SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 2
HALF YEARLY EXAMINATION

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- **S**how ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the **simplest exact form**, unless otherwise stated.
- NESA approved calculators may be used
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: SG

Name: _____

Class	Teacher	
9Ma**A**	B. Kilmore	
9Ma**B**	J. Bui	
9Ma**C**	R. Wang	
9Ma**P**	A. Wang	
9Ma**L**	M. Gainford	
9Ma**U**	J. James	
9Ma**S**	T. Broockmann	

Section	Marks
1	**/15**
2	**/15**
3	**/15**
4	**/11**
5	**/11**
Total	**/67**

Section 1

1. Evaluate, correct to 3 significant figures, $\sqrt{\dfrac{523.3-11.2^2}{15.7}}$. **1**

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2. Convert 250 cm^2 into m^2. **1**

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3. Express the following in scientific notation:

(a) 380.5 **1**

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(b) 0.00903 **1**

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4. Simplify $\left(5a^2\right)^3$. **1**

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5. Divide \$360 in the ratio $2:4:9$. **1**

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6. Write $4^{444}+4^{444}+4^{444}+4^{444}$ as a single term. **1**

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7. Expand and simplify $3(x-2)+4x-7$. **1**

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8. Simplify $\frac{1}{k} - \frac{1}{k+1}$. **1**

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9. Simplify $\frac{4}{x^2-9} \div \frac{2}{5x+15}$. **2**

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10. Expand, and if necessary simplify:

(a) $5x(x-5)$ **1**

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(b) $(a+3)(b-4)$ **1**

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(c) $(3c+4)(2c+5)$ **1**

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11. Write $\frac{706}{825}$ as a recurring decimal. **1**

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Extra Working Page for Section 1 only

Section 2

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/15**

1. Simplify the following:

(a) $5\sqrt{6}-8\sqrt{6}$ **1**

……

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(b) $\sqrt{45}$ **1**

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(c) $8\sqrt{2}+2\sqrt{8}$ **1**

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(d) $\left(3-9\sqrt{10}\right)\left(\sqrt{5}+4\sqrt{2}\right)$ **1**

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(e) $\left(\sqrt{3}-3\sqrt{7}\right)^2$ **1**

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2. Write $\sqrt{\sqrt[3]{36}}$ as a fractional power of 6. **1**

……

……

3. Solve $2^{x+2} = 8^{2x}$. **2**

……

……

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4. The driving distance from Alex's home to her work is 20 km. **1**
She drives to and from work five times each week.
Her car uses fuel at the rate of 8 L/100 km.
How much fuel does she use driving to and from work each week?

A. 16 L B. 20 L C. 25 L D. 40 L

5. Simplify the ratio $3\sqrt{6} : \frac{2}{\sqrt{6}}$. **1**

……

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6. Simplify, using positive indices if necessary:

(a) $4a^3 \times 5a^5$ **1**

……

(b) $18b^{10} \div \frac{9}{b}$ **1**

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(c) $\left(64c^{\frac{1}{2}}\right)^{\frac{1}{3}}$ **1**

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(d) $d^{-\frac{7}{2}} \div d^{-2}$ **2**

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Extra Working Page for Section 2 only

Section 3

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/15**

1. Rationalise the denominator in the following, simplifying if possible:

(a) $\dfrac{4+\sqrt{2}}{\sqrt{5}}$ **1**

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(b) $\dfrac{8}{\sqrt{11}-\sqrt{5}}$ **1**

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2. A rectangular field is marked out on Moore Park. Its dimensions are 32 m by 14 m, each measured to the nearest metre.

