NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL

2016 Half-Yearly Examination

FORM III

MATHEMATICS

Wednesday 18th May 2016

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Calculators are not to be used.

Total — 104 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

3A: LRP	3B: LYL	3C: WJM	3D: SJE
3E: DWH/BR	3F: SG/RCF	3G: SDP	3H: FMW
3I: PKH	3J: REJ		

Checklist

- Writing paper required.
- Candidature — 179 boys

Examiner

SDP

QUESTION ONE (13 marks) Start a new page.

(a) Simplify:

(i) $8x \times 3y$

(ii) $\sqrt{z^2}$

(b) Simplify $\sqrt{20}$.

(c) Factorise $xy - 2x$.

(d) Solve:

(i) $2x - 8 = 20$

(ii) $x + 5 < 15$

(e) Evaluate:

(i) 3^{-2}

(ii) $49^{\frac{1}{2}}$

(iii) 10^0

(f) Express the following numbers in scientific notation.

(i) 5 688

(ii) 0·000 823

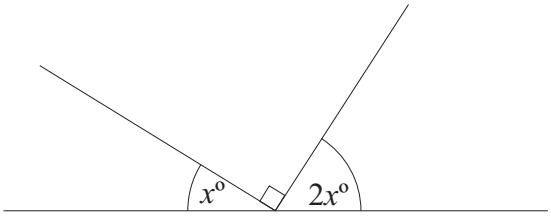
(g) Expand and simplify $(x + 2)(x + 5)$.

Examination continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Simplify $3\sqrt{3} + 2\sqrt{3}$.

(b)



Form and solve an equation to find the value of x in the above diagram, giving a geometric reason.

(c) Express the following numbers in decimal form, rounding to 3 significant figures.

(i) $4{\cdot}3842 \times 10^2$

(ii) $4{\cdot}07836 \times 10^{-4}$

(d) Below is a stem and leaf plot of results from a test, where 1|2 represents a mark of 12.

Stem	Leaf
1	2 2 6 8
2	3 5 7

Use the table of results to find the:

(i) mode

(ii) median

(iii) mean

(e) Factorise:

(i) $x^2 + 8x + 12$

(ii) $x^2 - 16$

(iii) $x^2 + 6x + 9$

Examination continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

(a) Simplify $(3x)^2 \times 2x^3$.

(b) Evaluate $\dfrac{a}{3b^2}$ for $a = 36$ and $b = -2$.

(c) (i) Solve $10 - 2x \leq 4$.

(ii) Graph your solution on a number line.

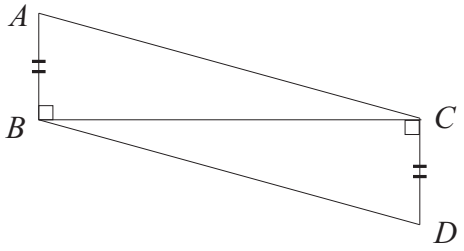
(d) Write each of the following as a single power of x.

(i) $\sqrt[3]{x}$

(ii) $\dfrac{1}{x^4}$

(iii) $\left(x^{\frac{1}{2}}\right)^5$

(e)



State which test should be used to show that $\triangle ABC$ is congruent to $\triangle DCB$. You need not prove congruence.

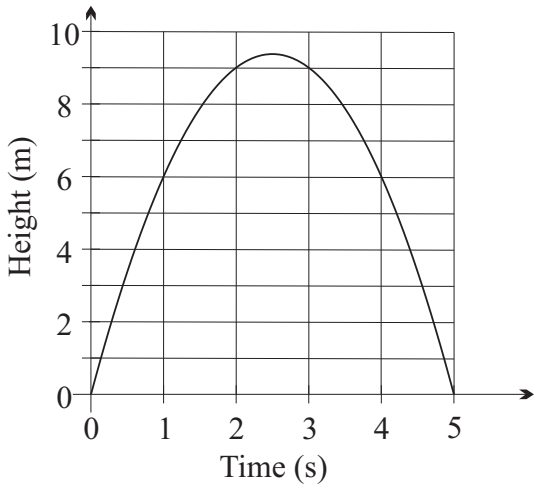
(f) Make x the subject of the equation $b = \dfrac{3x}{2}$.

