SYDNEY BOYS HIGH SCHOOL

2021

YEAR 9
HALF YEARLY
EXAMINATION

Mathematics Stage 5.3

General Instructions

- All questions may be attempted.
- Write using black or blue pen.
- Marks may be deducted for careless or badly arranged work. This can include insufficient working out.
- All working and answers are to be written in this test booklet.
- NESA approved Calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on Extra working pages provided.
  Show the number and part of the answer being rewritten and write "**See Back**" at the original question.

Time Allowed: 90 minutes

Examiner: BK

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Section	Marks
A	/ 12
B	/ 12
C	/ 13
D	/ 12
E	/ 13
F	/ 13
Total	/ 75

1. Simplify $(a^2)^3$ **(1m)**

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2. Write with a positive index: 3^{-2} **(1m)**

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3. Write 14 830 000 000 in scientific notation. **(1m)**

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4. Simplify:

(a) $4\sqrt{7} - \sqrt{7}$ **(1m)**

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(b) $3\sqrt{72}$ **(1m)**

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5. Expand and simplify: $12 - 4(2x + 4)$ **(1m)**

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6. Factorise $10ab - 35b^2 + 20bc$ **(1m)**

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7. Evaluate $4.7 \times \left(\frac{23 + 8.5}{5.6} \right)$ correct to 2 significant figures. **(1m)**

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8. Simplify $\frac{6y^5 \times y^7}{10y^2}$ **(1m)**

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9. A length of fabric is measured to be 12.56 metres. Given the possible error in the measurement, between which two measurements does the true length lie? **(1m)**

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10. A teacher must mark the papers of her 28 students. **(2m)**
In the first 40 minutes she marked 5 papers.
If she keeps up at this rate, how long will it take to finish the task?

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Use this space to re-write any questions for Part A.

Part B

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Mark	/12

1. Simplify, leaving answers as simplified surds.

(a) $3\sqrt{20}+4\sqrt{5}$ **(1m)**

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(b) $-\sqrt{6}\times\sqrt{2}$ **(1m)**

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(c) $18\sqrt{6}\div 9\sqrt{3}$ **(1m)**

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2. Expand and simplify:

(a) $2a^3(4a^{-2}-5)$ **(1m)**

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(b) $(4b-3)(5b-2)$ **(1m)**

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3. Simplify $\left(\frac{3}{x}+\frac{5}{2x}\right)\div\frac{11}{x}$ **(2m)**

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4. Simplify $\frac{x+3}{6} - \frac{2x-5}{8}$ **(2m)**

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5. The Dragons and the Sharks played their annual match of softball.
After 3 innings the Dragons had scored runs in the ratio $3:1:2$ for a total of 18 runs.

(a) How many runs did they score in the second innings? **(1m)**

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(b) If the Sharks scored runs in the ratio $1:1:4$ with 4 runs in the second innings, which team is winning and by how much? **(2m)**

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Use this space to re-write any questions for Part B.

Part C

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Mark	/13

1. Which of the following is the same as $27^{\frac{4}{3}}$? **(1m)**

A. $\sqrt[4]{27^3}$ B. $\sqrt[3]{27^4}$ C. $\sqrt{\left(\frac{27}{3}\right)^4}$ D. $\sqrt[4]{27}$

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2. The mass of one atom of oxygen is 2.7×10^{-23} grams. **(2m)**
What is the mass of 8.5×10^{29} atoms of oxygen?
Answer in scientific notation correct to 3 significant figures.

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3. Find the exact value of $0.\dot{8}\dot{3}-\frac{2}{11}$ without a calculator. Show all working. **(2m)**

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4. Subtract $\left(\frac{1}{3}x-4\frac{3}{5}\right)$ from $\frac{1}{4}\left(-6\frac{2}{3}+2x\right)$ **(2m)**

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5. If $\sqrt{28}+\sqrt{7}=\sqrt{A}$, show that $A=63$ **(2m)**

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6. Simplify $(8x^6)^{\frac{1}{3}}x^{-3}$ **(2m)**

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7. Simplify $\sqrt{32}-\sqrt{98}-\sqrt{50}\times\sqrt{72}$ **(2m)**

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Use this space to re-write any questions for Part C.

