Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 9 STAGE 5.3 MATHEMATICS

Half Yearly Examination 2018

General Instructions:

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of this booklet. **Show the number and part of the answer being rewritten.**
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing an ✕ next to your class.

Time Allowed: 90 minutes

Examiner: TE

Name: _____

Class	Teacher	
9**A**	Ms Kilmore	
9**B**	Mr Gurjar	
9**C**	Ms Millar	
9**P**	Mr Choy	
9**L**	Mr Gainford	
9**U**	Mr Wang	
9S	Ms A Ward	

Section	Marks
A	/ 15
B	/ 13
C	/ 13
D	/ 13
E	/ 13
F	/13
G	/ 10
Total	/90

Section A - 15 Marks

(a)	Increase \$400 by 30%	**1**
(b)	What is the volume of a 6 cm cube?	**1**
(c)	Calculate: i) $\frac{6.873}{5.21\times3.06}$ to 4 decimal places ii) $\sqrt[3]{5.498\times10^6}$ to 2 significant figures	**1** **1**
(d)	Simplify: i) $6d-d+4e-5d$ ii) $3b^4\times2b\times4b^3$ iii) $15x^3y^4\div5x^2y^2$ iv) $\frac{3a}{4}-\frac{a}{2}$	**1** **1** **1** **1**

(e)	Write $\frac{4}{27}$ as a decimal	**1**
(f)	Evaluate $8^{\frac{2}{3}}$	**1**
(g)	Write 7 986 000 in standard form	**1**
(h)	Write 0.000001234 in scientific notation	**1**
(i)	Write $3x^{-4}$ without negative indices	**1**
(j)	How many metres in *y* kilometres?	**1**
(k)	What is the supplement of 72°?	**1**

End of section A

Section B – 13 Marks

(a)	Write in simplest surd form: i) $(8x)^{\frac{1}{3}}$ ii) $(16m^2)^{\frac{-3}{4}}$	**1** **1**
(b)	Expand and simplify: i) $(2x+5)^2$ ii) $6x(2x+1)-x(4x+3)$ iii) $(4x+3)(4x-3)$	**1** **1** **1**
(c)	Find the value of $\frac{c}{2}(a^2-b)(b^2-a)$ if $a=6.3,\ b=8.9,\ c=18.3$ to 2 decimal places	**2**

(d)	Simplify: i) $5x^5 \times 2x^2 + 3x^3$ ii) $m^{x+1} \times m^{x-2}$	**1** **1**
(e)	Circle the rational numbers: $\sqrt{7}, \quad \pi, \quad \left(\frac{1}{\sqrt{2}}\right)^2, \quad 4.\dot{6}, \quad \sqrt{\frac{16}{25}}$	**2**
(f)	If we record the height of a tree to be 8.0 metres. What is the greatest possible error of this measurement? What range could the height of the tree be?	**1** **1**

End of Section B

Section C – 13 Marks

(a)	If 2:30pm was 12 minutes ago, in how many minutes will it be 3:00pm?	**1**
(b)	Simplify: i) $\sqrt{63}$	**1**
	ii) $\sqrt{\frac{3}{8}}$	**1**
	iii) $5\sqrt{10}\times 4\sqrt{5}$	**2**
	iv) $12\sqrt{7}\div 4\times 2\sqrt{5}$	**1**
	v) $2\sqrt{12}+3\sqrt{27}$	**2**

(c)	Factorise: i) $3x+6y$ ii) $8b^2-16b$ iii) x^3-3x^2-3x iv) $y(x+3z)+4(x+3z)$	**1** **1** **1** **1**
(e)	A truck carries 15 tonne of coal. How many trucks would be needed to move 190 cubic metres of coal, if there are 1.2 cubic metres in one tonne of coal?	**1**

