Shore

Student Number:

Set:

Year 12
General Mathematics
Trial HSC
2012

General Instructions

- Reading time – 5 minutes
- Working time – 2.5 hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"

Note: Any time you have remaining should be spent revising your answers.

Total marks – 100

Section I Pages 2 - 10

25 marks

- Attempt Questions 1-25
- Allow about 30 minutes for this section

Section II Pages 11 - 22

75 marks

- Attempt Questions 26–30
- Allow about 2 hours for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

25 marks
Attempt Questions 1–25
Allow about 30 minutes for this section
Use the multiple-choice answer sheet for questions 1-25

Assume 52 weeks in a year, and 365 days in a year, where necessary.

1 What is the mode of the following scores?

5, 12, 12, 14, 14, 15, 15, 15, 16

(A) 11

(B) 13.1

(C) 14

(D) 15

2 James invested $3000 for 4 years at a flat rate of 6% p.a. How much was his total investment worth at the end of 4 years?

(A) $180

(B) $720

(C) $3180

(D) $3720

3 A grain hopper is in the shape of an upside down square pyramid. The perpendicular height of the grain hopper is 13 m, and the area of its square base is 144 m^2. What is the volume of the grain hopper?

(A) 624 m^3

(B) 1872 m^3

(C) 89 856 m^3

(D) 269 568 m^3

4 Which of the following is 4 256 870 000, correctly written using scientific notation?

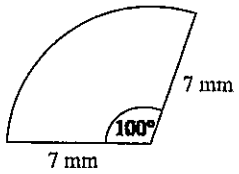
(A) 4.26×10^{-9}

(B) 42.5687×10^{8}

(C) 4.25687×10^{9}

(D) 4260000000

5 This is a sketch of a sector of a circle.



What is the area of this sector (correct to one decimal place)?

(A) 12.2 mm^2

(B) 42.8 mm^2

(C) 44.0 mm^2

(D) 153.9 mm^2

6 Which of the following is the correct simplification of $4y^4 - 2y^4$?

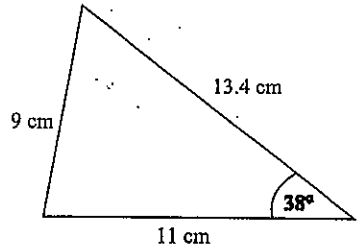
(A) $2y^8$

(B) $2y^4$

(C) $2y$

(D) 2

7 What is the area of the triangle shown below, correct to the nearest square centimetre?



NOT TO SCALE

(A) 37 cm^2

(B) 45 cm^2

(C) 50 cm^2

(D) 58 cm^2

8 Peter bought a new car at the beginning of 2009 for $15 000. At the end of 2009, the value of the car had depreciated by 25%. In 2010, the value of the car depreciated by 20% of the value of the car at the end of 2009.

What was the value of the car at the end of 2010?

(A) $750

(B) $8250

(C) $9000

(D) $11 250

9 If $A = 8b^2$, what is a possible value of b when $A = 3200$?

(A) –20

(B) 40

(C) 200

(D) 400

10 The table below is used to calculate monthly loan repayments.

Monthly loan repayments (in dollars) per $1000 borrowed

Interest Rate % p.a.	5 years	10 years	15 years	20 years
4%	18.42	10.14	7.40	6.07
5%	18.87	10.61	7.91	6.60
6%	19.33	11.10	8.44	7.16
7%	19.80	11.61	8.99	7.75
8%	20.28	12.13	9.56	8.36

Thomas has borrowed $40 000 at 7% p.a. for 10 years.

What is his monthly loan repayment?

(A) $11.61

(B) $116.10

(C) $464.40

(D) $3870

11 Patrick rides his bicycle downhill at a speed of 38 km/h.

What is this speed in m/s (correct to one decimal place)?

(A) 0.2 m/s

(B) 0.38 m/s

(C) 10.6 m/s

(D) 136.8 m/s

12 The mean of a set of 6 scores is 20.

What is the new mean of the set of scores after a score of 55 is added?

(A) 20

(B) 25

(C) 37.5

(D) 55

13 The positions of President, Vice President, Treasurer and Secretary of a committee are to be chosen from a group of 10 people.

In how many ways can the four positions be filled?