(g) Express $\dfrac{2}{\sqrt{5}}$ with a rational denominator.

Examination continues next page ...

QUESTION FOUR (13 marks) Start a new page.

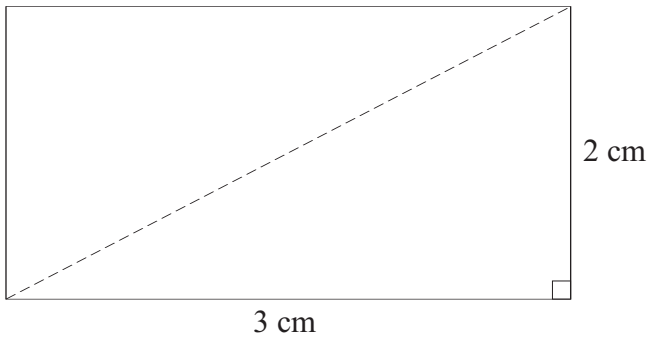
(a) The graph below shows the height of a ball thrown into the air.



(i) How high is the object at 1 second?

(ii) For what length of time was the object above a height of 6 m?

(b)



A rectangle with length 3 cm and height 2 cm is shown above. Find the length of the diagonal, giving your answer in simplest exact form.

(c) Expand and simplify:

(i) $(3x+4)^2$

(ii) $(2x-3)(x-5)$

(d) Simplify:

(i) $\dfrac{20x-15y}{5}$

(ii) $\dfrac{x^2-5x+6}{2x-6}$

Examination continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Solve $4(y+5) = 42 - 2(y-4)$.

(b) Expand and simplify $(3+\sqrt{5})(3-\sqrt{5})$.

(c) Simplify, expressing your answers in scientific notation:

(i) $\dfrac{8 \times 10^{-10}}{2 \times 10^{8}}$

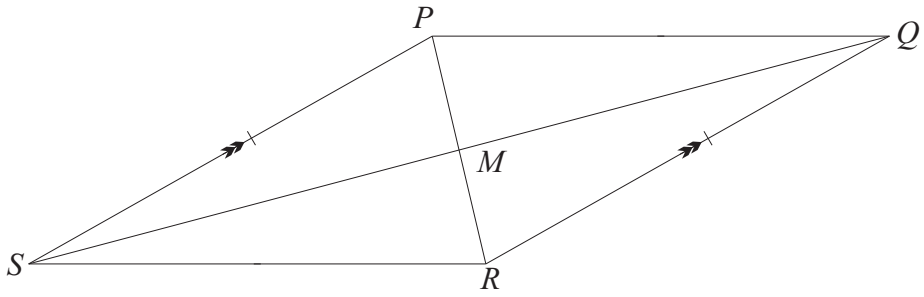
(ii) $\left(4 \times 10^{-5}\right)^{2}$

(d) What is the sum of the exterior angles of a polygon?

(e) Solve $3(x+5) = 48$.

(f) Evaluate $\left(\dfrac{1}{81}\right)^{-\frac{1}{4}}$.

(g) Below is a quadrilateral $PQRS$ with $PS \parallel QR$ and $PS = QR$. The diagonals PR and QS meet at M.



(i) Show that $\triangle PMS$ is congruent to $\triangle RMQ$. Clearly state your geometric reasons.

(ii) Hence prove that M is the midpoint of PR.

Examination continues next page ...

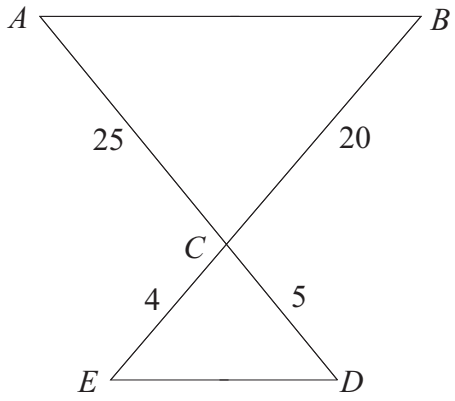
QUESTION SIX (13 marks) Start a new page.

(a) Make a the subject of the following:

(i) $b = \dfrac{a}{5} - b$

(ii) $s = ut + \dfrac{1}{2}at^2$

(b)



In the diagram above, $AC = 25$, $BC = 20$, $CD = 5$ and $CE = 4$.