End of section C

Section D – 13 Marks

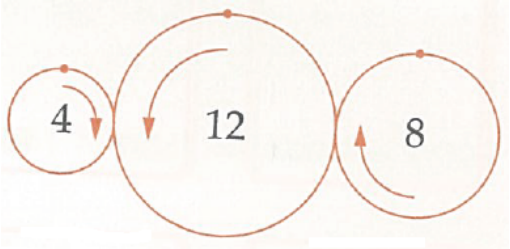
(a)	Rationalise the denominator of the following surds: i) $\frac{3}{4\sqrt{2}}$ ii) $\frac{5}{4-\sqrt{3}}$	**1** **2**
(b)	Express $3.\dot{3}0\dot{4}$ as a fraction in its simplest form.	**1**
(c)	Expand and simplify $(x^2+2x+1)(x^2-2x+1)$	**2**

(d)	Simplify the following algebraic fractions: i) $\frac{x}{3}+\frac{2}{3x}$ ii) $\frac{4}{3x}+\frac{2}{9x^2}$ iii) $1-\frac{1}{x}$ iv) $\frac{4x^4y^3}{(2xy)^3}\div\frac{(2x^2y)^2}{16x^2y^3}$	**1** **1** **1** **2**
(e)	An equilateral triangle and a square have the same perimeter. The sides of the triangle are 2 cm longer than the sides of the square. What is the perimeter?	**2**

End of section D

Section E – 13 Marks

(a)	Solve for x: $$\frac{1}{2\sqrt{5}+\sqrt{x}}+\frac{1}{2\sqrt{5}-\sqrt{x}}=2\sqrt{5}$$	**3**
(b)	If $y=\frac{1}{x^2}+\frac{1}{x}$ and $x=\frac{t}{1-t}$, show that $y=\frac{1-t}{t^2}$	**3**

(c)	Burt expanded a product using the distributive law but, unfortunately, he erased the terms in the second pair of brackets. $$(3x+1)(.....)=15x^2-13x-6$$ What are the missing terms in the second pair of brackets?	**2**
(d)	Simplify $\dfrac{m^{-1}+n^{-1}}{m+n}$	**3**
(e)	In a system of 3 gear wheels, the first wheel has 4 teeth, the second 12 teeth, and the third 8 teeth. How many revolutions will the small wheel have to make before all wheels are back in their starting positions?  	**2**

End of section E

Section F – 13 Marks

(a)	Jeremiah had a book of raffle tickets. On Monday he sold 10 tickets to his family. On Tuesday he sold ½ of the remaining tickets to her neighbours. On Wednesday he sold 5 tickets to his school friends and his maths teacher bought the two tickets remaining. How many tickets were in the book to start with?	**2**
(b)	A painter had 3 tins A, B and C holding 1, 3 and 5 litres respectively. A is empty, B is full of blue paint and C is full of white paint. The painter fills A from B, he then fills B from C and pours the contents of A into C. He then repeats the process. How much blue paint is there now in C?	**3**
(c)	At a school 15 students were absent on Monday, 12 absent on Tuesday and 9 absent on Wednesday. If 22 students were absent at least once during these three days, what is the maximum number of students who could have been absent on all three days?	**2**

(d)	Three people bought a pet one day. Their names were Sonic, Mario and Princess Peach. The animals they bought were a turtle, a parrot and a guinea pig. They bought their pets for \$25, \$50 and \$100. Use the following clues to figure out which person bought which animal and for how much. (i) Sonic is allergic to birds. (ii) Princess Peach couldn't get the \$100 pet, so she bought the turtle. (iii) Mario saved \$10 a week for 10 weeks in order to get his pet. (iv) The mammal was \$25.	**3**
(e)	Angela and Barry share a piece of land. The ratio of the area of Angela's portion to the area of Barry's portion is 3 : 2. They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio 7 : 3. On Angela's portion of land, the ratio of corn to peas is 4 : 1. What is the ratio of corn to peas for Barry's portion?	**3**

End of section F

Section G – 10 Marks

(a)	(i) Show that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	**2**
	(ii) Hence show that $x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$ is a solution of the equation $x^3 - 6x - 6 = 0$.	**3**