(A) 4

(B) 210

(C) 5040

(D) 10 000

14 Hamish wants to buy a new plasma TV from Terry's Televisions.

TERRY'S TELEVISIONS

Cash Price $3000

OR

Terms
10% deposit PLUS
$160 per month for 2 years

What is the amount of interest Hamish will have to pay if he buys the plasma TV on terms?

(A) $840

(B) $1140

(C) $1500

(D) $3840

15 Which of the following correctly expresses k as the subject of $M = \frac{k^2 - w}{w}$?

(A) $k = \pm\sqrt{w(M+1)}$

(B) $k = w(M+1)$

(C) $k = \pm w\sqrt{(M+1)}$

(D) $k = \pm\sqrt{M}$

16 Elliot used the capture-recapture technique to estimate the number of koalas on the island of St Bees.

- He caught, tagged and released 40 koalas.
- A week later he caught 100 koalas on the island.
- Elliot found that 8 of the 100 koalas had been tagged.

What is the correct estimate for the total number of koalas living on the island, using the capture-recapture technique?

(A) 140

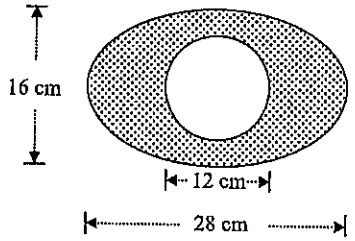
(B) 320

(C) 500

(D) 4000

17 The figure shows an ellipse and a circle.

What is the area of the shaded part of the figure, to the nearest square centimetre?



NOT TO SCALE

(A) 239 cm^2

(B) 352 cm^2

(C) 955 cm^2

(D) 1294 cm^2

18 Anne purchased a pair of jeans for $320 while on holiday in America. The price included 12% sales tax. What was the original price of the jeans, before the sales tax was added?

(A) $38.40

(B) $281.60

(C) $285.71

(D) $358.40

19 Aaron calculates the present value (N) of an annuity. The interest rate is 6% per annum, compounded monthly. In 4 years the future value will be $120 000.

Which calculation will result in the correct answer?

(A) $N = \frac{120000}{(1+0.06)^4}$

(B) $N = \frac{120000}{(1+0.06\div 12)^4}$

(C) $N = \frac{120000}{(1+0.06)^{48}}$

(D) $N = \frac{120000}{(1+0.06\div 12)^{48}}$

20 Henry buys a $2 raffle ticket. There are 300 tickets in the raffle, and three prizes. First prize is $120, second prize is $80 and third prize is $50.

What is Henry's financial expectation?

(A) – $1.17

(B) – $1.15

(C) +$0.83

(D) +$1.15

21 Adam measured the width of his study to be 1.80 m, correct to 2 decimal places.

What percentage error is the width of his study closest to?

(A) 0.03%

(B) 0.28%

(C) 2.78%

(D) 27.78%

22 The following table shows tax brackets for the tax year 2011 – 2012.

Taxable income	Tax on this income
0 - $6000	Nil
$6001 - $37 000	15c for each $1 over $6000
$37 001 - $80 000	$4650 plus 30c for each $1 over $37 000
$80 001 - $180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000

Charlie paid $7650 in tax. What was his taxable income?

(A) $47 000

(B) $57 000

(C) $67 000

(D) $87 000

23 Harriet is travelling through the city, and will pass through 3 sets of traffic lights. The probability that the lights are green at any one of the three junctions is 0.65.

Which of the following is closest to the probability that Harriet can go through all three sets without stopping?

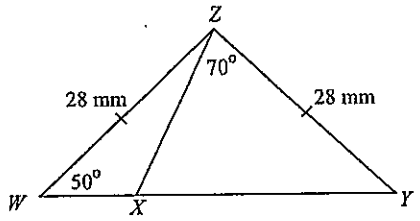
(A) 0.27

(B) 0.35

(C) 0.65

(D) 1.95

24 In the diagram, WZ and ZY are equal to 28 mm.



NOT TO SCALE

What is the length of XY to the nearest millimetre?

(A) 26 mm

(B) 30 mm

(C) 34 mm

(D) 40 mm

25 The population of dingoes on Laser Island is found to be directly proportional to the cube root of the number of tourists who visit the island in the summer months. The population of dingoes is 26 in a year when 320 tourists visited the island over the summer.

