Sydney Boys High School

MOORE PARK

YEAR 9 STAGE 5.3 MATHEMATICS

Half Yearly Examination 2019

General Instructions:

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer in the appropriate spaces.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing an ✕ next to your class.

Time Allowed: 90 minutes

Examiner: PSP

Name: _____

Class	Teacher	
9**A**	Mr A. Wang	
9**B**	Mr James	
9**C**	Ms H. Chan	
9**P**	Mr Fuller	
9**L**	Ms J. Chan	
9**U**	Ms Kilmore	
9S	Ms Millar & Mr R. Wang	

Section	Marks
A	/ 10
B	/ 10
C	/ 10
D	/ 10
E	/ 13
F	/ 13
G	/ 11
Total	/ 77

Section A – 10 Marks

(a) Write 678.9 in scientific notation. **1**

(b) Write 4.8945 correct to two significant figures. **1**

(c) Write 5^{-3} as a fraction in simplest form. **1**

(d) Simplify $4u - 5v + 6v - 6u$. **1**

(e) Simplify $4\sqrt{7} + 3\sqrt{7} - \sqrt{7}$. **1**

(f) Expand $-4(5j - 3)$. **1**

(g) Simplify $\sqrt{28}$. **1**

Section A (continued)

(h) Simplify $5m^5 \times 5m^5$. **1**

(i) Write the answer to $(13.8 \times 10^9) \times (5.4 \times 10^{-12})$ as an ordinary number correct to two significant figures. **2**

Use this space to rewrite any questions for Section A only.

Section B – 10 Marks

(a) Which of the following is rational? Circle the correct response. **1**

(A) $\sqrt[3]{18}$ (B) $\sqrt{8}$ (C) $0\cdot\dot{4}\dot{9}$ (D) π

(b) Write an expression for twice the sum of x and 3. **1**

(c) Simplify $-7y^0$. **1**

(d) Evaluate (i) $64^{\frac{1}{3}}$. **1**

(ii) $\left(2\sqrt{5}\right)^2$. **1**

(e) Expand and simplify

(i) $(x + 5)(2x – 1)$. **2**

(ii) $(x – 8)^2$. **1**

Section B (continued)

(f) Express $\frac{2}{\sqrt{6}}$ with a rational denominator, fully simplified. **1**

(g) If $h = k^3 - \frac{6m^2}{k}$, find the value of h when $k = -1$ and $m = 2$. **1**

Use this space to rewrite any questions for Section B only.

Section C – 10 Marks

(a) Which of the following is equal to $\sqrt{36y^{36}}$? Circle the correct response. **1**

(A)	$18y^6$	(B)	$6y^{18}$	(C)	$6y^6$	(D)	$18y^{18}$

(b) 30% of the people at Nahc's concert are girls. 1295 of the people at the concert are boys. How many girls are at her concert? **1**

(c) Express $\frac{3}{2x}+\frac{2x}{3}+3+2x$ as a single fraction. **2**

(d) Semaj cycles at a speed of 18 km/h. It takes him 55 minutes to cycle from one suburb to another. What is the distance that Semaj has travelled? **1**

(e) Simplify $p^{-2} \div p^{-4}$. **1**

Section C (continued)

(f) Simplify $2x^6y^{-3} \times 4xy^{-6}$, leaving your answer with non-negative indices. **2**

(e) Simplify $\dfrac{20r^5u^2}{2r^4u^2}$. **2**

Use this space to rewrite any questions for Section C only.

Section D – 10 Marks

(a) Simplify $2\sqrt{32} - 5\sqrt{50} - 2\sqrt{48}$. **2**

(b) Rallim has blue socks and pink socks in her drawer mixed in a ratio of 4 : 5, how many socks does she have to take out of the drawer to make sure she has a pair of the same colour? **1**

(b) Factorise $-6x^2 + 6x$. **1**

(c) Simplify $81^{\frac{3}{4}}$. **1**

(d) Simplify $x^{\frac{2}{3}} \div x^{-\frac{4}{3}}$. **1**

(e) Simplify $\left(\frac{8}{y^6}\right)^{-\frac{1}{3}}$. **1**

Section D (continued)

(f) Expand and simplify $\left(1-3\sqrt{3}\right)\left(2+6\sqrt{3}\right)$. **2**

(g) If $x < 0$, simplify $\sqrt{x^2}$. **1**

Use this space to rewrite any questions for Section D only.