Part D

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Mark	/12

1. Rationalise the denominator: $\frac{5}{2\sqrt{3}}$ **(1m)**

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2. Evaluate $(2^3 + 2^{-3})^{-2}$ and round to 2 significant figures. **(1m)**

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3. Simplify and write with positive indices: $\frac{-30a^7b^4c^{-1}}{18a^3b^{-2}c}$ **(2m)**

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4. Simplify: $\sqrt{75c^3d^4} + 10c^{\frac{1}{2}}d^2\sqrt{3}\,c$ **(2m)**

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5. Expand and simplify:

(a) $(3x-2)(3x+2)-(4x-1)(2x-5)$ **(2m)**

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(b) $(7\sqrt{5}-2\sqrt{3})^2$ **(2m)**

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6. Find the exact value of the sum: $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}$ **(2m)**

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Use this space to re-write any questions for Part D.

Part E

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Mark	/13

1. Rationalise the denominator and simplify: $\dfrac{\sqrt{6}}{2\sqrt{2}+3\sqrt{5}}$ **(2m)**

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2. Expand and simplify: $(x+y)(x^2+3xy+2y^2)-(x+y)^2$ **(3m)**

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3. Simplify $\dfrac{2^{a+2}-4}{2^a-1}$ **(2m)**

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4. The sums of money in the hands of 3 card players, A, B and C, are in the ratio $7:6:5$ at the beginning of the game and $6:5:4$ at the end of the game.

(a) Who won and who lost? **(2m)**

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(b) If one of the players won \$120 find how much each of the players had at the beginning of the game. **(2m)**

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5. Show that $\frac{x-y}{x+y}\sqrt{\frac{x+y}{x-y}} = \sqrt{\frac{x-y}{x+y}}$ **(2m)**

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Use this space to re-write any questions for Part E.

Part F

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Ms A. Ward & Ms J Bui	
9**C**	Ms Millar	
9**P**	Mr R. Wang	
9**L**	Mr Fuller	
9**U**	Mr J. James	
9**S**	Mr T. Broockmann	

Mark	/13

1. If $1.68\times10^{21} = 2^a \times 3^b \times 5^c \times 7^d$, find the value of $a+b+c+d$. **(2m)**

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2. Write $(1-d)^{2a}(d-1)^2(1-c)^b(c-1)^4$ in the form $(1-d)^n(1-c)^n$. **(2m)**

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3. Orange juice concentrate, which is 25% water, is imported. A twelve-litre container of the orange juice concentrate has 4 litres removed. The container is filled up with water, mixed thoroughly and a further 4 litres of this mixture is then removed. If the container is filled up with water again, find what percentage of the final mixture is water. **(3m)**

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4. Simplify $\left(\dfrac{(2d^6k^3)^{\frac{-1}{3}} \times 8d^{\frac{3}{4}}k^{\frac{2}{3}}}{3d^{\frac{-1}{4}} \times \sqrt[3]{(4k)^4}} \right)$ **(3m)**

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5. Racehorses Phar Lap and Kingston Town run a match race over 1 kilometre. Kingston Town is given a 48 metres head start. They run at their own constant speed and Phar Lap wins by 2 metres. How many metres had Phar Lap run when he passed Kingston Town? Show all reasoning. **(3m)**

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End of paper

Use this space to re-write any questions for Part F.

SYDNEY
BOYS
HIGH
SCHOOL

2021

YEAR 9
HALF YEARLY
ASSESSMENT TASK

Stage 5.3 Mathematics

Sample Solutions

NOTE: **Some of you may be disappointed with your mark.**
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

2021 YR9 HYE Section A Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

1. Simplify $(a^2)^3$. 1

Solution	Comment(s)
$(a^2)^3 = a^6$	No half marks. Students generally did well on this question.

2. Write with a positive index: 3^{-2} 1

Solution	Comment(s)
$3^{-2} = \frac{1}{3^2}$	Common error(s): • Writing the answer as $\frac{1}{9}$.

3. Write 14 830 000 000 in scientific notation. 1

Solution	Comment(s)
$14\,830\,000\,000 = 1.483 \times 10^{10}$	Common error(s): • Writing 1.483 as a number greater than 10, e.g. 1483×10^7.