(b)	The fraction $\frac{37}{13}$ can be written in the form $2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$, where x, y and z are integers. Find x, y, and z.	**3**
(c)	Suppose A, B and C are three numbers for which 1001C – 2002A = 4004 and 1001B + 3003A = 5005. What is the average of the three numbers A, B and C ?	**2**

End of the test

Additional Working Space:

SYDNEY
BOYS
HIGH
SCHOOL

2018

YEAR 9 HALF YEARLY
STAGE 5.3 MATHEMATICS

Sample Solutions

Sections	Teacher
A	AW
B	AYW
C	AMG
D	EC
E	JM
F	SG
G	BK

Section A - 15 Marks

(a) Increase $400 by 30% — **1**

$400 \times 1.3 = \$520$

VERY WELL DONE

(b) What is the volume of a 6 cm cube? — **1**

$6^3 = 216\ cm^3$

VERY WELL DONE

(c) Calculate:

i) $\dfrac{6.873}{5.21 \times 3.06}$ to 4 decimal places — **1**

$= 0.4311$ (4dp)

VERY WELL DONE

ii) $\sqrt[3]{5.498 \times 10^6}$ to 2 significant figures — **1**

$= 180$ (2sig fig)

POORLY DONE
CONFUSION OVER SIG FIG'S WAS COMMON

(d) Simplify:

i) $6d - d + 4e - 5d$ — **1**

$= 4e$

EXTREMELY WELL DONE

ii) $3b^4 \times 2b \times 4b^3$ — **1**

$= 24b^7$

MOST ANSWERED CORRECTLY

iii) $15x^3y^4 \div 5x^2y^2$ — **1**

$= 3xy^2$

MOST ANSWERED CORRECTLY

iv) $\dfrac{3a}{4} - \dfrac{a}{2}$ — **1**

$= \dfrac{3a}{4} - \dfrac{2a}{4}$

$= a/4$

POORLY DONE
MANIPULATION OF FRACTIONS WAS AN ISSUE FOR SOME

(e)	Write $\frac{4}{27}$ as a decimal $0.\overline{148}$	SOME DID NOT RECOGNISE THAT EXACT ANSWER WAS A REPEATING DECIMAL - FAIRLY WELL DONE	**1**
(f)	Evaluate $8^{\frac{2}{3}}$ 4	IT APPEARED THAT SOME DID NOT REALISE THEY COULD USE THEIR CALCULATOR - FAIRLY WELL DONE.	**1**
(g)	Write 7 986 000 in standard form 7.986×10^6	- VERY POORLY DONE - Standard and scientific are the same	**1**
(h)	Write 0.000001234 in scientific notation 1.234×10^{-6}	- REASONABLY WELL DONE - INCORRECT NOTATION WAS THE MOST COMMON ERROR.	**1**
(i)	Write $3x^{-4}$ without negative indices $\frac{3}{x^4}$	- VERY POORLY DONE $\frac{1}{2}$ FOR x^4 IN DENOMINATOR	**1**
(j)	How many metres in *y* kilometres? 1000y metres	VERY WELL DONE	**1**
(k)	What is the supplement of 72°? $180° - 72° = 108°$	REASONABLY WELL DONE Demonstration of correct equation but incorrect calculation $\frac{1}{2}$	**1**

End of section A

Section B – 13 Marks

(a) Write in simplest surd form:

i) $(8x)^{\frac{1}{3}}$ $2x\sqrt[3]{x}$ (1) mark for correct answer — **1**

ii) $(16m^2)^{\frac{-3}{4}}$ $\frac{1}{\sqrt[4]{(16m^2)^3}} = \frac{1}{8} \times \frac{1}{\sqrt[4]{m^6}} = \frac{1}{8\sqrt{m^3}}$ — **2**

(1) mark for correct working
(1) mark for correct answer

(b) Expand and simplify:

i) $(2x+5)^2 = 4x^2 + 20x + 25$ (1) mark for correct answer — **1**

ii) $6x(2x+1) - x(4x+3) = 12x^2 + 6x - 4x^2 - 3x$
$= 8x^2 + 3x$
(1) mark for correct answer — **1**

iii) $(4x+3)(4x-3) = 16x^2 - 9$
(1) mark for correct answer. — **1**

(c) Find the value of $\frac{c}{2}(a^2-b)(b^2-a)$ if $a=6.3,\ b=8.9,\ c=18.3$ to 2 decimal places — **1**

20540.82 (1) mark for correct answer.

