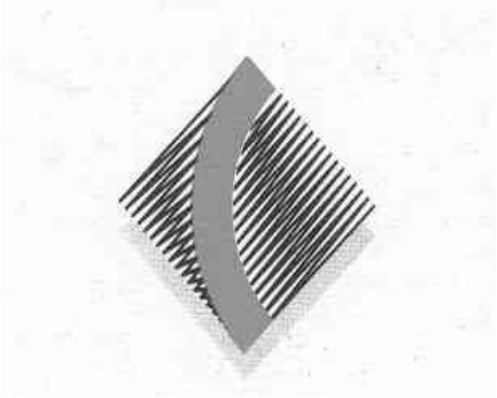
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NAME_____

CLASS 12MTX___

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2018

YEAR 12

AP4

MATHEMATICS EXTENSION 1

Time allowed – 2 HOURS + 5 Minutes Reading Time

DIRECTIONS TO CANDIDATES:

SECTION I: Use the multiple-choice answer sheet provided.

SECTION II:

- Each question is to be commenced in a new booklet clearly marked Question 11, Question 12, etc. on the front.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- If you do not attempt a question, you must submit a blank booklet clearly indicating the question number, your name and class.
- All questions should be placed in order – multiple choice answer sheet followed by Questions 11 to 14.
- NESA approved calculators may be used.
- The NESA Reference Sheet is provided.

Section I

10 Marks

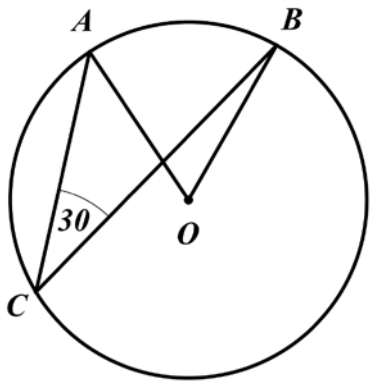
Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for questions 1 – 10.

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1 The points *A*, *B* and *C* lie on a circle with centre *O*, as shown in the diagram. The size of $\angle ACB$ is 30^o.



What is the size of $\angle AOB$?

(A) 30^o

(B) 60^o

(C) 15^o

(D) 45^o

2 Given that $\tan\alpha = \sqrt{2},\ \pi \le \alpha \le \frac{3\pi}{2}$, which of the following equates to $\sin 2\alpha$?

(A) $\frac{\sqrt{2}}{3}$

(B) $-\frac{3\sqrt{2}}{2}$

(C) $\frac{3\sqrt{2}}{3}$

(D) $\frac{2\sqrt{2}}{3}$

3 What is the domain of the function $f(x) = \cos^{-1}(1 - x^2)$?

(A) $-1 \leq x \leq 1$

(B) $0 \leq x \leq 1$

(C) $-\sqrt{2} \leq x \leq \sqrt{2}$

(D) $-2 \leq x \leq 2$

4 It is given that $\log_a 27 = x$. What is the value of $\log_a 9a$ in terms of x?

(A) $x + 1$

(B) $\frac{x}{3}$

(C) $\frac{x}{3} + 1$

(D) $\frac{2x}{3} + 1$

5 The point $P(2\sin t, \sin 2t)$ lies on the parabola $x^2 = 4y$. What is the equation of the tangent at P?

(A) $y = x - \sin t$

(B) $y = x \sin t - \sin 2t$

(C) $y = x \sin t + \sin 2t$

(D) $y = x \sin t + \sin t$

6 Two roots of the equation $x^3 + ax - b = 0$, $b \neq 0$ are reciprocals of each other. Which of the following is a correct statement?

(A) $b^2 = 1 - a$

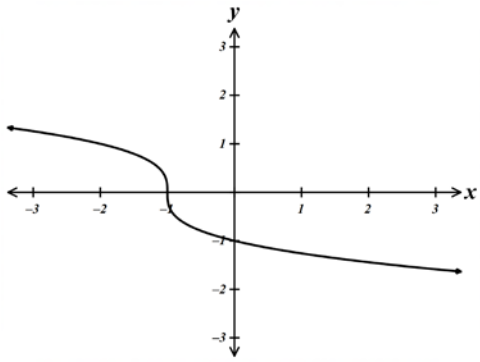
(B) $a^2 + b^2 = 1$

(C) $a = b = 1$

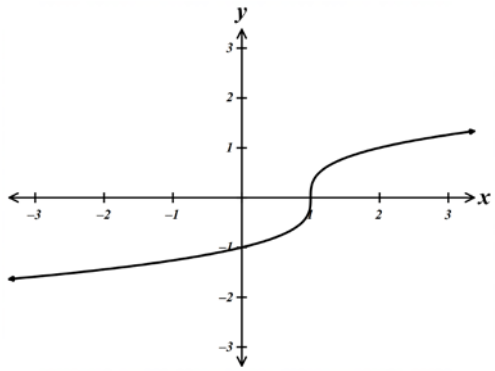
(D) $a^2 = b^2 - 1$

7 Given that $f(x) = x^3 + 1$, which of the following represents the graph of $y = f^{-1}(x)$?

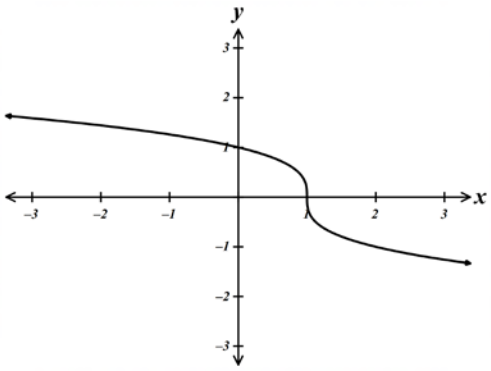
(A)



