SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 9

HY Examination 2016

Mathematics Stage 5.3

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Clearly indicate your class by placing an X, next to your class
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- Unless otherwise indicated, give all answers in simplest exact form
- Approved calculators may be used

Examiner: P. Parker

NAME:

Class	Teacher	
9 A	Mr Boros	
9 B	Miss Chan	
9 C	Ms Kilmore/ Ms Evans	
9 P	Ms Ward	
9 L	Mr Choy	
9 U	Mr Wang	
9 S	Mr Gainford	

Question	Mark
1	/14
2	/15
3	/15
4	/13
5	/13
6	/13
7	/9
Total	/92

Question 1 (14 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) Simplify $9m - 8 - 4m$.

(b) Factorise $3x^2 + x$.

(c) Simplify $x^8 \div x^2$.

(d) Expand and simplify $(x - 4)(x + 9)$. **2**

(e) True or False: $16^{\frac{1}{2}} = \pm 4$. (T) or (F)

(f) True or False: $5x^{-\frac{1}{2}} = \dfrac{1}{5x^2}$ (T) or (F)

(g) Evaluate pq^2 when $p = -3$ and $q = 4$. **2**

(h) Express 2.718 28 correct to three significant figures.

Question 1 (continued) All questions 1 mark unless indicated.

(i) Simplify $8y^0 - (4y)^0$.

(j) Simplify $4^3 \div 4^{-2}$.

(A) 1 (B) 4 (C) 2^5 (D) 2^{10}

(k) A fair die is rolled. What is the probability of obtaining a number less than three?

(l) A property was advertised for \$2.1 million. Which of the following is the same as this?

(A) \$210 000 (B) \$2 010 000 (C) \$2 100 000 (D) \$21 000 000

Question 2 (15 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) Simplify $\frac{x}{2}+\frac{3x}{5}$. **2**

(b) Simplify $(2x)^3$.

(c) Round $23.3\dot{7}$ correct to 3 decimal places.

(d) The number represented by a 1 followed by one hundred zeros is called a googol.
Which of the following is equal to a googol?
(A) 10^2 (B) 10^{100} (C) 100^{10} (D) 100^{100}

(e) A set of golf clubs valued at \$800 is sold at a discount of 22%. **2**
Find the selling price of the golf clubs

(f) Simplify $3p^2 \times 2p^5$.

Question 2 (continued) All questions 1 mark unless indicated

(f) Express $3a^{-2}$ with a positive index.

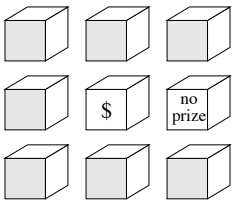
(g) What fraction, in its simplest form, is halfway between $-1\frac{1}{4}$ and $6\frac{2}{3}$?

(h) If the symbol $\begin{vmatrix} p & q \\ r & s \end{vmatrix}$ is defined by $p \times s - q \times r$, what is the value of $\begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix}$? **2**

(i) Complete the expression below

$$5.34 \times 10^{14} = 53\,400 \times _____$$

(j) In a game show there are nine boxes, three of which contain money.
All the other boxes are empty. Two boxes have already been chosen as shown ($ = money).
What is the probability that the next box chosen holds a money prize?



(k) A microsecond is one millionth of a second. Write 8 microseconds in scientific notation.

Question 3 (15 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) The diameter of a hydrogen nucleus is about 1 femtometre.
How many femtometres are there in one nanometre?

1 femtometre = 10^{-15} m and one nanometre = 10^{-9} m

(b) Simplify $\frac{12m^3n^2}{2mn^4} \div \frac{3m^3}{mn^2}$. **2**

(c) An integer from 10 to 99 inclusive is randomly chosen so that each such integer is equally likely to be chosen. **2**
What is the probability that at least one digit of the chosen integer is a 6?

(d) How many significant figures are there in 0.003010 ?

(e) Given that $x \neq 0$, simplify $\frac{1}{x^{-1}}$.

(f) Express $1+\frac{x-1}{2}$ as a single fraction in simplest form. **2**

(g) A soccer team played three games. Each game ended in a win (W), loss (L) or tie (T).
In total, the team scored more goals than were scored against them.
Which of the following combinations of outcomes is not possible for this team?

(A) 2 W, 0 L, 1 T (B) 1 W, 2 L, 0 T (C) 0 W, 1 L, 2 T

(D) 1 W, 1 L, 1 T (E) 1 W, 0 L, 2 T

(h) A student was asked to simplify $\frac{2}{x-1}-\frac{3}{x}$.
On which line of working was the first mistake made?

$$\frac{2}{x-1}-\frac{3}{x}=\frac{2x}{x(x-1)}-\frac{3(x-1)}{x(x-1)} \quad -(A)$$

$$=\frac{2x}{x(x-1)}-\frac{3x-3}{x(x-1)} \quad -(B)$$

$$=\frac{2x-3x-3}{x(x-1)} \quad -(C)$$

$$=\frac{-x-3}{x(x-1)} \quad -(D)$$

$$=-\frac{x+3}{x(x-1)} \quad -(E)$$

(i) Expand and simplify $6q(2q-9)-3(2q-9)$ **2**

(j) From a standard pack of 52 cards, divided into the four suits of Hearts, Diamonds, Clubs and Spades, a card is selected. Find the probability

(i) that the card is a King

(ii) that the card is a King or a red card.

