KAMBALA

Student Number: _____

HSC Trial Examination
August 2015

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- Answer questions 1 – 10 on the multiple choice answer sheet provided
- Answer questions 11-14 on the paper provided, showing relevant mathematical reasoning and/or calculations
- **Start each question on a new page**
- Write your Student Number on the top of this page and at the top of every writing page

Total marks – 69

Section I
10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 When the polynomial $P(x) = x^3 - 5x^2 + kx + 2$ is divided by $(x+1)$ the remainder is 3. What is the value of k?

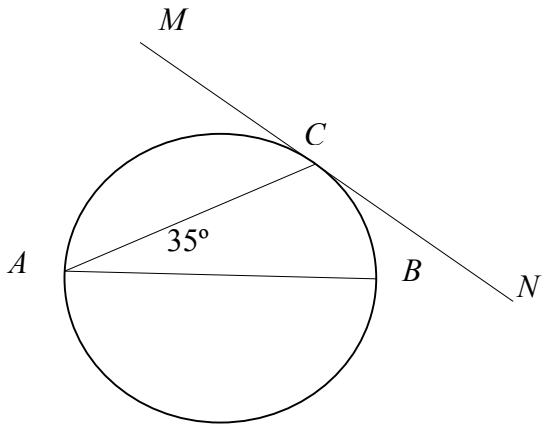
(A) -7 (B) -5 (C) 5 (D) 7

2 Which of the following is a simplification of $4\log_e \sqrt{e^x}$?

(A) $4\sqrt{x}$ (B) $\frac{1}{2}x$ (C) $2x$ (D) x^2

3 In the diagram, AB is a diameter of the circle and MCN is the tangent to the circle at C. $\angle CAB = 35^\circ$. What is the size of $\angle MCA$?

(A) 35º (B) 45º (C) 55º (D) 65º



4 The acute angle between the lines $2x - y = 0$ and $kx - y = 0$ is equal to $\frac{\pi}{4}$. What is the value of k?

(A) $k = 3$ or $k = -\frac{1}{3}$

(B) $k = -3$ or $k = \frac{1}{3}$

(C) $k = 3$ or $k = -\frac{1}{3}$

(D) $k = 3$ or $k = \frac{1}{3}$

5 Which of the following is a simplification of $\frac{1-\cos 2x}{\sin 2x}$?

(A) $1 - \cot 2x$ (B) 1 (C) $\cot x$ (D) $\tan x$

6 The statement $7^n - 3^n$ *is always divisible by 10* is true for

(A) all integers $n \geq 1$

(B) all integers $n \geq 2$

(C) all odd integers $n \geq 1$

(D) all even integers $n \geq 2$

7 What is the value of $\int_1^2 \frac{1}{\sqrt{4-x^2}} dx$?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

8 The radius *r* of a circle is increasing at a constant rate of 0.1 cms $^{-1}$. What is the rate at which the area of the circle is increasing when $r = 10$ cm?

(A) π cm ^{2} s $^{-1}$ (B) 2π cm ^{2} s $^{-1}$ (C) 10π cm ^{2} s $^{-1}$ (D) 20π cm ^{2} s $^{-1}$

9 If $x + \frac{1}{x} = 2$ what is the value of $x^2 + \frac{1}{x^2}$?

(A) 2 (B) 4 (C) 6 (D) 8

10 Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

(A) 0 (B) ∞ (C) 1 (D) ½

End of Section I

Section II

60 Marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on the writing paper provided. Start each question on a new page.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page

(a) Solve the inequality $\frac{1}{|x-1|} > \frac{1}{2}$. **2**

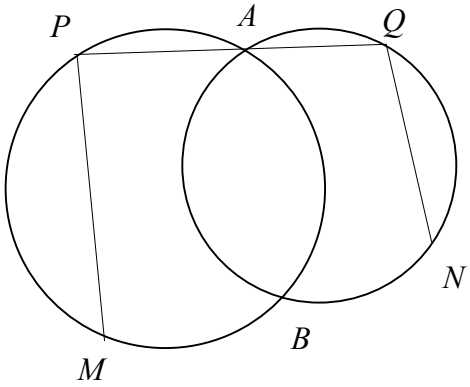
(b) A (-2, 5) and B (6, -7) are two points. Find the coordinates of the point P (x, y) that divides the interval AB internally in the ratio 3:1. **2**

(c) Find $\frac{d}{dx}(x^2 \tan^{-1} x)$. **2**

(d) Use Mathematical Induction to show that for all positive integers $n \geq 1$
$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \ldots + n \times 2^{n-1} = 1 + (n-1)2^n$. **3**

(e) Use the substitution $x = u^2 - 1,\ u \geq 0,$ to evaluate $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$. **3**

(f) In the diagram the two circles intersect at A and B. PAQ is a straight line and PM is parallel to QN. Copy the diagram. Show that MBN is a straight line. **3**



Question 12 (15 marks) Start a new page

(a) Given that $\int_0^a e^{1-2x}\,dx = \frac{e}{4}$, find a in terms of $\ln k$. **2**

(b) The equation $x^3 - 6x^2 + 4x + 2 = 0$ has roots α, β and γ. Find the value of