Section E – 13 Marks

(a) In a raffle, the total prize money is shared among the first three tickets drawn in the ratio 5 : 3 : 2. **2**
The prize for the third ticket drawn is $300. What is the total prize money?

(b) The sides of Draw's triangle are 5.2 cm, 6.3 cm and 9.4 cm, each correct to the nearest millimetre. By considering the error in the above measurements, calculate the smallest possible perimeter of her triangle. **2**

(c) Show that $0.4\dot{3}\dot{6}$ can be written as $\frac{24}{55}$. **2**

(d) Factorise the following

(i) $9pq - 6p^2q + 12pq^2$. **1**

(ii) $7(h + k)^2 - 21(h + k)$. **1**

Section E (continued)

(c) Simplify $\frac{3x}{x^2+3x}$. **2**

(d) Express $\frac{1}{3x-2}-\frac{1}{3x+2}$ as a single fraction. **2**

(e) Gnaw performs a typing task for 7 hours. His average typing speeds for the first 3 hours and the last 4 hours are 63 words per minute (wpm) and 56 wpm respectively. **1**
Find his average typing speed for the 7 hours. Circle the correct response.

(A)	17 wpm	(B)	35 wpm	(C)	59 wpm	(D)	60 wpm

Use this space to rewrite any questions for Section E only.

Section F – 13 Marks

(a) The table shows the average energy used, in kilojoules per kilogram of body mass, by a person walking for 30 minutes at different speeds. **2**

Walking speed	*Energy used in 30 minutes*
3 km/h	5.53 kJ/kg
5 km/h	7.37 kJ/kg

Di Ett, who weighs 65 kg, drinks a regular cappuccino made with full cream milk.
It contains 73 kilocalories.
For approximately how long must Di walk at 5 km/h to burn off the energy contained in the cappuccino? (1 kilocalorie = 4.184 kJ). Leave your answer to the nearest minute.

(b) $\dfrac{\left(3-4\sqrt{x}\right)^2}{\sqrt{x}}$ can be written in the form $9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, for constants A and B. **2**

Write down the values of A and B.

(c) You are given that n, $n + 1$ and $n + 2$ are three consecutive integers.

(i) Expand and simplify $n^2 + (n + 1)^2 + (n + 2)^2$. **2**

(ii) For what values of n will the sum of the squares of these three consecutive integers be an even number? Justify. **1**

Section F (continued)

(d) Simplify $\frac{y^2 - x^2}{x^2 - y^2}$. **1**

(e) (i) Show that 126 km/h is the same speed as 35 m/s. **1**

(ii) Eromlik's train, of length 220 m, is travelling at 126 km/h and crosses a bridge of length 1400 m. How long, to the nearest second, did it take her train to completely cross the bridge? **2**

(f) There are only blue pens, green pens and red pens in a Relluf's box. **2**
The ratio of the number of blue pens to the number of green pens is 2 : 5.
The ratio of the number of green pens to the number of red pens is 4 : 1.
There are less than 100 pens in the box.
What is the greatest possible number of red pens in his box?

Use this space to rewrite any questions for Section F only.

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Any writing on this page will NOT be marked.

Section G – 11 Marks

(a) (i) Expand $\left(\sqrt{x}-\sqrt{y}\right)^2$. **2**

(ii) Using part (i), or otherwise, simplify $\sqrt{9-6\sqrt{2}}$. **2**

Section G (continued)

(b) (i) Consider the following pattern:

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$

Write down a simple expression for $1+2+3+...+n$. **1**

(ii) In the picture below, the side of each small square has length 1 cm. **2**
Using part (i) above, or otherwise, find how many lines of length 1 cm there are in the nth diagram?