What is the population of dingoes in a year when 200 tourists visit Laser Island over the summer?

(A) 3

(B) 6

(C) 16

(D) 22

Section II

75 marks
Attempt Questions 26–30
Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Assume 52 weeks in a year, and 365 days in a year where necessary.

Question 26 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Arthur is paid $21.70 per hour, working as a barista. He is paid overtime at time-and-a-half when he works on weekends.

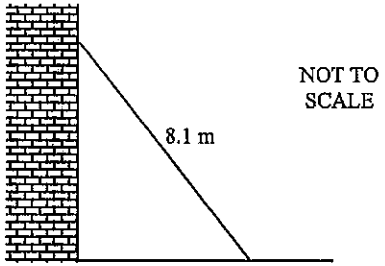
(i) If he works an 8-hour shift per day, Monday to Friday, calculate his fortnightly gross pay. **1**

(ii) In one week, Arthur works an 8-hour shift on Thursday, Friday, Saturday and Sunday. **2**

Find the amount he will earn for the four days of work during that week.

(b) Sarah is rescuing someone from the second floor window of a house. Her ladder is 8.1 metres long, as shown, and for safety reasons the angle of elevation of the ladder must not be greater than 50 degrees. **2**

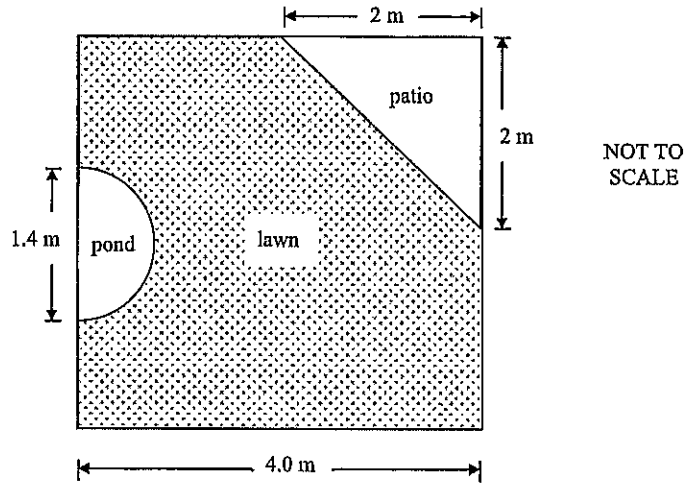
Calculate the maximum height above ground that Sarah's ladder will reach. Give your answer correct to two decimal places.



Question 26 continues on page 12

Question 26 (continued)

(c) Ed is a garden designer. He has drawn a plan for a garden, shown below.



The garden is a square, 4 metres by 4 metres. The pond is semi-circular, and the patio is triangular.

(i) Calculate the area of the lawn, correct to 2 decimal places. **3**

(ii) Turf is sold in rolls which each cover 1.2 square metres, and cost $9.20. Find the total cost of the rolls of turf required to cover the lawn. **2**

(iii) Before the turf is laid, Ed must put a layer of top soil down, to a depth of 2 centimetres. Find the volume of soil that Ed must use as top soil, in cubic centimetres. **2**

Question 26 continues on page 13

(d) Harriet wishes to survey 50 students at her school on how many hours they spend watching television each night. She wishes to choose a stratified sample from Years 10, 11, and 12. The table shows how many students are in each year group.

Year Group	Number of students
10	120
11	103
12	112

(i) How many students must she select from Year 11 to survey? 2

(ii) What kind of data is she collecting? 1

End of question 26

Question 27 (15 marks) Use a SEPARATE writing booklet Marks

(a) Jane keeps socks in a draw. She has 24 socks, in three different colours. Eight socks are red, eight socks are green, and eight socks are black. In the morning, she picks two socks at random from the draw.