(i) Prove $\triangle ABC \;|||\; \triangle DEC$.

(ii) Hence, prove that $AB \parallel DE$, giving a reason.

(c) Expand and simplify:

(i) $(x^2 - 5x)^2$

(ii) $\left(\dfrac{a}{3} + 4\right)\left(\dfrac{2a}{5} - 2\right)$

(d) Simplify $\dfrac{6}{2 + \sqrt{7}}$ by rationalising the denominator.

Examination continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

(a) Solve $\dfrac{20-3a}{5} \leq a-4$.

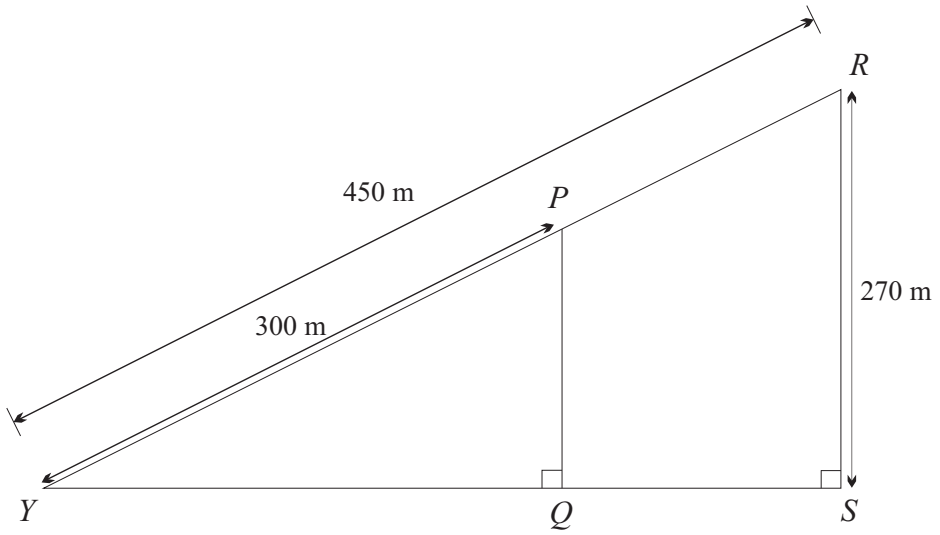
(b) Simplify $\dfrac{8a^{-4}}{9b^2} \times \dfrac{a^2b}{4b^{-2}}$. Express your answer with positive indices.

(c) Fully factorise:

(i) $-2x^2-16x-30$

(ii) $\dfrac{y^2}{3}-12$

(d)



From the wharf at Y, Sam looks at the tops of towers PQ and RS. The points Y, P and R lie in a straight line. Points Y, Q and S all lie on level ground. Using his laser measuring device, he measures the distance to the tops of the towers as YP= 300 m and YR= 450 m. Sam knows that the height of the taller tower RS is 270 m.

(i) Prove that $\triangle YRS$ is similar to $\triangle YPQ$.

(ii) Find the distance QS.

(e) Consider the formula

$$q = \left(\frac{r^2-s^2}{9p} + \sqrt[4]{10t}\right)^2.$$

Find all the values of p given $q = 196$, $r = 11$, $s = 7$ and $t = \dfrac{8}{5}$.

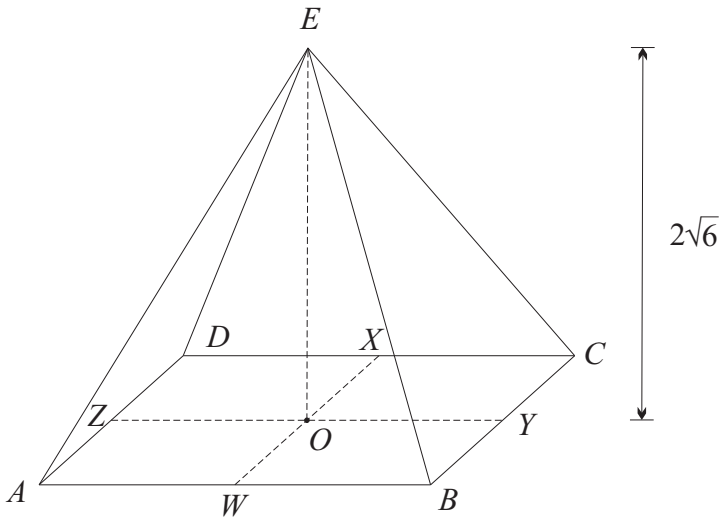
Examination continues next page ...

QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $\dfrac{x^2-x-6}{20x+100} \div \dfrac{x^2-3x}{5x^2+25x}$.

(b) Find which number is greater: 8^{200} or 2^{800}. Justify your answer.

(c)



The pyramid above has a square base $ABCD$ with area $8\,\text{cm}^2$ and a perpendicular height OE of $2\sqrt{6}\,\text{cm}$. The points W, Y, X and Z are the midpoints of AB, BC, CD and DA respectively.

(i) Find the length of OA.

(ii) Hence, find the length of edge AE, showing clear working.

(d) Write x as the subject of the equation:

$$\frac{x-3}{4} - \frac{2(x-y)}{3} + \frac{x+9}{12} = 1$$

(e) A cooling system of a car contains 16 litres of antifreeze, which is a solution of ethylene glycol and water.

Initially the solution contained 40% ethylene glycol. A certain amount of solution is drained from the system and topped back up to 16 litres with a 60% ethylene glycol solution, so that the final system contains 55% ethylene glycol.

Using an equation, determine how much of the original solution was drained.

END OF EXAMINATION

F3 HY Solutions

1ai) $24xy$ ✓

ii) z ✓ (accept $\pm z$)

b) $2\sqrt{5}$ ✓

c) $x(y-2)$ ✓

di) $2x - 8 = 20$ ✓
$2x = 28$ ✓
$x = 14$

ii) $x + 5 < 15$ ✓
$x < 10$

ci) $\frac{1}{9}$ ✓

ii) 7 ✓

iii) 1 ✓

gi) 5.688×10^{3} ✓

ii) 8.23×10^{-4} ✓

g) $x^2 + 7x + 10$ ✓

2a) $5\sqrt{3}$ ✓

b) $x + 90 + 2x = 180$ ✓ (angles on a straight line add to 180°) / (straight angle) ✓

$3x + 90 = 180$

$3x = 90$

$x = 30^\circ$ ✓

ci) 438 ✓

ii) 0·000408 ✓

di) 12 ✓

ii) 18 ✓

iii) $12+12+16+18+23+25+27 = 133$ ✓

$133 \div 7 = 19$ ✓

ei) $(x+2)(x+6)$ ✓

ii) $(x+4)(x-4)$ ✓

iii) $(x+3)^2$ ✓ allow $(x+3)(x+3)$

3a) $(3x)^2 \times 2x^3 = 9x^2 \times 2x^3$ ✓

$= 18x^5$ ✓

b) $\frac{36}{3\times(-2)^2} = \frac{36}{3\times 4}$ ✓ substitution

$= \frac{36}{12}$

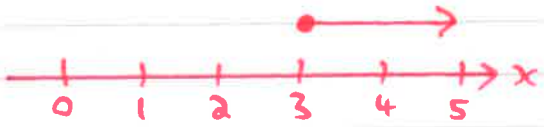
$= 3$ ✓

ci) $10 - 2x \leq 4$

$-2x \leq -6$

$x \geq 3$ ✓ number ✓ correct inequality

ii)