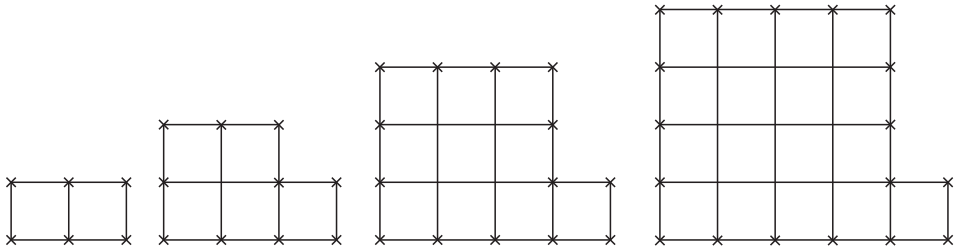


Diagram 1 Diagram 2 Diagram 3 Diagram 4

Section G (continued)

(c) (i) Show that $(x-1)(x^2+x+1)=x^3-1$. **1**

(ii) Consider $x+\frac{1}{x}=-1$. Do NOT try to find the value of x. **3**

By re-arranging this equation and by using part (i), or otherwise, find the value of

$$x^{2019}+\frac{1}{x^{2019}}.$$

End of paper

Use this space to rewrite any questions for Section G only.

SYDNEY
BOYS
HIGH
SCHOOL

2019

YEAR 9 HALF YEARLY
STAGE 5.3 MATHEMATICS

Sample Solutions

Sections	Teacher
A	AF
B	RW
C	JC
D	BK
E	HC
F	JJ
G	AYW

Section A – 10 Marks

(a) Write 678.9 in scientific notation. **1**

6.789×10^2

(b) Write 4.8945 correct to two significant figures. **1**

4.9

(c) Write 5^{-3} as a fraction in simplest form. **1**

$\frac{1}{5^3} = \frac{1}{125}$

(d) Simplify $4u - 5v + 6v - 6u$. **1**

$-2u + v$

(e) Simplify $4\sqrt{7} + 3\sqrt{7} - \sqrt{7}$. **1**

$6\sqrt{7}$

(f) Expand $-4(5j - 3)$. **1**

$-20j + 12$

(g) Simplify $\sqrt{28}$. **1**

$\sqrt{4 \times 7} = 2\sqrt{7}$

Section A (continued)

(h) Simplify $5m^5 \times 5m^5$. **1**

$25m^{10}$

(i) Write the answer to $(13.8 \times 10^9) \times (5.4 \times 10^{-12})$ as an ordinary number correct to two significant figures. **2**

$0.07452 = 0.075$ (2 sig. figures)

Use this space to rewrite any questions for Section A only.

Section B – 10 Marks

(a) Which of the following is rational? Circle the correct response. **1**

(A) $\sqrt[3]{18}$	(B) $\sqrt{8}$	(C) $0.\dot{4}\dot{9}$	(D) π

(b) Write an expression for twice the sum of x and 3. **1**

$2(x+3)$	Equivalent forms such as $2x+6$ accepted No half marks

(c) Simplify $-7y^0$. **1**

$-7y^0 = -7 \times 1$ $= -7$	No half marks

(d) Evaluate (i) $64^{\frac{1}{3}}$. **1**

$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$	No half marks

(ii) $\left(2\sqrt{5}\right)^2$. **1**

$\left(2\sqrt{5}\right)^2 = 2 \times \sqrt{5} \times 2 \times \sqrt{5}$ $= 4 \times 5$ $= 20$	No half marks

(e) Expand and simplify

(i) $(x + 5)(2x – 1)$. **2**

$(x+5)(2x-1)$ $= x(2x-1) + 5(2x-1)$ $= 2x^2 - x + 10x - 5$ $= 2x^2 + 9x - 5$	Students who did not expand the expression first could only score a maximum of 1 mark

(ii) $(x – 8)^2$. **1**

$(a-b)^2 = a^2 - 2ab + b^2$ $\therefore (x-8)^2 = x^2 - (2 \times x \times 8) + 8^2$ $= x^2 - 16x + 64$	Students who expanded incorrectly could not score half marks if no working out was provided.

Section B (continued)

(f) Express $\frac{2}{\sqrt{6}}$ with a rational denominator, fully simplified. **1**

$\frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ $= \frac{2\sqrt{6}}{6}$ $= \frac{\sqrt{6}}{3}$	Many students forgot to simplify after rationalising

(g) If $h = k^3 - \frac{6m^2}{k}$, find the value of h when $k = -1$ and $m = 2$. **1**

$h = k^3 - \frac{6m^2}{k}$ $= (-1)^3 - \frac{6 \times 2^2}{-1}$ $= 23$	Students who did not substitute values for k and m into the expression correctly could not score half marks

General Feedback:

Students generally performed well in this section.