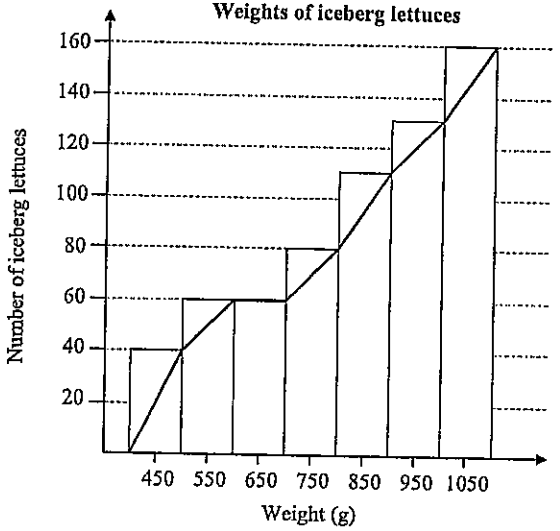
(i) What is the probability that the first sock she chooses is red? 1

(ii) Draw a tree diagram and write the correct probability on each branch. 2

(iii) Use your probability tree to determine the probability that Jane selects two socks of the same colour. 2

(iv) What is the probability that at least one of the socks is green? 2

(b) The weights of 160 iceberg lettuces were recorded. The results were grouped and recorded in a cumulative frequency histogram and polygon, shown below. The lightest iceberg lettuce weighed 400 grams and the heaviest weighed 1.1 kilograms.



(i) How many iceberg lettuces weighed between 500 and 600 grams? 1

(ii) Draw an accurate box and whisker plot to represent the data. 3

Question 27 continues on page 15

(c) Natasha invested money in a bank for 6 years, in an account that paid interest of 5.8% per annum, *compounded annually*. The equivalent effective simple interest for this bank account is 6.71% per annum. 4

Natasha used the formula shown below to calculate the effective simple interest rate.

$$E = \frac{(1+r)^n - 1}{n}$$

Where r is the given interest rate per period (expressed as a decimal)
E is the effective simple interest rate per period (expressed as a decimal)
n is the number of periods

Stephen invested his money for 6 years in an account at a different bank. His account pays 5.4% per annum *compounded monthly*.

Stephen thinks his account is a better deal than Natasha's. Is he correct? Justify your answer by comparing their effective simple interest rates.

End of question 27

Question 28 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Hazel purchased a motorboat for $40 000. It depreciated in value by $3000 per year for the first 5 years. At the end of the fifth year, Hazel changed to the declining balance method of depreciation at a rate of 20% per year.

(i) Calculate the value of the motorboat at the end of the second year. 1

(ii) Calculate the value of the motorboat six years after Hazel purchased it. 2

(b) The table below gives income tax rates.

Taxable income	Tax payable
$1 to $6000	$0
$6001 to $34 000	15c for each dollar over $6000
$34 001 to $80 000	$4200 plus 30c for each $1 over $34 000
$80 001 to $180 000	$18 000 plus 40c for each $1 over $80 000
Over $180 000	$58 000 plus 45c for each $1 over $180 000

Robert's salary is $73 500 for the tax year. In the same year, he earns $422 of interest from an investment. He pays union fees of $120 per year.

(i) Calculate Robert's taxable income. 1

(ii) Robert must pay the Medicare levy of 1.5% of taxable income. What is the total amount of tax Robert will have to pay, including the Medicare levy? 2

(iii) Robert has $1240 tax deducted from his salary per month. How much does Robert owe the tax department at the end of the financial year? 2

(c) A motorcycle service centre uses three battery chargers for 6 hours every day of the year. The purchase price of each battery charger is $135, and the running costs for each charger are $\$m$ per hour.

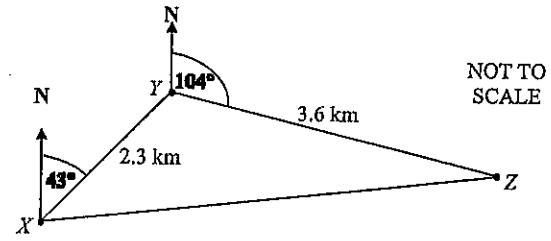
(i) Write an equation for the total cost of purchasing and running the three battery chargers for one year in terms of m. 2

(ii) Find the value of m (correct to 3 decimal places) if the total cost of purchasing and running the three battery chargers is $580, over one year. 1

Question 28 continues on page 17

Question 28 (continued)

(d) David designed a walking track in the shape of a triangle, shown in the diagram. The track from X to Y is 2.3 km on a true bearing of 043°. The track changes direction at Y. The track from Y to Z is 3.6 km and the bearing of Z from Y is 104°.