Question 4 (13 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) Simplify $(2a + b)^2 - (b - 2a)^2$. **3**

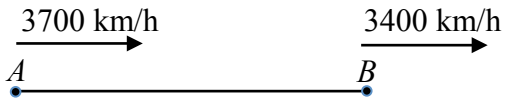
(b) Simplify $\frac{n-m}{m-n}$.

(c) Evaluate $(-1)^{2n+1} + 1^n$, where n is an integer.

(d) Express as a power of 2 in simplest form $8^{2x-1} \times 2^{\frac{x-2}{2}} \div 4^{\frac{5x-8}{4}}$ **3**

(e) Write $2x^{-1}y^{-\frac{3}{2}}$ without negative and fractional indices **2**

(f) Two planes, *A* and *B*, set off in the same direction. **2**
Initially, *A* is 100 km west of *B* and both fly in an easterly direction.
A flies at a constant speed of 3700 km/h and *B* lies at a constant speed of 3400 km/h.
Find the distance A must travel in order to intercept *B* to refuel it.
Give your answer correct to the nearest kilometre.



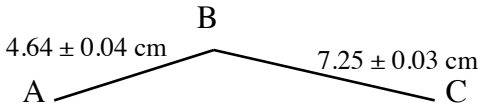
(g) Given that x is negative, simplify $(x^2)^{\frac{1}{2}}$.

Question 5 (13 marks) All questions 1 mark unless indicated
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

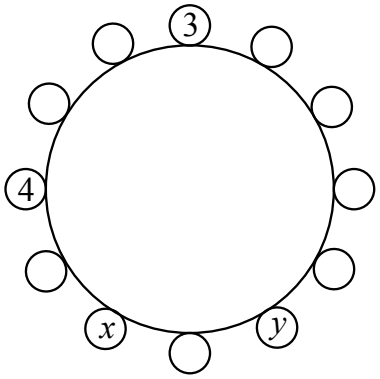
(a) Simplify $\frac{4ab}{6x^2} \times \frac{xy}{2a} \div \frac{b}{5}$ **2**

(b) Simplify $\frac{a+2b}{2a^{-1}+b^{-1}}$ **2**

(c) The distances *AB* and *BC* are shown on the right.
What is the maximum possible distance from *A* to *C*?



(e) Each integer from 1 to 12 is to be placed around the outside of a circle so that the positive difference between any two integers next to each other is at most 2. The integers 3, 4, x and y are placed as shown, what is the value of $x + y$? **2**



(f) When Eromlik places her marbles into bags with 6 marbles per bag, she has 4 marbles left over. When Snave does the same with her marbles, she has 3 marbles left over. Eromlik and Snave pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be left over? **2**

(g) What is the value of the fraction on the right? $\dfrac{2^{2016}+2^{2014}}{2^{2016}-2^{2014}}$ **2**

(h) Natural fruit juice contains 80% water. In concentrating the juice, 75% of the water is removed. What is the percentage of water in the concentrated juice? **2**

Question 6 (13 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) Two fair dice are thrown at random. What is the probability that the two numbers obtained are the two digits of a perfect square? **2**

(b) If $x^2 = x + 6$, show that $x^3 = 7x + 6$. Do NOT solve the equation. **2**

Question 6 (continued) All questions 1 mark unless indicated.

(c) The numbers on the six faces of the cube below are consecutive even numbers. **2**



If the sums of the numbers on each of the three pairs of opposite faces are equal, find the sum of all six numbers on the cube.

(d) If $x+\frac{1}{x}=\frac{13}{6}$, find the value of $\left(x-\frac{1}{x}\right)^2$. DO NOT solve the equation. **2**

(e) Three pipes lead into the Yohc dam. The pipes may be opened or closed separately with taps. There is an upper pipe, a middle pipe and a lower pipe. The property owner finds that he could fill the dam from empty in a number of ways: **3**

1. By only opening the lower and upper pipes, the dam fills in 3 days.
2. By only opening the upper and middle pipes, the dam fills in 4 days.
3. By only opening the lower and middle pipes, the dam fills in 6 days.

Assume that the water flows in each pipe at a constant rate which is not affected by the flow in other pipes.

How many hours does it take to fill the Yohc dam if all three pipes are open?

(f) Simplify $\left(\frac{x^2}{y^4}\right)^{\frac{1}{m}} \times \left(\frac{y^4}{x^2}\right)^{\frac{1}{m}-\frac{1}{2}}$. **2**

Question 7 (9 marks) All questions 1 mark unless indicated.
Please note that correct answers *without* working for questions worth more than 1 mark will only score a maximum of 1 mark.

(a) Sorob has arranged the positive integers in a zigzag fashion across five rows as follows: **2**

A	1							9										17	
B		2					8		10								16		18
C			3			7				11					15				19
D				4	6						12		14						
E					5							13							

In which row will 2016 appear?

(b) Factorise $(x+7)^{\frac{1}{2}}(2x-1)^3 - (x+7)^{-\frac{1}{2}}(2x-1)^4$. **2**
Leaving your answer with no fractional or negative powers.

Question 7 (continued) All questions 1 mark unless indicated.

(c) (i) Show that $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$.

(ii) Hence show that $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$. **2**

(f) If $2^m = 3^n = 6^x$, prove that $x = \dfrac{mn}{m+n}$. (Do NOT use logarithms) **2**

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Use this space if you wish to **REWRITE** any answers
Clearly *indicate* the **QUESTION** number.

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