The most common error occurred in Part F due to forgetting to simplify after rationalising.

Section C – 10 Marks

$(36y^{36})^{\frac{1}{2}} = 6y^{18}$

Some students took $\sqrt{36}$ for the power.

(a) Which of the following is equal to $\sqrt{36y^{36}}$? Circle the correct response. **1**

(A)	$18y^6$	(B)	$6y^{18}$ ✓	(C)	$6y^6$	(D)	$18y^{18}$

(b) 30% of the people at Nahc's concert are girls. 1295 of the people at the concert are boys. How many girls are at her concert? **1**

70% = 1295 people.

Girls = $\frac{1295}{70} \times 30$

= 555 girls ✓

(c) Express $\frac{3}{2x}+\frac{2x}{3}+3+2x$ as a single fraction. **2**

$\frac{3}{6x}(3) + \frac{2x}{6x}(2x) + \frac{3(6x)}{6x} + \frac{2x(6x)}{6x}$

$= \frac{9+4x^2+18x+12x^2}{6x}$

$= \frac{16x^2+18x+9}{6x}$

(d) Semaj cycles at a speed of 18 km/h. It takes him 55 minutes to cycle from one suburb to another. What is the distance that Semaj has travelled? **1**

$\frac{18}{60} \times 55$

= 16.5 km ✓

(e) Simplify $p^{-2} \div p^{-4}$. **1**

$p^{-2-(-4)} = p^2$ ✓

Some careless mistake of negative signs. Answer is not p^{-6}

Section C (continued)

(f) Simplify $2x^6y^{-3} \times 4xy^{-6}$, leaving your answer with non-negative indices. **2**

$8x^7y^{-9}$ ✓

$= \frac{8x^7}{y^9}$ ✓

(e) Simplify $\frac{20r^5u^2}{2r^4u^2}$. **2**

$10r$

✓ ✓

Use this space to rewrite any questions for Section C only.

Section C

c) Major problem with the $12x^2$ term.
Many students had $2x \times 6x = 12x$.

$\frac{1}{2}$ mark only for $\frac{9+4x^2}{6x}$.

Section D – 10 Marks

(a) Simplify $2\sqrt{32}-5\sqrt{50}-2\sqrt{48}$. **2**

$2\times 4\sqrt{2}-5\times 5\sqrt{2}-2\times 4\sqrt{3}$ ✓

$=8\sqrt{2}-25\sqrt{2}-8\sqrt{3}$

$=-17\sqrt{2}-8\sqrt{3}$ ✓

(b) Rallim has blue socks and pink socks in her drawer mixed in a ratio of 4 : 5, how many socks does she have to take out of the drawer to make sure she has a pair of the same colour? **1**

3 socks → 2 in a pair.

(b) Factorise $-6x^2+6x$. **1**

$-6x(x-1)$ or $6x(-x+1)$

(c) Simplify $81^{\frac{3}{4}}$. **1**

$=\left(\sqrt[4]{81}\right)^3=3^3=27$

(d) Simplify $x^{\frac{2}{3}}\div x^{-\frac{4}{3}}$. **1**

$x^{\frac{2}{3}-\left(-\frac{4}{3}\right)}=x^2$.

(e) Simplify $\left(\frac{8}{y^6}\right)^{-\frac{1}{3}}$. **1**

$\left(\frac{y^6}{8}\right)^{\frac{1}{3}}=\frac{y^2}{2}$.

Section D (continued)

(f) Expand and simplify $\left(1-3\sqrt{3}\right)\left(2+6\sqrt{3}\right)$. **2**

$2+6\sqrt{3}-6\sqrt{3}-18\times3$ ✓

$= 2-54$

$=-52$ ✓

(g) If $x<0$, simplify $\sqrt{x^2}$. eg. $\sqrt{(-3)^2} = \sqrt{9} = 3$ **1**

$-x$ (which is positive)

Use this space to rewrite any questions for Section D only.

Section E

(a) In a raffle, the total prize money is shared among the first three tickets drawn in the ratio 5 : 3 : 2. 2
The prize for the third ticket drawn is \$300. What is the total prize money?