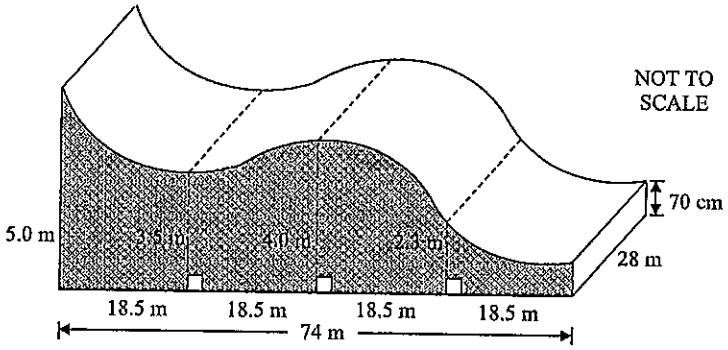
(i) Show that $\angle XYZ = 119°$. 1

(ii) What is the total length of the walking track? Give your answer in kilometres, correct to one decimal place. 3

End of question 28

Question 29 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) An indoor ski slope has been designed so that it has the dimensions shown below (and a uniform cross section, which is shaded in the diagram).



(i) Use two applications of Simpson's Rule to approximate the area of the cross section. 3

(ii) The total curved surface area is 2352 m^2. Thomas skis the most direct route from the top to the bottom of the slope. How far will he have skied? 1

(iii) On another ski slope, Thomas can travel 500 metres in 2 minutes. What is this speed in kilometres per hour? 1

Question 29 continues on page 19

(b) In an experiment, Terry uses two unbiased dice with faces numbered 1, 2, 3, 4, 5 and 6. Terry rolls the dice 72 times and records the sum of the uppermost faces of the two dice each time. The results of his experiment are shown in the table below.

Sum of the dice	Frequency
2	1
3	4
4	7
5	9
6	11
7	13
8	9
9	7
10	5
11	4
12	2
TOTAL	72

(i) Before conducting the experiment, Terry makes a note of the theoretical probabilities of each outcome. What is the theoretical probability that the two faces of the dice will add to 8? **2**

(ii) Terry repeats his experiment with the same dice. He rolls the dice 40 times.

(1) Based on his results from the first experiment, how many times can he expect the two faces of the dice to add to 5 in the second experiment? **2**

(2) Based on his results from the first experiment how many times can he expect to get a sum which is greater than 10? **2**

Question 29 continues on page 20

(c) Tim is saving to buy a car. He deposits $4200 into an account at the end of every year for 6 years. The investment pays 4% per annum interest, compounding annually.

The table shows future values of an annuity of $1.

Future values of an annuity of $1

End of year	*Interest Rate*				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.301
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

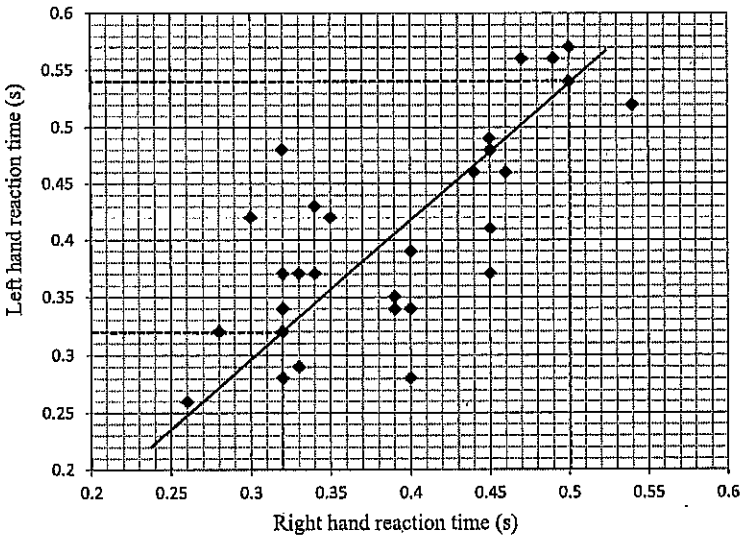
(i) Use the table to find the future value of Tim's investment at the end of 6 years. **2**

(ii) How much interest does Tim earn on his investment over the 6 years? **2**

End of Question 29

Question 30 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Lucy measured and recorded the left-hand and right-hand reaction times of a group of men and women. She graphed the results and drew in a regression line, as shown.



