frac{2}{3}$

$9(x+3) - 10(x+7) = 8$ ✓

$9x + 27 - 10x - 70 = 8$ ✓

$-x - 43 = 8$

$-x = 51$

$x = -51$ ✓

e). $\frac{8 \times 23.5 + x}{9} = 25$ ✓

$188 + x = 225$

$x = 37$ ✓

f). $a(c+d) + b(c+d) = 72$

$6a + 6b = 72$

$a + b = 12$

$a+b+c+d = 12+6$ ✓

$= 18$

$(c+d)(a+b) = 72$

$6(a+b) = 72$

$a+b = 12$

g). (i). Let x be Angela's age.

$4x$ is Michael's age.

$x - 7 + 4x - 7 = 56$ ✓ either

$5x - 14 = 56$

(ii).

$5x = 70$

$x = 14$ ✓

∴ Angela is 14.

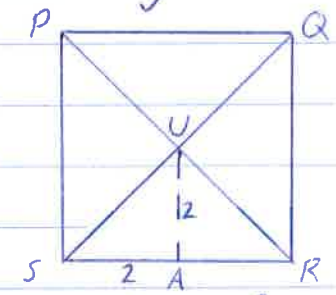
Question Eight

a) $9^m \times 3^{m-2} = (3^2)^m \times 3^{m-2}$
$= 3^{2m} \times 3^{m-2}$
$= 3^{3m-2}$ ✓

b) $\sqrt[4]{p \times \sqrt[3]{p^{-1}}} = \sqrt[4]{p \times p^{-1/3}}$
$= \sqrt[4]{p^{2/3}}$ ✓
$= (p^{2/3})^{1/4}$
$= p^{1/6}$
$= \sqrt[6]{p}$ ✓

c) (i) $SQ^2 = 4^2 + 4^2$
$= 32$
$SQ = \sqrt{32}$
$= 4\sqrt{2}$
$SU = \frac{1}{2} \times 4\sqrt{2}$
$= 2\sqrt{2}$ cm ✓ must have reason
(diagonals of rhombus bisect each other)

OR Let A be the point on SR directly below U.

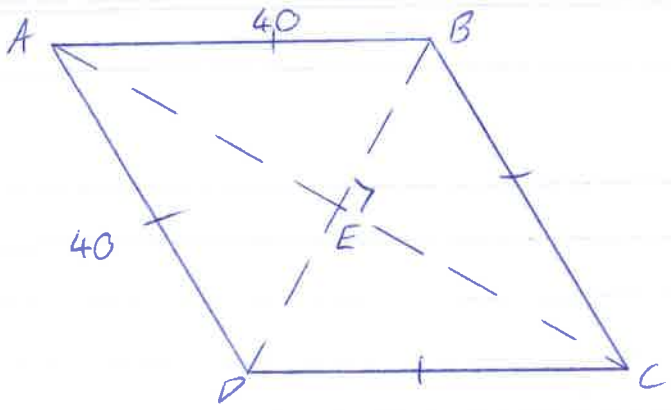


$SU^2 = SA^2 + UA^2$
$= 2^2 + 2^2$
$= 8$
$SU = \sqrt{8}$
$= 2\sqrt{2}$ cm

(ii) $TU^2 = TS^2 + SU^2$
$= 4^2 + (2\sqrt{2})^2$
$= 24$
$TU = \sqrt{24}$
$= 2\sqrt{6}$ cm ✓

d) $\frac{x^{-1}+y}{x+y^{-1}} = \frac{\frac{1}{x}+y}{x+\frac{1}{y}}$
$= \frac{\frac{1+xy}{x}}{\frac{xy+1}{y}}$ ✓
$= \frac{1+xy}{x} \times \frac{y}{xy+1}$
$= \frac{y}{x}$ ✓ Must have working to get any marks.

e).



/ reasons

Let $BE = x$.

$AE = 2x$ (diagonals of rhombus bisect each other)

$AB^2 = AE^2 + BE^2$ (diagonals of rhombus bisect at 90°)

$40^2 = (2x)^2 + x^2$ /

$1600 = 4x^2 + x^2$

$= 5x^2$

$x^2 = 320$

$x = \sqrt{320}$ $(x > 0)$

$= 8\sqrt{5}$

$BD = 16\sqrt{5}$ / no deduction if not simplified.

$AC = 32\sqrt{5}$

f). 18 km/h = 18 000 m/h
= 300 m/min
= 5 m/s



After 6s:

Train moves $6 \times 5 = 30$ m
Back of train must move 15m ahead of A to pass.
∴ Moves 15m past point where it meets A (30-15=15).
i.e. A runs 15m in 6s = 2.5 m/s ✓
32s later:
Front of train is $15 + 32 \times 5 = 175$ m further on.
A is $32 \times 2.5 = 80$ m further on.
Front of train is 95m ahead of A. ✓



In 2s: Train moves $2 \times 5 = 10$ m.
B must move 5m to make up the 15m to the back of the train.
B runs 5m in 2s = 2.5 m/s.
A runs $2 \times 2.5 = 5$ m.
A and B are $95 - 5 - 5 = 85$ m apart.
Same speed, so each must go 42.5m.
$42.5 \div 2.5 = 17$ seconds. ✓