4. Simplify:

A. $4\sqrt{7} - \sqrt{7}$ 1

Solution	Comment(s)
$4\sqrt{7} - \sqrt{7} = 3\sqrt{7}$	No half marks. Students generally did well on this question.

B. $3\sqrt{72}$ 1

Solution	Comment(s)
$3\sqrt{72} = 3\sqrt{36 \times 2}$ $= 3 \times \sqrt{6^2} \times \sqrt{2}$ $= 3 \times 6 \times \sqrt{2}$ $= 18\sqrt{2}$	Common error(s): • Leaving the answer as $9\sqrt{8}$ without further simplification.

2021 YR9 HYE Section A Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

5. Expand and simplify: $12 - 4(2x + 4)$ 1

Solution	Comment(s)
$12 - 4(2x + 4) = 12 - 8x - 16$ $= -4 - 8x$	Common error(s): • Incorrectly expanding the minus sign. • Misinterpreting the question as a binomial expansion. • Incorrectly expanding the expression to $8(2x + 4)$.

6. Factorise $10ab - 35b^2 + 20bc$. 1

Solution	Comment(s)
$10ab - 35b^2 + 20bc = 5b(2a - 7a + 4c)$	No half marks. Common error(s): • Not factorising completely. • Attempting to factorise the expression into binomial products.

7. Evaluate $4.7 \times \left(\frac{23+8.5}{5.6}\right)$ correct to 2 significant figures. 1

Solution	Comment(s)
$4.7 \times \left(\frac{23 + 8.5}{5.6}\right) = 26\ (2\ s.f.)$	Students generally did well on this question.

8. Simplify $\frac{6y^5 \times y^7}{10y^2}$. 1

Solution	Comment(s)
$\frac{6y^5 \times y^7}{10y^2} = \frac{6y^{12}}{10y^2}$ $= \frac{3y^{10}}{5}$	Common error(s): • Forgetting to simplify the $\frac{6}{10}$. • Incorrectly simplifying the indices.

2021 YR9 HYE Section A Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

9. A length of fabric is measured to be 12.56 metres. 1
   Given the possible error in the measurement, between which two measurements does the true length lie?

Solution	Comment(s)
Absolute error: $$\frac{0.01}{2} = 0.005$$ Hence, for the true length l, the range of measurements are: $$12.56 - 0.005 \leq l < 12.56 + 0.005$$ $$12.555 \leq l < 12.565$$	Common error(s): • Not specifying that 12.555 is included, but 12.565 is excluded in the range of values. • Incorrectly using the inequality symbols, e.g. $12.555 \geq l > 12.565$ and $12.555 \leq l > 12.565$.

10. A teacher must mark the papers of her 28 students. 2
    In the first 40 minutes, she marked 5 papers.
    If she keeps up at this rate, how long will it take to finish the task?

Solution	Comment(s)
$$40:5 = 8:1$$ Hence, the rate is 8 minutes per paper. $$8:1 = x:28$$ $$\therefore x = 224\ min$$ $$= 3\ hr\ 40\ min$$	Students who answered 3 hr 4 min or equivalent were also awarded full marks. Students who rounded their answer to 3.73 or equivalent were penalised due to it not being exact.

Part B

1. Simplify, leaving answers as simplified surds.

(a) $3\sqrt{20}+4\sqrt{5}$ **(1m)**

$= 3\sqrt{4\times5} + 4\sqrt{5}$

$= 6\sqrt{5} + 4\sqrt{5}$

$= 10\sqrt{5}$

(b) $-\sqrt{6}\times\sqrt{2}$ **(1m)**

$= -\sqrt{3}\times\sqrt{2}\times\sqrt{2}$

$= -2\sqrt{3}$

(c) $18\sqrt{6}\div9\sqrt{3}$ **(1m)**

$= \frac{18\sqrt{6}}{9\sqrt{3}}$

$= 2\sqrt{2}$

2. Expand and simplify:

(a) $2a^3(4a^{-2}-5)$ **(1m)**

$= 8a - 10a^3$

(b) $(4b-3)(5b-2)$ **(1m)**

$= 20b^2 - 8b - 15b + 6$

$= 20b^2 - 23b + 6$

3. Simplify $\left(\frac{3}{x}+\frac{5}{2x}\right)\div\frac{11}{x}$ **(2m)**

$= \left(\frac{6+5}{2x}\right)\div\frac{11}{x}$

$= \frac{11}{2x}\times\frac{x}{11}$

$= \frac{1}{2}$

4. Simplify $\frac{x+3}{6}-\frac{2x-5}{8}$ **(2m)**

$= \frac{4(x+3)-3(2x-5)}{24}$

$= \frac{4x+12-6x+15}{24}$

$= \frac{-2x+27}{24}$

5. The Dragons and the Sharks played their annual match of softball.
After 3 innings the Dragons had scored runs in the ratio 3 : 1 : 2 for a total of 18 runs.