Let the total prize be x

$$\frac{2x}{(5+3+2)} = 300$$

$$x = 1500$$

$\therefore$ the total prize money is \$1500

(b) The sides of Draw's triangle are 5.2 cm, 6.3 cm and 9.4 cm, each correct to the nearest millimetre. By considering the error in the above measurements, calculate the smallest possible perimeter of her triangle. 2

$(5.2 - 0.05 + 6.3 - 0.05 + 9.4 - 0.05)$ cm

$= 20.75$ cm.

(c) Show that $0.4\dot{3}\dot{6}$ can be written as $\frac{24}{55}$. 2

$x = 0.436363636\ldots$

$10x = 4.363636\ldots$ ①

$1000x = 436.363636\ldots$ ②

② − ①

$990x = 432$

$x = \frac{432}{990}$

$x = \frac{24}{55}$

(d) Factorise the following

(i) $9pq - 6p^2q + 12pq^2$. 1

$3pq(3 - 2p + 4q)$

(ii) $7(h+k)^2 - 21(h+k)$. 1

$7(h+k)[(h+k) - 3]$

Section E (continued)

(c) Simplify $\frac{3x}{x^2+3x}$. **2**

$$\frac{3x}{x(x+3)}$$

$$= \frac{3}{x+3}$$

(d) Express $\frac{1}{3x-2} - \frac{1}{3x+2}$ as a single fraction. **2**

$$\frac{3x+2-(3x-2)}{(3x-2)(3x+2)}$$

$$= \frac{3x+2-3x+2}{9x^2-4} = \frac{4}{9x^2-4}$$

(e) Gnaw performs a typing task for 7 hours. His average typing speeds for the first 3 hours and the last 4 hours are 63 words per minute (wpm) and 56 wpm respectively. Find his average typing speed for the 7 hours. Circle the correct response. **1**

(A) 17 wpm (B) 35 wpm (C) 59 wpm (D) 60 wpm

Use this space to rewrite any questions for Section E only.

$$(3\times 60\times 63 + 4\times 60\times 56) \div (7\times 60)$$

$$= 59 \text{ wpm}.$$

Section F – 13 Marks

(a) The table shows the average energy used, in kilojoules per kilogram of body mass, by a person walking for 30 minutes at different speeds. **2**

Walking speed	*Energy used in 30 minutes*
3 km/h	5.53 kJ/kg
5 km/h	7.37 kJ/kg

Di Ett, who weighs 65 kg, drinks a regular cappuccino made with full cream milk.
It contains 73 kilocalories.
For approximately how long must Di walk at 5 km/h to burn off the energy contained in the cappuccino? (1 kilocalorie = 4.184 kJ). Leave your answer to the nearest minute.

73 kC x 4.184 J/C = 305.432 kJ & 65 kg x 7.37 kJ/kg ÷ 30mins = 15.96 kJ/min

305.432 kJ ÷ 15.96 kJ/min = 19.4 mins ($\frac{1}{2}$ mark off if you answered 19.4 mins or 20 mins)

Closest minute is 19 mins ($\frac{1}{2}$ mark give to each operation done appropriately but $\frac{1}{2}$ mark deducted for an absurd answer – completely unreasonable)

(b) $\dfrac{\left(3-4\sqrt{x}\right)^2}{\sqrt{x}}$ can be written in the form $9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, for constants A and B. **2**

Write down the values of A and B.

$\dfrac{\left(3-4\sqrt{x}\right)^2}{\sqrt{x}} = \dfrac{(3-4\sqrt{x})(3-4\sqrt{x})}{\sqrt{x}}$ 1 mark

$= \dfrac{9-24\sqrt{x}+16x}{\sqrt{x}}$

$= \dfrac{9}{\sqrt{x}} + 16\sqrt{x} - 24$ $\frac{1}{2}$ mark

From below this can be rewritten as:

$9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$, ∴ A = 16 & B = -24

$\frac{1}{2}$ mark

(c) You are given that n, $n + 1$ and $n + 2$ are three consecutive integers.