✓

di) $x^{\frac{1}{3}}$ ✓

ii) x^{-4} ✓

iii) $x^{\frac{5}{2}}$ ✓

e) SAS ✓

f) $b = \frac{3x}{2}$

$2b = 3x$

$x = \frac{2b}{3}$ ✓

g) $\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ ✓

4ai) 6m ✓

ii) 3 seconds ✓

b) $2^2 + 3^2 = x^2$ ✓ using pythagorus

$4 + 9 = x^2$

$x^2 = 13$

$x = \sqrt{13}$ cm ✓

ci) $(3x+4)(3x+4) = 9x^2 + 12x + 12x + 16$ ✓

$= 9x^2 + 24x + 16$ ✓

ii) $(2x-3)(x-5) = 2x^2 - 10x - 3x + 15$ ✓

$= 2x^2 - 13x + 15$ ✓

di) $\frac{20x - 15y}{5} = \frac{5(4x - 3y)}{5}$ ✓

$= 4x - 3y$ ✓

ii) $\frac{x^2 - 5x + 6}{2x - 6} = \frac{(x-3)(x-2)}{2(x-3)}$ ✓ ✓

$= \frac{x-2}{2}$ ✓

5a) $4(y+5) = 42 - 2(y-4)$

$4y + 20 = 42 - 2y + 8$ ✓ expand

$4y + 20 = 50 - 2y$

$6y = 30$

$y = 5$ ✓

b) $(3+\sqrt{5})(3-\sqrt{5}) = 9 - 5$

$= 4$ ✓

ci) $\frac{8 \times 10^{-10}}{2 \times 10^{8}} = 4 \times 10^{-18}$ ✓

ii) $(4 \times 10^{-5})^2 = 16 \times 10^{-10}$ ✓

$= 1.6 \times 10^{-9}$ ✓

d) 360° ✓

e) $3(x+5) = 48$

$x + 5 = 16$

$x = 11$ ✓

f) $\left(\frac{1}{81}\right)^{-\frac{1}{4}} = 81^{\frac{1}{4}}$ ✓ some correct working

$= \sqrt[4]{81}$

$= 3$ ✓

gi) in ΔPMS and ΔRMQ:

$PS = RQ$ (given)

$\angle MPS = \angle MRQ$ (alternate angles $PS \parallel QR$)

$\angle PMS = \angle RMQ$ (vertically opposite angles)

✓ 2 correct statements

so ΔPMS is congruent to ΔRMQ (AAS) ✓ with reason

ii) As ΔPMS is congruent to ΔRMQ,

$PM = RM$ (matching sides in congruent triangles)

therefore M is the midpoint of PR ✓

6ai) $b = \frac{a}{5} - b$

$2b = \frac{a}{5}$

$a = 10b$ ✓

ri) $s = ut + \frac{1}{2}at^2$

$s - ut = \frac{1}{2}at^2$

$2(s-ut) = at^2$ ✓ some correct working

$a = \frac{2(s-ut)}{t^2}$ (allow $\frac{2s-2ut}{t^2}$) ✓

bi) in $\triangle ABC$ and $\triangle DEC$

$\angle ACB = \angle DCE$ (vertically opposite angles)

$\frac{BC}{EC} = \frac{20}{4}$ (given)

$= 5$

$\frac{AC}{DC} = \frac{25}{5}$ (given) ✓ 2 correct

$= 5$

matching as $\wedge$ sides are in the same ratio

$\triangle ABC \,|||\, \triangle DEC$ (SAS) ✓ with reason

ii) As $\triangle ABC \,|||\, \triangle DEC$

$\angle CAB = \angle CDE$ so they are alternate angles ✓

which shows that $AB \parallel DE$

ci) $(x^2 - 5x)^2 = x^4 - 5x^3 - 5x^3 + 25x^2$ ✓

$= x^4 - 10x^3 + 25x^2$ ✓

ii) $\left(\frac{a}{3} + 4\right)\left(\frac{2a}{5} - 2\right) = \frac{2a^2}{15} - \frac{2a}{3} + \frac{8a}{5} - 8$ ✓

$= \frac{2}{15}a^2 - \frac{10a}{15} + \frac{24a}{15} - 8$ ✓

$= \frac{2}{15}a^2 + \frac{14}{15}a - 8$ ✓

or $\frac{2(a-5)(a+12)}{15}$ or $\frac{2a^2+14a-120}{15}$

d) $\frac{6}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}}$ ✓ $= \frac{12-6\sqrt{7}}{4-7}$

$= \frac{12-6\sqrt{7}}{-3}$

$= -4+2\sqrt{7}$ or $2\sqrt{7}-4$ ✓
or $2(\sqrt{7}-2)$ etc

7a) $\frac{20-3a}{5} \leq a-4$

$20-3a \leq 5a-20$ ✓

$40 \leq 8a$ ✓

$5 \leq a$

or $a \geq 5$

b) $\frac{8a^{-4}}{9b^2} \times \frac{a^2b}{4b^{-2}} = \frac{8}{9a^4b^2} \times \frac{a^2b^3}{4}$ ✓