(a) How many runs did they score in the second innings? **(1m)**

(3 + 1 + 2) parts is 18 runs

6 parts is 18 runs

1 part is 3 runs

∴ 3 runs are scored in 2nd innings

(b) If the Sharks scored runs in the ratio 1 : 1 : 4 with 4 runs in the second innings, which team is winning and by how much? **(2m)**

1 part is 4 runs

6 parts is 24 runs

24 − 18 = 6

∴ Sharks are winning by 6 runs

Part C

1. Which of the following is the same as $27^{\frac{4}{3}}$? **(1m)**

A. $\sqrt[4]{27^3}$ B. $\sqrt[3]{27^4}$ C. $\sqrt{\left(\frac{27}{3}\right)^4}$ D. $\sqrt[4]{27}$

Answer is B

2. The mass of one atom of oxygen is 2.7×10^{-23} grams. **(2m)**
What is the mass of 8.5×10^{29} atoms of oxygen?
Answer in scientific notation correct to 3 significant figures.

$2.7\times10^{-23}\times8.5\times10^{29}=22.95\times10^{6}$
$22.95\approx23.0$ to 3 significant figures
$23.0\times10^{6}=2.30\times10^{7}$ grams

Note: Nearly all students made at least one mistake by not writing their answer correct to 3 significant figures and also in scientific notation. Many did not do both of these things. Some students failed to realise that scientific notation requires the number in front to be greater or equal to one and less than 10.

3. Find the exact value of $0.\dot{8}\dot{3}-\frac{2}{11}$ without a calculator. Show all working. **(2m)**

Let $x=0.\dot{8}\dot{3}$, then $100x=83.\dot{8}\dot{3}$.
$100x-x=83.\dot{8}\dot{3}-0.\dot{8}\dot{3}$
$99x=83$
$x=\frac{83}{99}$

Then we have $\frac{83}{99}-\frac{2}{11}=\frac{83}{99}-\frac{18}{99}$, which equals $\frac{65}{99}$.

Note: This was generally done well but some students wrote the fraction $\frac{2}{11}$ as $0.\dot{1}\dot{8}$ without showing any working and received no marks for this part of the question.

4. Subtract $\left(\frac{1}{3}x-4\frac{3}{5}\right)$ from $\frac{1}{4}\left(-6\frac{2}{3}+2x\right)$ **(2m)**

$$\frac{1}{4}\left(-6\frac{2}{3}+2x\right)-\left(\frac{1}{3}x-4\frac{3}{5}\right)=\frac{1}{4}\left(-\frac{20}{3}+2x\right)-\left(\frac{x}{3}-\frac{23}{5}\right)$$
$$=-\frac{5}{3}+\frac{x}{2}-\frac{x}{3}+\frac{23}{5}$$
$$=\frac{x}{2}-\frac{x}{3}+\frac{23}{5}-\frac{5}{3}$$
$$=\frac{3x}{6}-\frac{2x}{6}+\frac{69}{15}-\frac{25}{15}$$
$$=\frac{x}{6}+\frac{44}{15}$$

Note: This is another question that most students lost marks on. Some students could not turn a mixed fraction into an improper fraction correctly. Often the sign of the constant term in the second pair of brackets was incorrect when students took the brackets away and a small number of students subtracted the wrong expression.

5. If $\sqrt{28}+\sqrt{7}=\sqrt{A}$, show that $A=63$ **(2m)**

$$\begin{aligned}2\sqrt{7}+\sqrt{7}&=\sqrt{A}\\3\sqrt{7}&=\sqrt{A}\\\sqrt{3^2\times 7}&=\sqrt{A}\\\sqrt{63}&=\sqrt{A}\end{aligned}$$

Note: This was mostly done very well.