(a) What are the limits of accuracy of each measurement? **1**

……

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(b) What is the difference between the largest possible and smallest possible *actual* areas of the rectangle? **1**

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(c) Considering that the length and width are each measured to 2 significant figures, explain, with a short rationale, what you consider the most appropriate approximation of the area of the rectangular field. **1**

……

……

3. Define a rational number. **1**

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4. Find the value of x if $p^x = \left(\frac{\sqrt{p}}{\sqrt[5]{p}}\right)^3$. **1**

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5. (i) Simplify $\frac{p^9 q^{-3}}{p\left(p^2 q\right)^3}$ **2**

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(ii) Hence express $\sqrt{\frac{p^9 q^{-3}}{p\left(p^2 q\right)^3}}$ in simplest form, and without fractions. **1**

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6. A TV program which has a file size of 495 MB (megabytes) is to be downloaded. **2**
The download speed is 82.7 megabits per second.
How long will it take to download this program?
Give your answer correct to the nearest second.
(Note: 1 megabit = 2^{20} bits and 1 byte = 8 bits)

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7. With a recent surge in the Deltacron variant, it is known that, per capita, Melbourne has a higher rate of infection than Sydney. **1**

This is reported in the newspapers as follows.

The Daily Telegraph	*The Herald Sun*
Sydney: A / 100 000 people infected	Sydney: 1 / C people infected
Melbourne: B / 100 000 people infected	Melbourne: 1 / D people infected

Select the correct answer below.

A. $A < B$ and $C < D$

B. $A < B$ and $C > D$

C. $A > B$ and $C < D$

D. $A > B$ and $C > D$

8. My phone plan has a fixed monthly credit per month. **2**

At the end of September, I had $\frac{1}{6}$ of my monthly credit left over.

At the end of October, I had $\frac{1}{5}$ of my monthly credit left over.

What was the ratio of my phone account usage in these two months?

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Extra Working Page for Section 3 only

Section 4

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/11**

1. If $x < 0$, simplify $x - \sqrt{x^2}$. **1**

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2. Write $\left(\frac{1}{x} + \frac{y}{x}\right)^{-1}$ as a simplified fraction. **2**

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3. Expand and simplify $\left(a^2 - 2a + 4\right)\left(a^2 + 2a - 4\right)$. **2**

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4. List these numbers in increasing order, explaining your reasoning. **2**

$2^{800}, \quad 3^{600}, \quad 5^{400}, \quad 6^{200}$

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5. Linda's trailer has two wheels on its axle and it carries a spare wheel. **2**
The three wheels are changed around from time to time.
The three tyres have been worn for 25 000 km, 28 000 km,
and 31 000 km respectively.
How many kilometres has Linda's trailer travelled? Explain carefully.

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6. If $a:b=8:13$, and $a:(b+c)=1:3$, find $a:c$. **2**

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Extra Working Page for Section 4 only

Section 5

Name: _____

Class	Teacher	
9**A**	B. Kilmore	
9**B**	J. Bui	
9**C**	R. Wang	
9**P**	A. Wang	
9**L**	M. Gainford	
9**U**	J. James	
9**S**	T. Broockmann	

Mark	**/11**

1. By first squaring both sides of $x = \sqrt{4-\sqrt{7}} + \sqrt{4+\sqrt{7}}$, find x as a single surd. **2**
Carefully show all reasoning.

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2. (i) Expand $(a+b)^3$. **1**

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(ii) If $a^{\frac{1}{3}} + a^{-\frac{1}{3}} = 3$, find the value of $a^3 + a^{-3}$. **3**

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4. The value of $\sqrt{5}$ is approximately 2.236.

Consider the problem of finding an approximate value of $\frac{1}{\sqrt{5}}$ without a calculator.

Let the number x be the difference between $\sqrt{5}$ and the approximation 2.236.
This means that $x = \sqrt{5} - 2.236$.

Let $J = \frac{1}{\sqrt{5}} - \frac{\sqrt{5}-x}{5}$ and $M = \frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}}$.

(a) Show that $M - J = \frac{x^2\left(\sqrt{5}+x\right)}{5\left(5-x^2\right)}$. **3**

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(b) Which of $\frac{\sqrt{5}-x}{5}$ and $\frac{1}{\sqrt{5}-x}$ is a better approximation to $\frac{1}{\sqrt{5}}$? **2**
Explain your reasons carefully without reference to a calculator.