(d) Simplify:

i) $5x^5 \times 2x^2 + 3x^3$ = $10x^7 + 3x^3$ **1**

(1) mark for correct answer

ii) $m^{x+1} \times m^{x-2}$ **1**

$= m^{x+1+x-2}$

$= m^{2x-1}$

(1) mark for correct answer

(e) Circle the rational numbers: **2**

$\sqrt{7}$, π, $\left(\frac{1}{\sqrt{2}}\right)^2$, $4.\dot{6}$, $\sqrt{\frac{16}{25}}$

(2) marks for all 3 correct answers circled

(1) mark for only 2 correct answers circled

or 3 correct answers and extra answer circled.

(f) If we record the height of a tree to be 8.0 metres. **1**

What is the greatest possible error of this measurement?

0.1m ÷ 2 = 0.05m

(1) mark for correct answer.

1

What range could the height of the tree be?

7.95m to 8.05m

(1) mark for correct answer.

End of Section B

Section C – 13 Marks Many erroniously wrote $\sqrt[3]{7}$ for $3\sqrt{7}$, etc

(a)	If 2:30pm was 12 minutes ago, in how many minutes will it be 3:00pm? 18	**1**
(b)	Simplify: i) $\sqrt{63}$ $= \sqrt{9\times7} = 3\sqrt{7}$	**1**
	ii) $\sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{6}}{3}$ [both answers accepted]	**1**
	iii) $5\sqrt{10}\times4\sqrt{5} = 5\sqrt{2}\sqrt{5}\times4\sqrt{5} = 100\sqrt{2}$ [$20\sqrt{50}$ received 1 mark]	**2**
	iv) $12\sqrt{7}\div4\times2\sqrt{5} = \frac{12\sqrt{7}}{4}\times2\sqrt{5} = 6\sqrt{35}$	**1**
	v) $2\sqrt{12}+3\sqrt{27} = 4\sqrt{3}+9\sqrt{3} = 13\sqrt{3}$	**2**

(c) Factorise:

i) $3x+6y$ = $3(x+2y)$ — 1

ii) $8b^2-16b$ = $8b(b-2)$ — 1

iii) x^3-3x^2-3x = $x(x^2-3x-3)$ — 1

iv) $y(x+3z)+4(x+3z)$ = $(x+3z)(y+4)$ — 1

(e) A truck carries 15 tonne of coal. How many trucks would be needed to move 190 cubic metres of coal, if there are 1.2 cubic metres in one tonne of coal? — 1

$190\ m^3 = \frac{190}{1.2}\ t$

$= 158.3\ t$

No. of trucks $= \frac{158.3}{15}$

$= 10.5$

$\therefore$ 11 trucks req'd.

End of section C

Section D (13)

(a) (i) $\frac{3}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{8}$ (1)

(ii) $\frac{5}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$

$= \frac{5(4+\sqrt{3})}{13} = \frac{20+5\sqrt{3}}{13}$ (2)

(b) Let $x = 3\dot{0}4\dot{3}04\ldots$

$\therefore 1000x = 304.\dot{3}0\dot{4}$

$-x = -0.\dot{3}0\dot{4}$ (1)

$\therefore 999x = 304$

$\therefore x = \frac{304}{999}$

7978
22

ie $3.\dot{3}0\dot{4} = 3\frac{304}{999}$ (2)

(c) $(x^2+2x+1)(x^2-2x+1)$

$= (x+1)^2(x-1)^2$

$= (x^2-1)^2 = x^4-2x^2+1$

(d) $\frac{x}{3} + \frac{2}{3x} = \frac{x^2+2}{3x}$ (1)