6. Simplify **(2m)**

$$\begin{aligned}8^{\frac{1}{3}}x^{\frac{6}{3}}x^{-3}&=2x^2x^{-3}\\&=2x^{-1}\\&=\frac{2}{x}\end{aligned}$$

Note: This was done reasonably well. Some students did not apply the cubed root to the 8 and some did not know how to correctly interpret the negative power.

7. Simplify $\sqrt{32}-\sqrt{98}-\sqrt{50}\times\sqrt{72}$ **(2m)**

$$\begin{aligned}\sqrt{2\times 4^2}-\sqrt{2\times 7^2}-\sqrt{2\times 5^2}\times\sqrt{2\times 6^2}&=4\sqrt{2}-7\sqrt{2}-5\sqrt{2}\times 6\sqrt{2}\\&=-3\sqrt{2}-5\times 6\times\sqrt{2}\times\sqrt{2}\\&=-3\sqrt{2}-30\times 2\\&=-3\sqrt{2}-60\end{aligned}$$

Note: This was done reasonably well by most. A common mistake was that students failed to realise the last term becomes an integer.

Part D:

1. Rationalise the denominator: $\frac{5}{2\sqrt{3}}$ **(1m)**

$$\frac{5}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{2\times 3} = \frac{5\sqrt{3}}{6}$$

or

$$\frac{5}{2\times\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{10\sqrt{3}}{4\times 3} = \frac{10\sqrt{3}}{12} = \frac{5\sqrt{3}}{6} \quad [1]$$

Comment:
Well done, apart from leaving your answer in the simplest form.

2. Evaluate $(2^3+2^{-3})^{-2}$ and round to 2 significant figures. **(1m)**

$$(2^3+2^{-3})^{-2} \doteqdot 0.015\,147\,928\,99$$

$$= 0.015 \quad (2.\ \text{sig. fig.}) \quad [1]$$

Comment:
Many students rounded to 2 decimal places instead of 2 sig. fig.

3. Simplify and write $\frac{-30a^7b^4c^{-1}}{18a^3b^{-2}c}$ with positive indices: **(2m)**

$$\frac{-30a^7b^4c^{-1}}{18a^3b^{-2}c} = -\frac{5}{3}\times(a^{7-3})\times(b^{4--2})\times(c^{-1-1})$$

$$= \frac{-5a^4b^6c^{-2}}{3}$$

$$= -\frac{5a^4b^6}{3c^2} \quad [2]$$

Comment:
Many students forgot about the c^2.

4. Simplify: $\sqrt{75c^3d^4} + 10c^{\frac{1}{2}}d^2\sqrt{3}\,c$ **(2m)**

$\sqrt{75\,c^3\,d^4} + 10\,c^{1/2}\,d^2\sqrt{3}\,c$

$= 5\sqrt{3}.\,c\sqrt{c}.\,d^2 + 10\sqrt{c}\,d^2\sqrt{3}\,c$

$= 15cd^2\sqrt{3c}$ [2]

Comment:
Very poorly done.

5. Expand and simplify:

(a) $(3x-2)(3x+2)-(4x-1)(2x-5)$ **(2m)**

(a) $(3x-2)(3x+2) - (4x-1)(2x-5)$

$= 9x^2 + 6x - 6x - 4 - [8x^2 - 20x - 2x + 5]$

$= 9x^2 - 4 - 8x^2 + 22x - 5$

$= x^2 + 22x - 9$ [2]

(b) $(7\sqrt{5}-2\sqrt{3})^2$ **(2m)**

(b) $(7\sqrt{5} - 2\sqrt{3})^2$

$= (7\sqrt{5} - 2\sqrt{3})(7\sqrt{5} - 2\sqrt{3})$

$= 49 \times 5 - 14\sqrt{3} - 14\sqrt{3} + 4 \times 3$

$= 245 - 28\sqrt{3} + 12$

$= 257 - 28\sqrt{3}$ [2]

Comment:
Very well done.

6. Find the exact value of the sum: $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$ **(2m)**

6. $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$.