(i) Expand and simplify $n^2 + (n + 1)^2 + (n + 2)^2$. **2**

Expanding produces: $n^2 + (n^2 + 2n + 1) + (n^2 + 4n + 4)$ 1 mark

Simplifying produces: $3n^2 + 6n + 5$ 1 mark

(ii) For what values of n will the sum of the squares of these three consecutive integers be an even number? Justify. **1**

'n' must be odd. $6n$ is always even, 5 is always odd so $3n^2$ must be odd to make the sum even. OddxOdd is always odd, so n must be odd and can't be even (odd x even = even).
($\frac{1}{2}$ mark for stating n correctly and $\frac{1}{2}$ mark for a reasonable justification)

Section F (continued)

(d) Simplify $\frac{y^2 - x^2}{x^2 - y^2}$. **1**

$\frac{-(x^2 - y^2)}{x^2 - y^2} = \frac{-1}{1}$

= -1 (1 mark)

If you used the difference of 2 squares but didn't get this result and only got: $\frac{y-x}{x-y}$, you recieved $\frac{1}{2}$ mark

(e) (i) Show that 126 km/h is the same speed as 35 m/s. **1**

126 km = 126000 m & 1hr = 3600 s

126000 m ÷ 3600 s = 35 m/s (no half marks here)

(ii) Eromlik's train, of length 220 m, is travelling at 126 km/h and crosses a bridge of length 1400 m. How long, to the nearest second, did it take her train to completely cross the bridge? **2**

The train will end up travelling across the whole 1400 m and then the last 220 m of the train will have to cross the bridge: 1400 m + 220 m = 1620 m (1 mark)

Because the answer requires the nearest second, we should use the converted measure in part i). *time* = *distance/speed*, so *time* = 1620 m / 35 m/s
= 46.29 to 2 decimal places (1 mark)

Therefore the nearest second is 46 (47 was paid) ($\frac{1}{2}$ mark off if you answered 46.29 s)

(f) There are only blue pens, green pens and red pens in a Relluf's box. **2**
The ratio of the number of blue pens to the number of green pens is 2 : 5.
The ratio of the number of green pens to the number of red pens is 4 : 1.
There are less than 100 pens in the box.
What is the greatest possible number of red pens in his box?

Between the two ratios green is common. We need to adjust both ratios until green is equal for both. HCF between 5 and 4 is 20.

2:5 → 8:20 (multiply both by 4) & 4:1 → 20:5 (multiply both by 5)
We can now create a ratio for all three… 8:20:5 1 mark

Total number of parts in this ratio is 8+20+5=33 of which 5 pens are red.
33 fits *under* 100, 3 times. (3 x 33 = 99) $\frac{1}{2}$ mark
Therefore the maximum number of red pens is 3 x 5 = 15 $\frac{1}{2}$ mark

Use this space to rewrite any questions for Section F only.

Section G – 11 Marks

(a) (i) Expand $\left(\sqrt{x}-\sqrt{y}\right)^2$. **2**

$$\left(\sqrt{x}-\sqrt{y}\right)^2 = \left(\sqrt{x}\right)^2 - 2\left(\sqrt{x}\right)\left(\sqrt{y}\right) - \left(\sqrt{y}\right)^2$$
$$= x - 2\sqrt{xy} + y$$

1 mark for correct expansion.
1 mark for correct answer.

(ii) Using part (i), or otherwise, simplify $\sqrt{9-6\sqrt{2}}$. **2**

Let $\sqrt{x}-\sqrt{y} = \sqrt{9-6\sqrt{2}}$

$(\sqrt{x}-\sqrt{y})^2 = 9-6\sqrt{2}$

$x - 2\sqrt{xy} + y = 9 - 6\sqrt{2}$ (From a) i))

$x + y - \sqrt{4xy} = 9 - \sqrt{72}$ (*)

Comparing both sides of (*)

$x + y = 9$

$4xy = 72 \rightarrow xy = 18$

By inspection and $x > y$ (due to $\sqrt{x}-\sqrt{y} \geq 0$)

$\therefore\ x = 6$

$y = 3$

$\therefore \sqrt{9-6\sqrt{2}} = \sqrt{6}-\sqrt{3}$

1 mark for correct expansion.
1 mark for correct answer.

Note: No marks for typing the result in a calculator and giving a decimal answer, as the question is simplifying, not evaluating.