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End of paper

Extra Working Page for Section 5 only

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 9 STAGE 5.3
TERM 2
HALF YEARLY

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks. Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

Year 9 Half-Yearly 2022

Section 1

This whole section was generally well done by the whole cohort.

(1) 5.03 ✓

(2) $0.025\ m^2$ ✓

(3) (a) 3.805×10^2 ✓

(b) 9.03×10^{-3} ✓

(4) $125a^6$ ✓

(5) Total = 15 parts.

$\frac{2}{15} \times 360$, $\frac{4}{15} \times 360$, $\frac{9}{15} \times 360$

$= \$48 : \$96 : \$216$ ✓

(6) $= 4^1 \times 4^{444} = 4^{445}$ ✓

✓(½)

(7) $= 3x - 6 + 4x - 7$

$= 7x - 13$ ✓

(8) $\frac{k+1-k}{k(k+1)} = \frac{1}{k(k+1)}$ ✓ or $\frac{1}{k^2+k}$

(9) $\frac{4}{(x+3)(x-3)} \times \frac{5(x+3)}{2}$ ✓

$= \frac{10}{x-3}$ ✓

(10) (a) $5x^2 - 25x$ ✓

(b) $ab - 4a + 3b - 12$ ✓

(c) $6c^2 + 23c + 20$ ✓

(11) $0.855\dot{7}$ ✓

Section 2 Solutions

1. Simplify the following:

(a) $5\sqrt{6}-8\sqrt{6}$ 1

$=-3\sqrt{6}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by majority of the candidates.

(b) $\sqrt{45}$ 1

$=\sqrt{9}\times\sqrt{5}$

$=3\sqrt{5}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by majority of the candidates.

(c) $8\sqrt{2}+2\sqrt{8}$ 1

$8\sqrt{2}+2\sqrt{4}\times\sqrt{2}$

$=8\sqrt{2}+4\sqrt{2}$

$=12\sqrt{2}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by majority of the candidates.

(d) $\left(3-9\sqrt{10}\right)\left(\sqrt{5}+4\sqrt{2}\right)$ 1

$=3\sqrt{5}+12\sqrt{2}-9\sqrt{50}-36\sqrt{20}$

$=3\sqrt{5}+12\sqrt{2}-45\sqrt{2}-72\sqrt{5}$

$=-69\sqrt{5}-33\sqrt{2}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: This was not done well by many candidates.
Many students did not simplify their answer as asked in the question, did not know how to simplify surds, or made silly mistakes in expanding the brackets.

(e) $\left(\sqrt{3}-3\sqrt{7}\right)^2$ 1

$= 3 - 2\times\sqrt{3}\times 3\sqrt{7} + 63$

$= 66 - 6\sqrt{21}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: This was not done well by many candidates.
Many students did not simplify their answer as asked in the question or did not know how to expand these brackets properly.

2. Write $\sqrt{\sqrt[3]{36}}$ as a fractional power of 6. 1

$= \left((6^2)^{1/3}\right)^{1/2}$

$= 6^{1/3}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: This was not done well by many candidates.
Many candidates got penalised for not leaving their answer as fractional power of 6 as asked in the question, did not know how to convert into fractional powers or made silly mistakes.

3. Solve $2^{x+2} = 8^{2x}$. 2

$2^{x+2} = (2^3)^{2x}$

$2^{x+2} = 2^{6x}$

$\therefore x+2 = 6x$

$5x = 2$

$x = 2/5$

Marking Scheme: 1 mark for correct working out.
1 mark for correct answer.
Marker's Comments: This was not done well by many candidates.
Many candidates were not able to come up with the equation $x+2=6x$ as they did not how convert $8^{2x} = 2^{6x}$.