- $\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1-2}$

$= \frac{1-\sqrt{2}}{-1}$

$= \sqrt{2}-1$

- $\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2-3}$

$= \frac{\sqrt{2}-\sqrt{3}}{-1}$

$= \sqrt{3}-\sqrt{2}$

- $\frac{1}{\sqrt{99}+\sqrt{100}} \times \frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}} = \frac{\sqrt{99}-\sqrt{100}}{99-100}$

$= \frac{\sqrt{99}-\sqrt{100}}{-1}$

$= \sqrt{100}-\sqrt{99}$

$= 10-\sqrt{99}$

$\therefore \sqrt{2}-1+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\ldots+(10-\sqrt{99}$

$= -1+10$

$= 9$ [2]

Comment:
Very poorly done.

1. Rationalise the denominator and simplify: $\dfrac{\sqrt{6}}{2\sqrt{2}+3\sqrt{5}}$ **(2m)**

Sample answer:

$$= \frac{\sqrt{6}\,(2\sqrt{2}-3\sqrt{5})}{(2\sqrt{2}+3\sqrt{5})\,(2\sqrt{2}-3\sqrt{5})}$$

$$= \frac{2\sqrt{12}-3\sqrt{30}}{8-45}$$

$$= \frac{3\sqrt{30}-4\sqrt{3}}{37}$$

Comments:

- Most students recognise rationalising denominators by multiplying both numerator and denominator by $(2\sqrt{2}-3\sqrt{5})$ then use the formula $(a-b)(a+b)=a^2-b^2$
- A few students multiply by $(2\sqrt{2}+3\sqrt{5})$ or $\sqrt{6}$ and will be rewarded 0 marks
- Students should simplify $2\sqrt{12}=4\sqrt{3}$. The answer $2\sqrt{12}$ will be deducted 0.5 marks
- The negative should not be in the denominator

2. Expand and simplify: $(x+y)(x^2+3xy+2y^2)-(x+y)^2$ **(3m)**

Sample answer:

$$= x^3+3x^2y+2xy^2+x^2y+3xy^2+2y^3-(x^2+2xy+y^2)$$
$$= x^3+4x^2y+5xy^2+3xy^2+2y^3-x^2-2xy-y^2$$

Comments:

- This question is generally well done in the first line of working out particularly expanding $(x+y)^2$
- The most common mistake is when expanding the bracket $(x^2+2xy+y^2)$ with negative sign. The answer should be $-x^2-2xy-y^2$ $-\sqrt{6}\times\sqrt{2}$
- A few students do not collect like terms after expanding $(x+y)(x^2+3xy+2y^2)$ or make mistakes when collecting like terms

3. Simplify $\frac{2^{a+2}-4}{2^a-1}$ **(2m)**

Sample answer:

$$= \frac{2^a 2^2 - 4}{2^a - 1}$$
$$= \frac{4(2^a - 1)}{2^a - 1}$$
$$= 4$$

Comments:

- Some students recognise $2^{a+2} = 2^a 2^2 \; or \; 2^a 4$ and will be rewarded 0.5 marks
- A few students attempt to multiply both numerator and denominator by $2^a + 1$ but this question does not require rationalising
- The common mistake is $2^{a+2} = 2^a + 2^2$
- Most students did well in the first line of working out but did not take 4 out as a common factor in the numerator
- Answer 4 without correct working out will be awarded 1 mark

4. The sums of money in the hands of 3 card players, A, B and C, are in the ratio $7:6:5$ at the beginning of the game and $6:5:4$ at the end of the game.

(a) Who won and who lost? **(2m)**

Sample answers:

	A	B	C
Before	35	30	25
After	36	30	24

Therefore, A won and C lost

Comments:

- This question is generally well done and students has used their knowledge and understanding of ratio
- Many different solutions are provided, but B should remain the same
- Students provide the answers without reasoning will be rewarded 1 mark

(b) If one of the players won \$120 find how much each of the players had at the beginning of the game. **(2m)**

Sample answer:

1 part = \$120

$$A = 35 \times 120 = 4200$$
$$B = 30 \times 120 = 3600$$
$$C = 25 \times 120 = 3000$$

Comments:

- Students could use their answers in part (a) to solve part (b)
- This question is well demonstrated and most students provided correct solutions in different methods

5. Show that $\frac{x-y}{x+y}\sqrt{\frac{x+y}{x-y}}=\sqrt{\frac{x-y}{x+y}}$ **(2m)**

Sample answer:

Method 1

$$LHS = \frac{(x-y)\sqrt{x+y}}{(x+y)\sqrt{x-y}}$$
$$= \frac{\sqrt{x-y}}{\sqrt{x+y}}$$
$$= \sqrt{\frac{x-y}{x+y}} = RHS\ as\ required$$

Method 2

$$LHS = \sqrt{\frac{(x-y)^2(x+y)}{(x+y)^2(x-y)}}$$
$$= \sqrt{\frac{x-y}{x+y}} = RHS\ as\ required$$

Method 3

$$(LHS)^2 = \frac{(x-y)^2(x+y)}{(x+y)^2(x-y)}$$
$$= \frac{(x-y)}{(x+y)} = (RHS)^2$$

Therefore, LHS = RHS as required

Comments:

- Students have attempted to solve this question using various methods
- A few students did method 3 and expand the bracket after the first line of working out but made some common mistakes in cancelling the numerator and denominator, therefore, they will be deducted marks
- Most students used the second method and achieved the correct results
- A few students recognise that $\sqrt{\frac{x+y}{x-y}}=\frac{\sqrt{x+y}}{\sqrt{x-y}}$ and will be rewarded 0.5 marks for this working out
- Students cannot make assumptions that this is true then square both sides or solving the equations

Section F Solution

1. If $1.68\times10^{21}=2^a\times3^b\times5^c\times7^d$, find the value of $a+b+c+d$. **(2m)**

$1.68\times10^{21} = 168\times10^{19}$

$= 2^3\times3\times7\times(5\times2)^{19}$

$= 2^3\times3\times7\times5^{19}\times2^{19}$

$= 2^{22}\times3^1\times5^{19}\times7^1$

$\therefore a=22 \quad b=1 \quad c=19 \quad d=1$

$\therefore a+b+c+d = 22+1+19+1$

$= 43$

Marking Scheme:

1 mark for correct working out.
1 mark for the correct answer for $a + b + c + d$.

Marker's comments:

- Many candidates found $2\times3\times5\times7=210$ which doesn't lead to the correct answer. No mark was given.
- Some candidates whom successfully found a, b, c, d did not answer the question $a + b + c + d$. Half a mark was penalised.

2. Write $(1-d)^{2a}(d-1)^2(1-c)^b(c-1)^4$ in the form $(1-d)^n(1-c)^n$. **(2m)**

$(1-d)^{2a}(d-1)^2(1-c)^b(c-1)^4 = (1-d)^{2a}(-(1-d)^2(1-c)^b(-(1-c))^4$

$= (1-d)^{2a}(1-d)^2(1-c)^b(1-c)^4$

$= (1-d)^{2a+2}(1-c)^{b+4}$

Marking Scheme:

1 mark for correct working.
1 mark for correct answer.

Marker's comments:

- Many candidates took out a (-1) out from $(d - 1)^2$ and $(c - 1)^4$, however it should be $(-1)^2$ and $(-1)^4$ respectively which is taking out 1. Any candidates whom wrote out two negatives were penalised as part of incorrect working.
- Significant number of candidates need to take more care with multiplication index law i.e. multiplying two equal bases means adding the powers, not multiply the powers.
- Candidates who expanded $(d-1)^2$ and $(c-1)^4$ did not receive any extra marks for the working as it did not contribute to the correct solution.

3. Orange juice concentrate, which is 25% water, is imported. A twelve-litre container of the orange juice concentrate has 4 litres removed. The container is filled up with water, mixed thoroughly and a further 4 litres of this mixture is then removed. If the container is filled up with water again, find what percentage of the final mixture is water. **(3m)**

	Mixture	Water	Orange Juice	
Originally:	12L	$0.25 \times 12 = 3L$	$0.75 \times 12 = 9L$	
4L removed	8L	$0.25 \times 8 = 2L$	$0.75 \times 8 = 6L$	
4L water added	12L	$2 + 4 = 6L$	6L	(Ratio : 1 : 1)
4L removed	8L	$6 - 2 = 4L$	$6 - 2 = 4L$	
4L water added	12L	$4 + 4 = 8L$	4L	

$\therefore$ Percentage of water $= \frac{8}{12} \times 100\%$

$= 66\frac{2}{3}\%$

Marking Scheme

- **2 marks** for correct working.
- **1 mark** for the correct exact percentage.