(i) $\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}$ **1**

(ii) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\lambda}$. **2**

(c) (i) Show that for $0 < x < \frac{\pi}{4}$, $\tan x + \tan^3 x + \tan^5 x + ... = \frac{1}{2}\tan 2x$. **2**

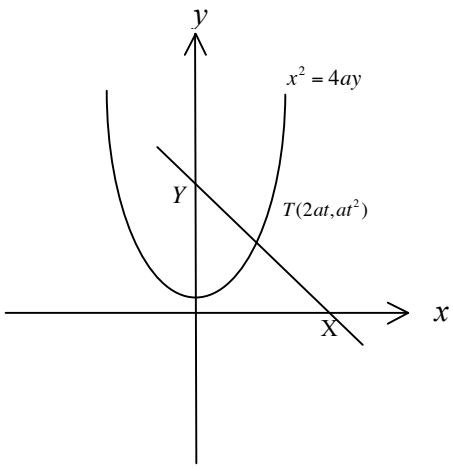
(ii) Hence find the exact value of $\tan\frac{\pi}{8} + \tan^3\frac{\pi}{8} + \tan^5\frac{\pi}{8} + ...$ **1**

(d) Solve the equation $\sin^{-1} x = 3\cos^{-1} x$, giving the solution correct to 2 decimal places. **3**

(e) In the diagram, $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$.

(i) Show that the normal to the parabola at t has equation $x + ty = 2at + at^3$. **2**

(ii) This normal cuts the x and y axes at X and Y respectively. Show $\frac{TX}{TY} = \frac{t^2}{2}$. **2**



Question 13 (15 marks) Start a new page

(a) Find the domain and range of the function $f(x) = \cos^{-1}(2x-1) - \frac{\pi}{2}$. **2**

(b) α is the real root of the equation $\log_e x - \frac{1}{x} = 0$. Use one application of Newton's Method with an initial approximation $\alpha_0 = 1.5$ to find the next approximation of α correct to 1 decimal place. **2**

(c) The region bounded by the curve $y = \cos x$ and the x axis between $x = 0$ and $x = \frac{\pi}{3}$ is rotated through one complete revolution about the x axis. Find the exact volume of the solid formed.

3

(d) The rise and fall of water in a harbour is simple harmonic. On a particular day in a harbour, high tide at its entrance occurs at noon and the water depth is then 11 m. Low tide occurs 6.25 hours later and the water depth is then 5 m.

(i) Find the amplitude and period of this motion and write the equation for the displacement of the motion. **2**

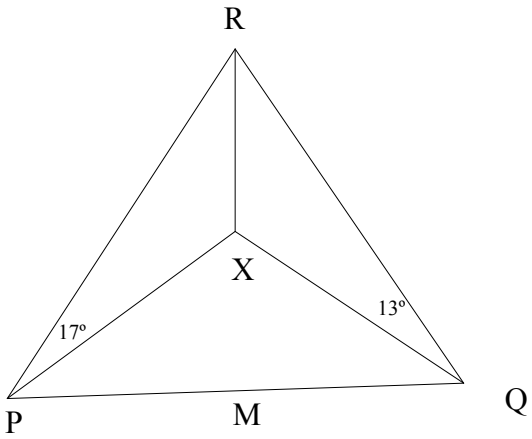
(ii) Find the time when the water level will be falling at its maximum rate and thus find the rate in metres per hour. Give your answer to 3 significant figures. **2**

(iii) A ship needs a depth of 7 m to enter the harbour. Find the latest time after noon at which it can enter without having to wait for low tide to pass. **2**

(e) Find the acute angle between the curve $y = x^2 + 3$ and the line $2x - y + 3 = 0$ at the point of intersection $x = 2$. Give your answer to the nearest degree. **2**

Question 14 (15 marks) Start a new page

(a)



(i) From a point P, the angle of elevation of the top of a vertical tower at X, due North of P, is 17º. From Q, due East of the tower, the angle of elevation is 13º. Given that P and Q are 130m apart, show that *h*, the height of the tower, can be given by

$$h = \frac{130}{\sqrt{\tan^2 73 + \tan^2 77}}.$$

2

(ii) Find *h* to the nearest metre. **1**

(b) At time *t* years the number *N* of individuals in a population is given by $N = \frac{a}{1+be^{-t}}$ for some constants $a > 0$ and $b > 0$. The initial population size is 20 and the limiting population size is 100.

(i) Show that $\frac{dN}{dt} = N\left(1 - \frac{N}{a}\right)$. **2**

(ii) Find the values of *a* and *b*. **2**

(c) Show that the series $\log_2 x + \log_4 x + \log_{16} x + \ldots$ is geometric and find the sum of the series for infinite terms. **2**

Question 14 continues on the next page

Question 14 continued

(d) Consider the function $f(x) = -\dfrac{x}{x^2+1}$

(i) Show that this function is odd. **1**

(ii) Find the equation of the horizontal asymptote. **1**

(iii) Find the coordinates of its stationary points and determine their nature. **2**

(iv) Sketch the graph of this function. **1**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

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Mathematics Extension 1

HSC Trial Examination
August 2014

Section I

Multiple-Choice Answer Sheet
Circle the correct response

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D