(i) Common den

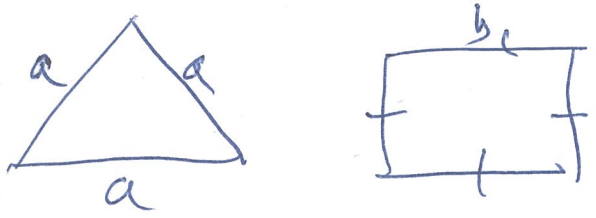
(ii) $\frac{4}{3x} + \frac{2}{9x^2} = \frac{12x+2}{9x^2} = \frac{2(6x+1)}{9x^2}$ (1)

(iii) $1 - \frac{1}{x} = \frac{x-1}{x}$ (1)

(iv) $\frac{4x^4y^3}{8x^3y^3} \times \frac{16x^2y^3}{4x^4y^2} = \frac{2y}{x^2}$ (1)

2 y x²

(e)



(2)

x^3y^4 / x^7y^5

$3a = 4b$ —— (1)

$a = b+2$ —— (2)

$3b+6 = 4b \quad \therefore b = 6$

$a = 8$.

$\therefore p = 24$.

Section E – 13 Marks

(a) Solve for x: **3**

$$\frac{1}{2\sqrt{5}+\sqrt{x}}+\frac{1}{2\sqrt{5}-\sqrt{x}}=2\sqrt{5}$$

$$\frac{2\sqrt{5}-\sqrt{x}+2\sqrt{5}+\sqrt{x}}{20-x}=2\sqrt{5}$$

$$4\sqrt{5}=2\sqrt{5}(20-x)$$

$$2=20-x$$

$$\therefore x=18$$

- very poorly done

- many students forgot that when adding fractions we need a common denominator

(b) If $y=\frac{1}{x^2}+\frac{1}{x}$ and $x=\frac{t}{1-t}$, show that $y=\frac{1-t}{t^2}$ **3**

$$y=\frac{1}{\left(\frac{t}{1-t}\right)^2}+\frac{1}{\left(\frac{t}{1-t}\right)}$$

$$=\frac{1-2t+t^2}{t^2}+\frac{1-t}{t}$$

$$=\frac{1-2t+t^2+t(1-t)}{t^2}$$

$$=\frac{1-2t+t^2+t-t^2}{t}$$

$$=\frac{1-t}{t^2}$$

- many students didn't 'show that'.

(c) Burt expanded a product using the distributive law but, unfortunately, he erased the terms in the second pair of brackets. **2**

$$(3x+1)(.....) = 15x^2 - 13x - 6$$

What are the missing terms in the second pair of brackets?

$3x \times \square = 15x^2$ $\quad$ $1 \times \triangle = -6$

$\square = 5x$ $\quad$ $\triangle = -6$

- 1 mark per term

(d) Simplify $\dfrac{m^{-1}+n^{-1}}{m+n}$ **3**

$$= \frac{\frac{1}{m}+\frac{1}{n}}{m+n}$$

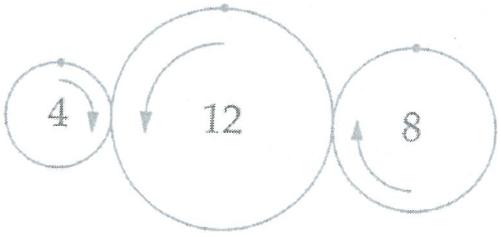
$$= \frac{\frac{m+n}{mn}}{m+n}$$

$$= \frac{m+n}{mn} \div \frac{m+n}{1}$$

$$= \frac{m+n}{mn} \times \frac{1}{m+n}$$

$$= \frac{1}{mn}$$

(e) In a system of 3 gear wheels, the first wheel has 4 teeth, the second 12 teeth, and the third 8 teeth. How many revolutions will the small wheel have to make before all wheels are back in their starting positions? **2**



LCM(4, 12, 8) = 24

24 ÷ 4 = 6

∴ 6 revolutions.