Marker's Comments

- Candidates whom showed the above working with actual amounts (rather than ratio or fractions) for each step were more successful in answering the question correctly.
- Significant number of candidates need to revise their percentages, the use of approximation symbol and recuring decimals. If candidates did not mention equal to $66\frac{2}{3}\%$ *or* $66.\dot{6}\%$ or *approximately* to 66.67%, they were penalised.

4. Simplify $\left(\frac{(2d^6k^3)^{\frac{-1}{3}} \times 8d^{\frac{3}{4}}k^{\frac{2}{3}}}{3d^{\frac{-1}{4}} \times \sqrt[3]{(4k)^4}} \right)$ **(3m)**

$= \frac{2^{-1/3} \times d^{-2} \times k^{-1} \times 8 \times d^{3/4} \times k^{2/3}}{3 \times d^{-1/4} \times 4^{4/3} \times k^{4/3}}$

$= \frac{2^{-1/3} \times 2^3 \times d^{-2} \times d^{3/4} \times k^{-1} \times k^{2/3}}{3 \times 2^{8/3} \times d^{-1/4} \times k^{4/3}}$ (as $4^{4/3} = 2^{2(4/3)} = 2^{8/3}$ & $8 = 2^3$)

$= \frac{2^{-1/3+3} \times d^{-2+3/4} \times k^{-1+2/3}}{3 \times 2^{8/3} \times d^{-1/4} \times k^{4/3}}$

$= \frac{2^{8/3} \times d^{-5/4} \times k^{-1/3}}{3 \times 2^{8/3} \times d^{-1/4} \times k^{4/3}}$

$= \frac{1 \times d^{-1} \times k^{-5/3}}{3} = \frac{1}{3dk^{5/3}}$

Marking Scheme

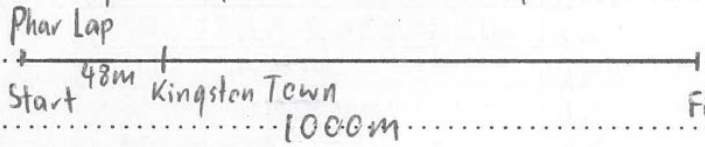
- **2 marks** for correct working out.
- **1 mark for** correct answer.

Marker's Comments

- This was done poorly by majority of the candidates.
- Significant number of errors occurred around index laws and silly mistakes.
- Candidates who changed all the numbers powers into its meaning e.g $d^{\frac{3}{4}} = \sqrt[4]{d^3}$ without any further progress did not gain any marks.
- Just because you have shown 'working' does not justify that your solution is worth any marks.

5. Racehorses Phar Lap and Kingston Town run a match race over 1 kilometre. Kingston Town is given a 48 metres head start. They run at their own constant speed and Phar Lap wins by 2 metres. How many metres had Phar Lap run when he passed Kingston Town? Show all reasoning. **(3m)**

let S_p = Speed of Phar Lap and S_K = Speed of Kingston Town.



Time = $\frac{\text{Distance}}{\text{Speed}}$

Time taken for Phar Lap to finish : $\frac{1000}{S_p}$

At the same for Kingston Town : $\frac{1000-48-2}{S_K} = \frac{950}{S_K}$

∴ Ratio of Speed : $\frac{1000}{S_p} = \frac{950}{S_K}$ (as time taken is equal)

$\frac{S_K}{S_p} = \frac{1000}{950} = \frac{20}{19}$ ∴ $S_K : S_p = 20 : 19$

Distance = Speed x time Phar Lap = $20 \times t$ Kingston Town = $48 + 19t$

Time when they meet:

$20t = 48 + 19t$ ∴ $t = 48$

∴ Distance for Phar lap $20 \times 48 = 960m$

End of paper

Marking scheme

- **1 mark** for finding the correct ratio of speeds (or equivalent).
- **1 mark** for the correct value of time that the racehorses meet with correct reasoning.
- **1 mark** for the correct answer.

Marker's comments:

- This was done poorly by many candidates.
- Significant number of candidates wrote that Phar Lap had travelled more than 1km, in a 1km race!
- Candidates who **only** wrote 950m in the difference of distance travelled between the two racehorses were not given any marks, as it was not enough to contribute to the correct solution.