4. The driving distance from Alex's home to her work is 20 km. 1
She drives to and from work five times each week.
Her car uses fuel at the rate of 8 L/100 km.
How much fuel does she use driving to and from work each week?

A. 16 L B. 20 L C. 25 L D. 40 L

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by majority of the candidates.

5. Simplify the ratio $3\sqrt{6} : \frac{2}{\sqrt{6}}$. 1

18 : 2

9 : 1

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: This was not done well by many candidates.
Common mistakes included candidates who did not simplify the ratio to its simplest form (where each term must be an integer and in the respective order); squared both sides of the ratio which changes the original ratio i.e. $81:1 \neq 9:1$; made arithmetic errors; or did not write their answer as a ratio.

6. Simplify, using positive indices if necessary:

(a) $4a^3 \times 5a^5$ 1

$20a^8$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by majority of the candidates.

(b) $18b^{10} \div \frac{9}{b}$ 1

$18b^{10} \times \frac{b}{9} = 2b^{11}$

Marking Scheme: 1 mark for the correct answer.
Marker's Comments: Done well by many of the candidates.
Most common mistake was the power of b in their final answer i.e. some wrote b^9 rather than b^{11}.

(c) $\left(64c^{\frac{1}{2}}\right)^{\frac{1}{3}}$ 1

$4c^{1/6}$

Marking Scheme: 1 mark for the correct answer.

Marker's Comments: This was not done well by many candidates.
Common mistakes included either not taking cube root of 64, not simplifying their answer enough to the requirements or not leaving their answer in index form i.e. writing $\sqrt[6]{c}$ rather than $c^{\frac{1}{6}}$

(d) $d^{-\frac{7}{2}} \div d^{-2}$ 2

$d^{-7/2+2} = d^{-1.5}$

$= \frac{1}{d^{1.5}} = \frac{1}{d^{3/2}}$

Marking Scheme: 1 mark for correct working out.
1 mark for correct answer.

Marker's Comments: This was not done well by many candidates. Many candidates did not follow the necessary requirement of leaving their answer in POSTIVE INDEX.
Final answers such as $d^{-\frac{3}{2}}$, $\frac{1}{d\sqrt{d}}$ or $\frac{1}{\sqrt{d^3}}$ does not meet the requirement as it is not positive or in index form.
Significant number of candidates made the mistake in the arithmetic of powers.

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

1. Rationalise the denominator in the following, simplifying if possible:

A. $\dfrac{4+\sqrt{2}}{\sqrt{5}}$ 1

Solutions	Comments
$\dfrac{4+\sqrt{2}}{\sqrt{5}} = \dfrac{4+\sqrt{2}}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$ $= \dfrac{4\sqrt{5}+\sqrt{10}}{5}$	No half marks. Common error(s): • Forgetting to multiply $\sqrt{5}$ to 4.

B. $\dfrac{8}{\sqrt{11}-\sqrt{5}}$ 1

Solutions	Comments
$\dfrac{8}{\sqrt{11}-\sqrt{5}} = \dfrac{8}{\sqrt{11}-\sqrt{5}} \times \dfrac{\sqrt{11}+\sqrt{5}}{\sqrt{11}+\sqrt{5}}$ $= \dfrac{8(\sqrt{11}+\sqrt{5})}{11-5}$ $= \dfrac{8(\sqrt{11}+\sqrt{5})}{6}$ $= \dfrac{4(\sqrt{11}+\sqrt{5})}{3}$ $= \dfrac{4\sqrt{11}+4\sqrt{5}}{3}$	Common error(s): • Not simplifying $\dfrac{8\sqrt{11}+8\sqrt{5}}{6}$. • Multiplying by $\dfrac{\sqrt{11}-\sqrt{5}}{\sqrt{11}-\sqrt{5}}$ instead of $\dfrac{\sqrt{11}+\sqrt{5}}{\sqrt{11}+\sqrt{5}}$

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

2. A rectangular field is marked out on Moore Park.
   Its dimensions are 32 m by 14 m, each measured to the nearest metre.