$= \frac{8a^2b^3}{36a^4b^2}$

$= \frac{2b}{9a^2}$ ✓

ci) $-2x^2-16x-30 = -2(x^2+8x+15)$

$= -2(x+3)(x+5)$ ✓

ii) $\frac{y^2}{3} - 12 = \frac{1}{3}(y^2-36)$ ✓

$= \frac{1}{3}(y-6)(y+6)$ ✓

di) in ΔPYQ and ΔRYS

only 2 needed

$\angle PYQ = \angle RYS$ (shared)

$\angle PQY = \angle RSY$ (given right angles)

$\angle YPQ = \angle YRS$ (corresponding angles, $PQ \parallel RS$)

so $\Delta YRS \parallel\parallel \Delta YPQ$ (AAA) ✓

dii) $\frac{PQ}{270} = \frac{300}{450}$ (matching sides in similar triangles in the same ratio)

$PQ = 180m$ ✓

Method 1:

$YQ^2 = 300^2 - 180^2$ ✓

$= 90,000 - 32,400$

$= 57,600$

$YQ = \sqrt{57,600}$

$= \sqrt{576} \times \sqrt{100}$

$= \sqrt{4} \times \sqrt{144} \times \sqrt{100}$

$= 2 \times 12 \times 10$

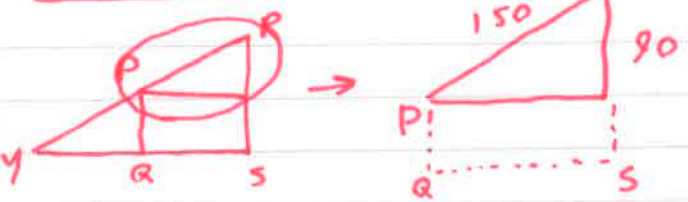
$= 240m$

$\frac{YS}{240} = \frac{450}{300}$ (sides in similar triangles)

$YS = 360m$

$360m - 240m = 120m$ ✓

Method 2:



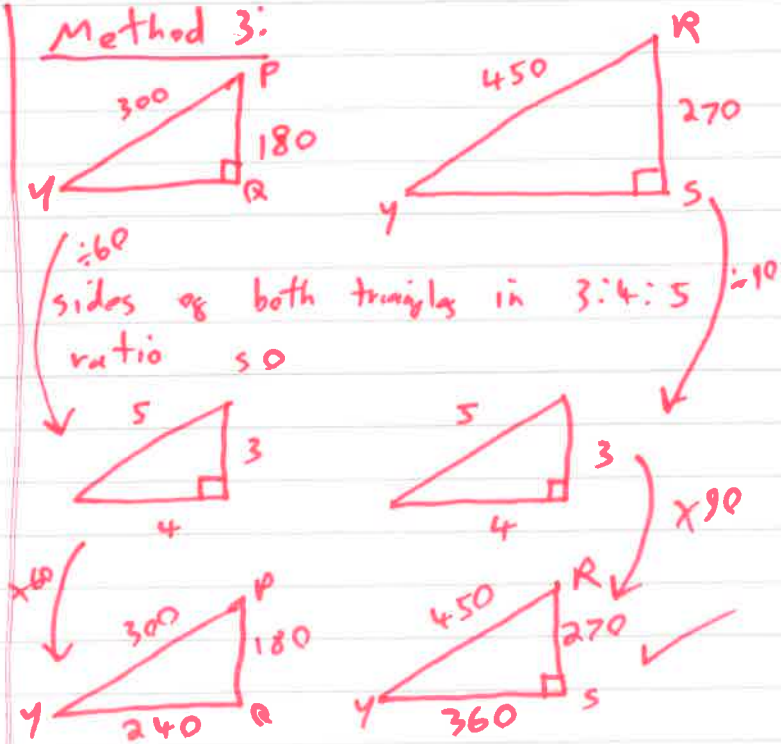
$150^2 - 90^2 = QS^2$ ✓

$22,500 - 8,100 = QS^2$

$QS^2 = 14400$

$QS = 120m$ ✓

Method 3:



$360m - 240m = 120m$ ✓

e) $196 = \left(\frac{11^2 - 7^2}{9p} + \sqrt[4]{10 \times \frac{8}{5}}\right)^2$