End of section E

Section F

a) Let x be the number of tickets

$x = 10 + \frac{x-10}{2} + 5 + 2$ ① for correctly constructing equation

$2x = 34 + x - 10$

$= x + 24$

$x = 24$ ①

b)

	A	B	C
Step 0:	0	3 B	5 W
1: B→A	1 B	2 B	5 W
2: C→B	1 B	2 B 1 W	4 W
3: A→C	0	2 B 1 W	1 B 4 W
4: B→A	$\frac{2}{3}$ B $\frac{1}{3}$ W	$\frac{4}{3}$ B $\frac{2}{3}$ W	1 B 4 W
5: C→B	$\frac{2}{3}$ B $\frac{1}{3}$ W	$\frac{23}{15}$ B $\frac{22}{15}$ W	$\frac{4}{5}$ B $\frac{16}{5}$ W
6: A→C	0	$\frac{23}{15}$ B $\frac{22}{15}$ W	$\frac{22}{15}$ B $\frac{53}{15}$ W

$\frac{4}{3} + \frac{1}{5} = \frac{23}{15}$, $\frac{2}{3} + \frac{4}{5} = \frac{22}{15}$

$\frac{4}{5} + \frac{2}{3} = \frac{22}{15}$, $\frac{16}{5} + \frac{1}{3} = \frac{53}{15}$

Final amount in Blue: $\frac{22}{15}$ L = $1\frac{7}{15}$ L

③ For final answer

② For correctly getting to step 4, or having only 1 error

① For correctly showing any 2 steps

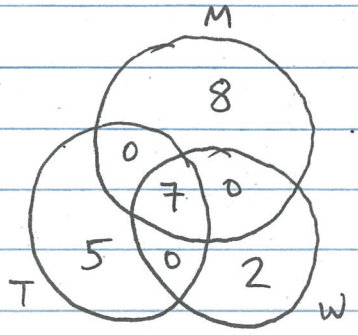
$\frac{1}{2}$ If didn't make a distinction between blue/white

c) Total days of absence = 15 + 12 + 9

= 36

22 students had absence of 1 or more days, so there are 36 − 22 = 14 "extra" days.

To maximise number that have all 3 days off, since 7 × 2 = 14, there are 7 students at a max. The Venn diagram opposite gives the possible solution.



② Correct solution ① Substantial correct working $\frac{1}{2}$ Answer of 9

d) Method 1: Draw a grid

	25	50	100	T	P	GP
Sonic	✓	×	×	×	×	✓
Peach	×	✓	×	✓	×	×
Mario	×	×	✓	×	✓	×
Turtle (T)	×	✓	×			
Parrot (P)	×	×	✓			
Guinea Pig (GP)	✓	×	×			

Method 2: Deduce directly from statements

eg. (iv) GP is $25

∴ (ii) Peach gets Turtle, for $50

∴ iii) Mario pays $100 for parrot

∴ By elimination, Sonic gets GP for $25.

Sonic : $25 : Guinea Pig

Peach : $50 : Turtle

Mario : $100 : Parrot

③ Fully correct

② 3 or more correct connections (of 6)

① 1 or more correct connections

e) Angela : Barry = 3 : 2

= 30 : 20

Corn : Peas = 7 : 3

= 35 : 15

Of Angela's 30 parts, $\frac{4}{5} \times 30 = 24$ are corn, and $\frac{1}{5} \times 30 = 6$ are peas.

∴ Barry has 35 - 24 = 11 parts corn, and 15 - 6 parts peas.