   A. What are the limits of accuracy of each measurement? 1

Solutions	Comments
The measurements are taken to the nearest metre, so the absolute error is $\frac{1}{2}\times 1 = 0.5$ m. Hence, the limits of accuracy are $31.5 \leq x < 32.5$ and $13.5 \leq y < 14.5$.	Some students chose to not use inequalities in writing their answer. Those who chose to do so can only score full marks if they explicitly stated both the inclusion of 31.5 and the exclusion of 32.5. Similarly, they must explicitly state both the inclusion of 13.5 and the exclusion of 14.5. Common error(s): • Mistaking the absolute error for the limit of accuracy.

   B. What is the difference between the largest possible and smallest possible *actual* areas of the rectangle? 1

Solutions	Comments
Largest area: $32.5 \times 14.5 = 471.25$ m^2 Smallest area: $31.5 \times 13.5 = 425.25$ m^2 Difference: $471.25 - 425.25 = 46$ m^2	0.5: Wrong areas, right diff, or right areas, wrong diff

   C. Considering that the length and width are each measured to 2 significant figures, explain, with a short rationale, what you consider the most appropriate approximation of the area of the rectangular field. 1

Solutions	Comments
$32 \times 14 = 448$ However, as length and width are measured to 2 s.f., the area can only be accurate to 2 s.f. at most. Hence, the area is 448 m^2 rounded to 2 s.f., which is 450 m^2.	Students should note that an explanation isn't simply writing an answer using words instead of numbers. Reasons must be given for why a particular number was chosen as the most appropriate approximation, which many students failed to do.

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

3. Define a rational number. 1

Solutions	Comments
A number in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.	Students who simply stated that a rational number is a fraction in the form $\frac{a}{b}$ without stating that a and b are integers couldn't score any marks. Students risked being penalised by not specifying that $b \neq 0$ or by providing certain non standard definitions.

4. Find the value of x if $p^x = \left(\frac{\sqrt{p}}{\sqrt[5]{p}}\right)^3$ 1

Solutions	Comments
$\left(\frac{\sqrt{p}}{\sqrt[5]{p}}\right)^3 = \left(\frac{p^{\frac{1}{2}}}{p^{\frac{1}{5}}}\right)^3$ $= \left(p^{\frac{1}{2}-\frac{1}{5}}\right)^3$ $= \left(p^{\frac{3}{10}}\right)^3$ $= p^{\frac{3}{10}\times 3}$ $= p^{\frac{9}{10}}$ $\therefore x = \frac{9}{10}$ Alternatively: $\left(\frac{\sqrt{p}}{\sqrt[5]{p}}\right)^3 = \left(\frac{p^{\frac{1}{2}}}{p^{\frac{1}{5}}}\right)^3$ $= \frac{p^{\frac{3}{2}}}{p^{\frac{3}{5}}}$ $= p^{\frac{3}{2}-\frac{3}{5}}$ $= p^{\frac{9}{10}}$ $\therefore x = \frac{9}{10}$	Many students couldn't make any meaningful progress in this question due to a variety of mistakes involving index laws.

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

5.

A. Simplify $\dfrac{p^9q^{-3}}{p\left(p^2q\right)^3}$ 2

Solutions	Comments
$\dfrac{p^9q^{-3}}{p(p^2q)^3} = \dfrac{p^9}{pq^3 \times p^6q^3}$ $= \dfrac{p^9}{p^7q^6}$ $= \dfrac{p^2}{q^6}$	Common error(s): • Not knowing how to simplify $\dfrac{q^{-3}}{q^3}$ or incorrectly simplifying it. • Various mistakes involving index laws.

B. Hence, express $\sqrt{\dfrac{p^9q^{-3}}{p\left(p^2q\right)^3}}$ in simplest form, and without fractions. 1

Solutions	Comments
$\sqrt{\dfrac{p^9q^{-3}}{p(p^2q)^3}} = \sqrt{\dfrac{p^2}{q^6}}$ $= \dfrac{p}{q^3}$ $= pq^{-3}$	Common error(s): • Not writing answers without using fractions.