(i) What is the gradient of the regression line that Lucy has drawn? 1

(ii) Use the regression line to estimate the left-hand reaction time for someone who has a right hand reaction time of 0.37 seconds. 1

(iii) Describe the correlation of the results shown in the graph. 1

Question 30 continues on page 22

(b) James takes out a $380 000 home loan. The loan is offered at an interest rate of 7.5% per annum over 25 years, with monthly repayments.

(i) Use the present value formula to show that the minimum monthly repayment is $2808.17. 2

(ii) Calculate the total interest paid on the loan over 25 years. 2

(iii) James decides he can afford to pay $3000 a month to repay the loan. Can he pay off the loan in 20 years? Justify your answer with suitable calculations. 3

(c) Peter decided to analyse his results in two assessments.

	Mean	Standard Deviation	Peter's Mark
History	73	7	81
Geography	56	17	81

(i) Using the table above, calculate Peter's z-scores for History and Geography. Give your answers correct to 3 significant figures. 2

(ii) In which task did Peter perform better? Justify your answer. 1

(d) In a swimming carnival, the mean result for the men's 200 metres freestyle was 2.26 minutes. The standard deviation was 0.03 minutes. What percentage of the competitors in this event had times between 2.2 minutes and 2.29 minutes? 2

End of Paper

Year 12 General Trial HSC 2012

MULTI CHOICE

1	D	6	B	11	C	16	C	21	B
2	D	7	B	12	B	17	A	22	A
3	A	8	C	13	C	18	C	23	A
4	C	9	A	14	B	19	D	24	B
5	B	10	C	15	A	20	A	25	D

26 a) i) Pay $= 21.70 \times 10 \times 8$
$= \$1736$

ii) Total hours $= 16 + 16 \times 1.5$
$= 40h$

pay $= 40 \times 21.70$
$= \$868$

b) $\sin 50° = \frac{h}{8.1}$
$h = 8.1 \sin 50°$
$= 6.20495...$
$\doteq 6.20m$

c) i) Area $= 4 \times 4 - \left(\frac{1}{2} \times 2 \times 2\right) - \frac{1}{2}\pi \times 0.7^2$
$= 13.2303...$
$= 13.23\ m^2$

ii) rolls required $= \frac{13.23}{1.2}$
$= 11.025$
$= 12$ rolls

cost $= 12 \times 9.20$
$= \$110.40$

iii) $V = 13.23 \times 0.02$
$= 0.2646\ m^3$
$= 264600\ cm^3$

$1m^3 = 100 \times 100 \times 100$
$= 1000000\ cm^3$

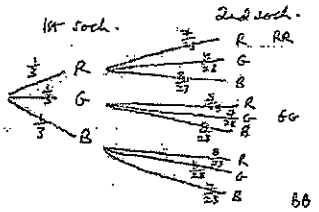
d) i) Year 11 $= \frac{103}{335} \times 51$
$= 15.373$
$= 15$ students

ii) quantitative discrete

Q27

a) i) $P(r) = \frac{8}{24}$
$= \frac{1}{3}$

ii)

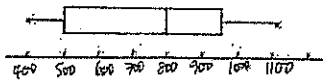


iii) $P(RR, GG, BB) = \frac{1}{3} \times \frac{7}{23} \times 3$
$= \frac{7}{23}$

iv) $P(\text{at least 1 G}) = \frac{1}{3} \times \frac{8}{23} + \frac{1}{3} + \frac{1}{3} \times \frac{8}{23}$
$= \frac{8}{69} + \frac{23}{69} + \frac{8}{69}$
$= \frac{13}{23}$

b) i) 20 (from graph)

ii)



c) $E = \frac{(1+r)^n - 1}{n}$
$= \frac{\left(1 + \frac{0.0854}{12}\right)^{12} - 1}{12}$
$= 0.00653...$ monthly
$\doteq 0.06367...$ yearly
$\doteq 6.37\%$