(b) (i) Consider the following pattern:

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$

Write down a simple expression for $1+2+3+...+n$. **1**

Pattern: Triangular Numbers - The working below in not necessary, but just for explanation.

Let $S_1 = 1$

$$S_1 = \frac{1+1}{2} = \frac{1\times 2}{2} = 1$$

Let $S_2 = 1 + 2 = 3$

$$S_2 = \frac{(1+2)+(2+1)}{2} = \frac{2\times 3}{2} = 3$$

Let $S_3 = 1+2+3 = 6$

$$S_3 = \frac{(1+2+3)+(3+2+1)}{2} = \frac{(1+3)+(2+2)+(3+1)}{2} = \frac{3\times 4}{2} = 6$$

$\therefore$ Let $S_n = 1+2+3+...+(n-2)+(n-1)+n$

$$S_n = \frac{[1+2+3+...+(n-2)+(n-1)+n]+[n+(n-1)+(n-2)+....+3+2+1]}{2}$$

$$= \frac{(1+n)+(2+(n-1))+(3+(n-2))+....+((n-2)+3)+((n-1)+2)+(n+1)}{2}$$

$$= \frac{(n+1)\times n}{2}$$

$$= \frac{n^2+n}{2}$$

1 mark for correct answer

(ii) In the picture below, the side of each small square has length 1 cm. **2**
Using part (i) above, or otherwise, find how many lines of length 1 cm there are in the *n*th diagram?

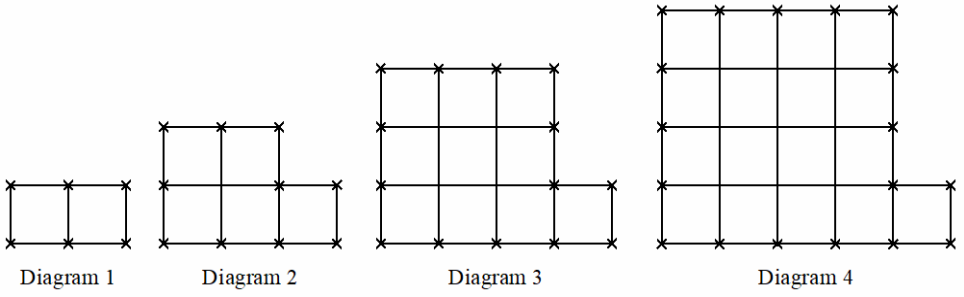


Diagram 1 = 7 lines $= \mathbf{1} \times 4 + 3$

Diagram 2 = 15 lines $= \mathbf{3} \times 4 + 3$

Diagram 3 = 27 lines = $\mathbf{6} \times 4 + 3$

Diagram 4 = 43 lines = $\mathbf{10} \times 4 + 3$

$$\therefore \text{ Diagram } n = \frac{n(n+1)}{2} \times 4 + 3$$
$$= 2n(n+1) + 3$$
$$= 2n^2 + 2n + 3$$

1 mark for finding the correct pattern
1 mark for the correct answer

(c) (i) Show that $(x-1)(x^2+x+1) = x^3 - 1$. **1**

$$\text{LHS} = x^3 + x^2 + x - x^2 - x - 1$$
$$= x^3 - 1$$
$$= \text{RHS}$$

1 mark for showing the correct expansion

(ii) Consider $x+\frac{1}{x}=-1$. Do NOT try to find the value of x. **3**

By re-arranging this equation and by using part (i), or otherwise, find the value of

$$x^{2019}+\frac{1}{x^{2019}}.$$

$x+\frac{1}{x}=-1$

$x^2+1=-x$ (Multiply the previous line by x)

$x^2+x+1=0$ (#)

From c) i),

$(x-1)(x^2+x+1)=x^3-1$

Sub in $(\#)$

$0\times(x^2+x+1)=x^3-1$

$\therefore x^3-1=0$

$x^3=1$

Now $x^{2019}=\left(x^3\right)^{673}=1^{673}=1$

$\therefore\ x^{2019}+\frac{1}{x^{2019}}=1+\frac{1}{1}$

$=2$

1 mark for rearranging $x+\frac{1}{x}=-1$ to get $x^2+x+1=0$.

1 mark for showing correct working.

1 mark for correct answer.