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

6. A TV program which has a file size of 495 MB (megabytes) is to be downloaded. 2
   The download speed is 82.7 megabytes per second.
   How long will it take to download this program?
   Give your answer correct to the nearest second (Note: 1 megabyte = 1 000 000 bytes).

Solutions	Comments
$495\ MB = 495 \times 10^6\ B$ $\frac{495 \times 10^6}{82.7 \times 10^6} = \frac{495}{82.7}$ $= 6\ s\ (1\ d.p.)$	Many students were penalised for not showing any working out despite the mark allocation. Some students gave the incorrect answer of 5985490 seconds due to forgetting that the speed is in megabits per second, not bits per second. Those who did so might have realised their mistake during the exam if they checked their answer, as 5985490 seconds is more than 2 months long.

7. With a recent surge in the Deltacron variant, it is known that, per capita, 1
   Melbourne has a higher rate of infection than Sydney.
   This is reported in the newspapers as follows:

Daily Telegraph		Herald Sun	
Sydney:	A/100 000 people infected	Sydney:	1/C people infected
Melbourne:	B/100 000 people infected	Melbourne:	1/D people infected

Select the correct answer below:

A. $A < B$ and $C < D$
B. $A < B$ and $C > D$
C. $A > B$ and $C < D$
D. $A > B$ and $C > D$

Solutions	Comments
If two fractions have the same denominator, then the one with a larger numerator is greater than the one with a smaller numerator. Furthermore, if two fractions have the same numerator, then the one with the smaller denominator is greater than the one with a larger denominator. For example, $\frac{1}{3} < \frac{2}{3}$ because $1 < 2$ and $\frac{1}{2} > \frac{1}{3}$ because $2 < 3$. Hence, as that Melbourne has a higher rate of infection, $A < B$ and $C > D$, so the correct choice is B.	No half marks. An incorrect answer that was chosen by many students was A.

2022 YR9 HYE Section 3 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

8. My phone plan has a fixed monthly credit per month. 2

   At the end of September, I had $\frac{1}{6}$ of my monthly credit left over.

   At the end of October, I had $\frac{1}{5}$ of my monthly credit left over.

   What was the ratio of my phone account usage in these two months?

Solutions	Comments
September: $\frac{1}{6}$ left, so $\frac{5}{6}$ used. October: $\frac{1}{5}$ left, so $\frac{4}{5}$ used. Hence, the ratio of usage of September to October is: $S:O = \frac{5}{6}:\frac{4}{5}$ $= 25:24$	Many students risked being penalised by not stating the order of the ratio. Common error(s): • Not simplifying $\frac{5}{6}:\frac{4}{5}$. • Finding the ratio of credit left instead of credit used.

Section 4 Solutions

1. If $x < 0$, simplify $x - \sqrt{x^2}$. **1**

Answer:

$$x - \sqrt{x^2} = x + x$$
$$= 2x$$

Comment: This question was poorly done by most students.

Significant number of students did not read the given information carefully and calculated as $x - x = 0$ or $-2x$.

2. Write $\left(\frac{1}{x} + \frac{y}{x}\right)^{-1}$ as a simplified fraction. **2**

Answer:

$$\left(\frac{1}{x} + \frac{y}{x}\right)^{-1} = \frac{1}{\frac{1+y}{x}}$$
$$= \frac{x}{y+1}$$

Comment: A few students were able to figure out the first line of working out, but they forgot simplifying their answers.

The common misconception was writing $\left(\frac{1}{x} + \frac{y}{x}\right)^{-1}$ as $\frac{x}{1} + \frac{x}{y} = \frac{xy + x}{y}$.