∴ Natasha has the best deal, because $6.71 > 6.37$

Q28 a) i) $S = 40000 - 3000 - 3000$
$= \$34000$

ii) End of 5 years $S = 40000 - (5 \times 3000)$
$= \$25000$

End of 6 years $S = 25000 \times 0.8$
$= \$20000$

b) i) taxable income $= 73500 + 422 - 120$
$= \$73802$

ii) Tax payable $= 0.015 \times 73802 + 4200 + 0.3(73802 - 34000)$
$= \$17247.63$

iii) Paid PAYG $= 1240 \times 12$
$= \$14880$

Tax owed $= 17247.63 - 14880$
$= \$2367.63$

c) i) Total cost $= 3 \times 135 + 3m \times 6 \times 365$
$= 405 + 6570m$
$C = 405 + 6570m$

ii) $580 = 405 + 6570m$
$6570m = 580 - 405$
$m = \frac{175}{6570}$
$= 0.026636...$
$\doteq 0.027\ \$/h$

d) bearing of Z from Y is 104° $\longleftarrow \angle XYN = 180 - 43 = 137°$ (corresponding ∠s in parallel lines)

i) $\angle XYZ = 360 - (104 + 137°)$
$= 119°$

ii) $XZ^2 = 2.3^2 + 3.6^2 - 2 \times 2.3 \times 3.6 \cos 119°$
$XZ = \sqrt{26.27...}$
$= 5.12625$

Total length $= 5.13 + 2.3 + 3.6$
$\doteq 10.96$
$\doteq 11km$

Q29 a) i) $A \doteq \frac{h}{3}\{d_f + 4d_m + d_l\}$
$= \frac{18-5}{3}\{5 + 4 \times 3.5 + 4\} + \frac{18.5}{3}\{4 + 4 \times 2.3 + 0.7\}$
$= \frac{18.5}{3}\{36.9\}$
$= 227.55$
$= 228\ m^2$

ii) $SA = 2352$
$2352 = l \times b$
$= l \times 28$
$l = \frac{2352}{28}$
$= 84m$

iii) $500m / 2\ min = 30 \times 500\ m/h$
$= 15000\ m/h$
$= 15\ km/h$

2 + 6	−2
3 + 5	−2
4 + 4	−1
5 + 3	
6 + 2	

b) i) $P(\text{sum of } 8) = \frac{2}{36} + \frac{2}{36} + \frac{1}{36}$
$= \frac{5}{36}$

ii) 1) Experimental prob (5) $= \frac{9}{72} \times 40$
$= 5$ times

2) Exp. prob (>10) $= \frac{4+2}{72} \times 40$
$= 3.\dot{3}$

c) i) $A = 4200 \times 6.633$
$= \$27858.60$

ii) contributions $= 6 \times 4200$
$= 25200$

$I = 27858.60 - 25200$
$= \$2658.60$

Q30 a) i) gradient $= \frac{0.54 - 0.32}{0.5 - 0.32}$

$= \frac{0.22}{0.18}$

$= 1.\dot{2}$

ii) form graph $= \underline{0.38 \text{ s}}$

iii) weak positive

b) i) $N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$

$M = 380\,000\left\{\frac{\frac{0.075}{12} \times \left(1 + \frac{0.075}{12}\right)^{300}}{\left(1 + \frac{0.075}{12}\right)^{300} - 1}\right\}$

$= 2808.16847$

$= \underline{\$2805.17}$

ii) Total paid $= 300 \times 2805.17$

$= 842\,449.942\ldots$

$= \underline{\$842\,449.94}$

$I = 842\,449.94 - 380\,000$

$= \underline{462\,449.94}$

iii) $M = 380\,000\left\{\frac{0.00625 \times (1.00625)^{240}}{(1.00625)^{240} - 1}\right\}$

$= \underline{3061.254}$ minimum monthly repayment.

No, he cannot pay it off at $3000 a month.

c) i) History z-score $= \frac{81 - 73}{7}$

$= 1.14285\ldots$

$\doteq 1.14$

Geography z-score $= \frac{81 - 56}{17}$

$= 1.47058\ldots$

$\doteq 1.47$

ii) Pete performed better in Geography since his z-score is higher, meaning his score was further from the mean, in the upper half of the scores.

d) $2.26 - 2.2 = 0.06$

$= 2$ standard dev

$2.29 - 2.26 = 0.03$

$= 1$ standard de

$\frac{95\% - 68\%}{2} = 13.5$

$13.5 + 68 = 81.5$