$\pm\sqrt{196} = \frac{121 - 49}{9p} + \sqrt[4]{16}$

$\pm 14 = \frac{72}{9p} + 2$ ✓

$14 = \frac{72}{9p} + 2$ or $-14 = \frac{72}{9p} + 2$

$12 = \frac{72}{9p}$ $-16 = \frac{72}{9p}$

$108p = 72$ $-144p = 72$

$p = \frac{2}{3}$ or $p = -\frac{1}{2}$ ✓ both

8a) $\frac{x^2-x-6}{20x+100} \div \frac{x^2-3x}{5x^2+25x} = \frac{(x-3)(x+2)}{20(x+5)} \div \frac{x(x-3)}{5x(x+5)}$

$= \frac{(x-3)(x+2)}{20(x+5)} \times \frac{5x(x+5)}{x(x-3)}$ ✓

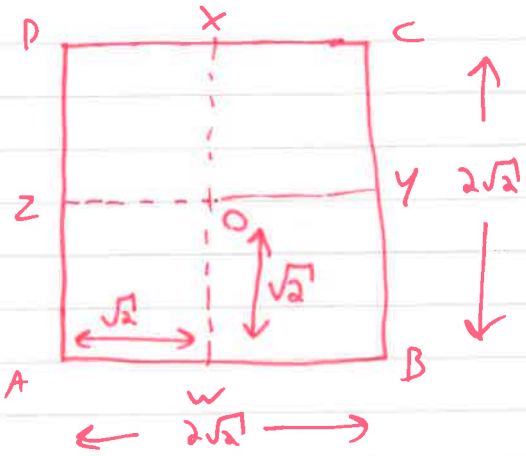
$= \frac{(x+2)}{4}$ ✓

b) $8^{200} = (2^3)^{200}$

$= 2^{600}$

$2^{600} < 2^{800}$ so 2^{800} is greater ✓ accept alternate methods

c i)



Area 8cm so side length $\sqrt{8} = 2\sqrt{2}$ ✓

so $AW = OW = \sqrt{2}$

$OA^2 = AW^2 + OW^2$

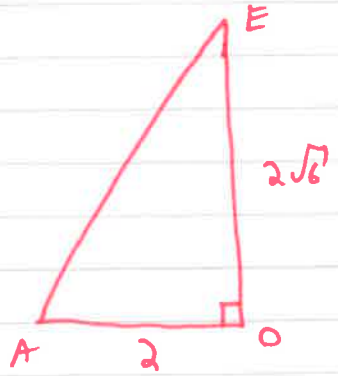
$= (\sqrt{2})^2 + (\sqrt{2})^2$

$= 2+2$

$OA^2 = 4$

so $OA = 2$ cm ✓ accept other methods

ii)



$AE^2 = AO^2 + EO^2$

$= 2^2 + (2\sqrt{6})^2$

$= 4 + 24$

$AE^2 = 28$ ✓

$AE = \sqrt{28}$

$= 2\sqrt{7}$ cm ✓ allow follow through marks from 8ci)

d) $\frac{x-3}{4} - \frac{2(x-y)}{3} + \frac{x+9}{12} = 1$

$\frac{3(x-3)}{12} - \frac{4\times 2(x-y)}{12} + \frac{x+9}{12} = 1$ ✓

$\frac{(3x-9)-(8x-8y)+(x+9)}{12} = 1$

$-4x + 8y = 12$ ✓

$8y = 12 + 4x$

$4x = 8y - 12$

$x = 2y - 3$ ✓

e) Let x be the amount taken away and then added.

So $(16-x)\times 0.4 + x \times 0.6 = 16 \times 0.55$ ✓

$\frac{2}{5}(16-x) + \frac{3}{5}x = \frac{11}{20}\times 16$

$\times 5$ $2(16-x) + 3x = \frac{11}{20}\times 80$ ✓

$32 - 2x + 3x = 44$

$x = 12$ L ✓

OR: 40% of 16 L is 6.4 L

55% of 16 L is 8.8 L

so an increase of 2.4 L

$0.6x - 0.4x = 2.4$ ✓

$0.2x = 2.4$

$\frac{1}{5}x = 2.4$

$x = 12$ L ✓

no equation but correct answer is one mark.