3. Expand and simplify $\left(a^2 - 2a + 4\right)\left(a^2 + 2a - 4\right)$. **2**

Answer:

<u>Method 1:</u>

$$\left(a^2 - 2a + 4\right)\left(a^2 + 2a - 4\right) = a^4 + 2a^3 - 4a^2 - 2a^3 - 4a^2 + 8a + 4a^2 + 8a - 16$$
$$= a^4 - 4a^2 + 16a - 16$$

<u>Method 2:</u>

$$\left(a^2 - 2a + 4\right)\left(a^2 + 2a - 4\right) = \left[a^2 - (2a - 4)\right]\left[a^2 + (2a - 4)\right]$$
$$= a^4 - (2a - 4)^2$$
$$= a^4 - 4a^2 + 16a - 16$$

Comment: Majority students attempted the first approach and made mistakes when expanding the brackets.

The common errors were either missing $16a$ or $-4a^2$.

4. List these numbers in increasing order, explaining your reasoning. **2**

$$2^{800},\quad 3^{600},\quad 5^{400},\quad 6^{200}$$

Answer:

Method 1:

$2^{800} = \left(2^4\right)^{200}$

$= 16^{200}$

$3^{600} = \left(3^3\right)^{200}$

$= 27^{200}$

$5^{400} = \left(5^2\right)^{200}$

$= 25^{200}$

Since $6 < 16 < 25 < 27$

$\therefore\ 6^{200},\ \ 2^{800},\ \ 5^{400},\ \ 3^{600}$

Method 2:

Similarly, we could also write these numbers with the same power 100.
Hence, we have:

$2^{800} = 256^{100}$

$3^{600} = 729^{100}$

$5^{400} = 625^{100}$

$6^{200} = 36^{100}$

Since $36 < 256 < 625 < 729$

$\therefore\ 6^{200},\ \ 2^{800},\ \ 5^{400},\ \ 3^{600}$

Comment: This question was poorly done by many students.

A few students listed these numbers in descending order instead of ascending order.

Some students only wrote the answers without reasoning. Thus, they were penalised.

5. Linda's trailer has two wheels on its axle and it carries a spare wheel. **2**
The three wheels are changed around from time to time.
The three tyres have been worn for 25 000 km, 28 000 km,
and 31 000 km respectively.
How many kilometres has Linda's trailer travelled? Explain carefully.

Answer:

Since Linda's trailer has two wheels on its axle, the total wheel run at any time will be twice to the distance that her trailer has travelled.

Therefore, the distance will be calculated as:

$$\frac{25\,000 + 28\,000 + 31\,000}{2} = 42\,000 \text{ km}$$

$\therefore$ Linda's trailer has travelled 42 000 km.

Comment: A significant number of students found the total distance, then divided it by 3 or multiplied the result by $\frac{2}{3}$ instead of dividing by 2. They should read the question more carefully that there are two wheels on her trailer's axle.

6. If $a:b=8:13$, and $a:(b+c)=1:3$, find $a:c$. **2**

Answer:

$$\begin{aligned} a:(b+c) &= 1:3 \\ &= 8:24 \end{aligned}$$

Since $a:b = 8:13$

Therefore, $c = 11$ parts

$\therefore\ a:c\ = 8:11$

Comment: This question was very well done.

A few students did not show clear working out. Therefore, they were penalised.

1. By first squaring both sides of $x=\sqrt{4-\sqrt{7}}+\sqrt{4+\sqrt{7}}$, find x as a single surd. Carefully show all reasoning. **2**

$x^2 = 4-\sqrt{7} + 2\sqrt{(4-\sqrt{7})(4+\sqrt{7})} + 4+\sqrt{7}$

$= 8 + 2\sqrt{16-7}$

$= 8+6$

$x^2 = 14$

$x = \pm\sqrt{14}$ but $x = \sqrt{A}+\sqrt{B}$ and $\sqrt{A}, \sqrt{B} > 0$

$\Rightarrow x > 0$

$\Rightarrow x = \sqrt{14}$

2. (i) Expand $(a+b)^3$. **1**

$a^3+3a^2b+3ab^2+b^3$

(ii) If $a^{\frac{1}{3}}+a^{-\frac{1}{3}}=3$, find the value of a^3+a^{-3}. **3**