∴ 11 : 9

③ Correct answer ② Correctly giving equivalent ratios and some subtraction

① Correctly giving some equivalent ratios.

e) Method 2:

Angela's corn $= \frac{3}{5} \times \frac{4}{5}$

$= \frac{12}{25}$ of the whole land

Angela's peas $= \frac{3}{5} \times \frac{1}{5}$

$= \frac{3}{25}$ of the whole land

Since $\frac{7}{10}$ of the land is corn

Barry's corn $= \frac{7}{10} - \frac{12}{25}$

$= \frac{11}{50}$ of the whole land

Since $\frac{3}{10}$ of the land is peas

Barry's peas $= \frac{3}{10} - \frac{3}{25}$

$= \frac{9}{50}$ of the whole land

$\therefore$ Corn : Peas $= \frac{11}{50} : \frac{9}{50}$

$= 11 : 9$

Marker's Comments

a) Very well done. Some errors after correctly establishing equation, where $\frac{x-10}{2} \Rightarrow \frac{x}{2} - 5$ instead of $\frac{x}{2} + 5$.

b) Poorly done. Some students didn't make the distinction between blue & white paint. Others didn't consider that the quantities would mix when combined. Some didn't maintain the restriction that A only fits 1L, B 3L and C 5L. Even most correct answers were poorly set out.

c) Poorly done. Some students divided 14 by 3, then rounded or ignored remainder.

d) Very well done. Some students lost a mark for not including cost of pets.

e) Poorly done. Many students incorrectly thought they can subtract ratios where the totals aren't the same, eg $7:3 - 4:1 = 3:2$.

Year 9

Section G – 10 Marks

(a) (i) Show that $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ **2**

$(a+b)^3 = (a+b)(a+b)(a+b)$

$= (a+b)(a^2+2ab+b^2)$ ✓

$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$= a^3+3a^2b+3ab^2+b^3$ ✓

(ii) Hence show that $x = 2^{\frac{1}{3}}+2^{\frac{2}{3}}$ is a solution of the equation $x^3-6x-6=0$. **3**

$x^3 = \left(2^{\frac{1}{3}}+2^{2/3}\right)^3$

$= \left(2^{1/3}\right)^3 + 3\left(2^{1/3}\right)^2\left(2^{2/3}\right) + 3\left(2^{1/3}\right)\left(2^{2/3}\right)^2 + \left(2^{2/3}\right)^3$

$= 2 + 3.2^{4/3} + 3.2^{5/3} + 4$

$= 6 + 3\left(2^{4/3}+2^{5/3}\right) = 6+6\left(2^{1/3}\right)+6\left(2^{2/3}\right)$ ✓

$-6x = -6\left(2^{1/3}+2^{2/3}\right)$ ✓

$\therefore x^3-6x-6 = 6+6\left(2^{1/3}\right)+6\left(2^{2/3}\right) - 6\left(2^{1/3}\right) - 6\left(2^{2/3}\right) - 6$

$= 0$ ✓

(b) The fraction $\frac{37}{13}$ can be written in the form $2+\cfrac{1}{x+\cfrac{1}{y+\frac{1}{z}}}$, where x, y and z are integers. **3**

$= 2\frac{11}{13}$

Find x, y, and z.

RHS $= 2 + \cfrac{1}{x + \cfrac{1}{\left(\frac{yz+1}{z}\right)}}$

$= 2 + \cfrac{1}{x + \frac{z}{yz+1}}$

$= 2 + \cfrac{1}{\left(\frac{xyz+x+z}{yz+1}\right)}$

$= 2 + \frac{yz+1}{xyz+x+z}$

$\therefore \frac{1+yz}{xyz+x+z} = \frac{11}{13}$

$yz+1 = 11$

$yz = 10$

$\Rightarrow 10x + x + z = 13$

$11x + z = 13$

Since $x, y, z \in \mathbb{Z} \Rightarrow$

Posisible values since $yz = 10$

$yz = 10$

$5 \times 2 = 10 \Rightarrow y = 5, z = 2$

Then $11x + 2 = 13$

$11x = 11$

$x = 1$

(c) Suppose A, B and C are three numbers for which **2**

1001C – 2002A = 4004 and 1001B + 3003A = 5005.

What is the average of the three numbers A, B and C ?

$1001C - 2002A = 4004$ (1)

$1001B + 3003A = 5005$ (2)

(1) + (2) $1001(B+C) + 1001A = 9009$

$B + C + A = 9$

$\therefore$ Avg $= \frac{A+B+C}{3} = \frac{9}{3} = 3$,

End of the test