$(a^{\frac{1}{3}}+a^{-\frac{1}{3}})^3 = 27$

$a+3a^{\frac{2}{3}}a^{-\frac{1}{3}}+3a^{\frac{1}{3}}a^{-\frac{2}{3}}+a^{-1}=27$

$a+a^{1}+3a^{\frac{1}{3}}+3a^{-\frac{1}{3}}=27$

$a+a^{-1}+3(a^{\frac{1}{3}}+a^{-\frac{1}{3}})=27$ But $a^{\frac{1}{3}}+a^{-\frac{1}{3}}=3$

$\Rightarrow a+a^{-1}+3(3)=27$

$\Rightarrow a+a^{-1}+9=27$

$(a+\frac{1}{a})^3 = 18^3$

$\Rightarrow a^3+3a+3a^{-1}+a^{-3}=5832$

$a^3+3(a+a^{-1})+a^{-3}=5832$

$a^3+3(18)+a^{-3}=5832$

$\Rightarrow a^3+a^{-3}=5832-54$

$a^3+a^{-3}=5778$

4. The value of $\sqrt{5}$ is approximately 2.236.

Consider the problem of finding an approximate value of $\frac{1}{\sqrt{5}}$ without a calculator.

Let the number x be the difference between $\sqrt{5}$ and the approximation 2.236.

This means that $x = \sqrt{5} - 2.236$.

Let $J = \frac{1}{\sqrt{5}} - \frac{\sqrt{5}-x}{5}$ and $M = \frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}}$.

(a) Show that $M - J = \frac{x^2(\sqrt{5}+x)}{5(5-x^2)}$. **3**

$J = \frac{\sqrt{5} - (\sqrt{5}-x)}{5} = \frac{x}{5}$ ✓

$M = \frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}}$

$= \frac{\sqrt{5} - (\sqrt{5}-x)}{\sqrt{5}(\sqrt{5}-x)}$

$= \frac{x}{\sqrt{5}(\sqrt{5}-x)}$ ✓

Then $M - J = \frac{x}{\sqrt{5}(\sqrt{5}-x)} - \frac{x}{5}$

$= \frac{\sqrt{5}x - x(\sqrt{5}-x)}{5(\sqrt{5}-x)}$

$= \frac{x^2}{5(\sqrt{5}-x)}$

Then

$M - J = \frac{x^2}{5(\sqrt{5}-x)} \times \frac{(\sqrt{5}+x)}{(\sqrt{5}+x)}$ ✓

$= \frac{x^2(\sqrt{5}+x)}{5(\sqrt{5}-x)(\sqrt{5}+x)}$

$= \frac{x^2(\sqrt{5}+x)}{5(5-x^2)}$

3

(b) Which of $\frac{\sqrt{5}-x}{5}$ and $\frac{1}{\sqrt{5}-x}$ is a better approximation to $\frac{1}{\sqrt{5}}$? **2**

Explain your reasons carefully without reference to a calculator.

'M' is difference between $\frac{1}{\sqrt{5}-x}$ and $\frac{1}{\sqrt{5}}$

'J' is difference between $\frac{1}{\sqrt{5}}$ and $\frac{\sqrt{5}-x}{5}$ ✓

2

Now $M - J = \frac{x^2(\sqrt{5}+x)}{5(5-x^2)}$

Numerator > 0 and Denominator > 0 since x^2 is approx for 5 and $x^2 < 5$ (since $x = \sqrt{5} - 2.236 \Rightarrow x < \sqrt{5}$) ✓

$\therefore M - J > 0$

$M > J$

$\therefore J$ is better approximation

ie. $\frac{\sqrt{5}-x}{5}$ is better approximation.

End of paper