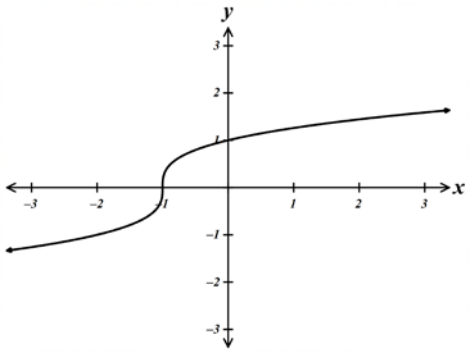
(B)



(C)



(D)



8 A particle is moving in simple harmonic motion in a straight line. At *t* seconds, it has a displacement of *x* metres from a fixed point *O* on the line, where *x* is given by $x = 4\sin^2 t - 1$. What is the centre of motion?

(A) $x = -1$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

9 The velocity of a particle moving with simple harmonic motion is given by $v^2 = 27 - 3x^2$. What is the amplitude, *A*, and the period, *T*, of the motion?

(A) $A = 3 \quad T = \frac{2\pi}{3}$

(B) $A = \sqrt{3} \quad T = \frac{2\pi}{3}$

(C) $A = 3 \quad T = \frac{2\pi}{\sqrt{3}}$

(D) $A = 9 \quad T = \frac{2\pi}{3}$

10 The maximum range of a baseball projected into the air with an initial velocity V is x metres. An investor considering building a baseball stadium on Mars would like to estimate the size of stadium required.

Given that acceleration due to gravity on Mars is approximately $\frac{1}{3}$ of its value on Earth, and ignoring the effects of air resistance, what is the maximum range of the baseball if it is projected into Mars' atmosphere with initial velocity V?

(A) $\sqrt{3}x$

(B) $9x$

(C) $\frac{x}{3}$

(D) $3x$

End of Section 1

Section II

60 Marks

Attempt Questions 11 – 14.
Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate writing booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Start a new writing booklet. **Marks**

a) Find the ratio into which the point $C(1,0)$ divides the line segment $A(0,2)$ to $B(3,-4)$. **2**

b) (i) Sketch the graph of $y = |x^2 - 6x + 5|$. **2**

(ii) Hence find the value of c so that $|x^2 - 6x + 5| = c$ has exactly three solutions. **2**

c) Evaluate $\lim \dfrac{\sin(\pi + x)}{x}$. **2**

d) Solve $\dfrac{x}{x-1} \leq \dfrac{4}{x}$. **4**

e) The roots of $x^2 - 2x - 1 = 0$ are $\tan\alpha$ and $\tan\beta$. Show that if α and β are acute, then $\alpha + \beta = \dfrac{\pi}{4}$. **3**

End of Question 11

Question 12 (15 Marks) Start a new writing booklet. **Marks**

a) Evaluate $\int_1^e \frac{dx}{x\sqrt{1-(\ln x)^2}}$ using the substitution $u = ln\, x\ (x \geq 0)$. **3**

b) By considering the graph of $y = \sin^{-1} x$, evaluate $\int_0^{\frac{\sqrt{3}}{2}} \sin^{-1} x\, dx$. **3**

c) (i) Show that $\frac{n}{2(3n+2)} = \frac{1}{6} - \frac{1}{3(3n+2)}$. **1**

(ii) Use mathematical induction to prove that **3**

$$\frac{1}{2\times 5} + \frac{1}{5\times 8} + \ldots\ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}.$$

(iii) Hence evaluate $\sum_{r=1}^{\infty} \frac{1}{(3r-1)(3r+2)}$ using the previous results. **1**

d) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of the chord PQ is given by $(p+q)x - 2y - 2apq = 0$. (Do NOT prove this.)

(i) If PQ is a focal chord, show that $pq = -1$. **1**

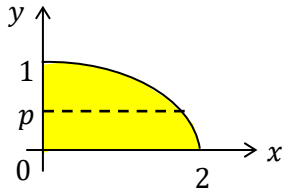
(ii) Show that the locus of the midpoint of PQ is the parabola $x^2 = 2a(y-a)$. **3**

End of Question 12

Question 13 (15 Marks) Start a new writing booklet. **Marks**

a) Differentiate $\sin^{-1}(x^2)$. **2**

b) The region enclosed by the curve $y^3 = 1 - \frac{x^2}{4}$, the x axis and the y axis in the first quadrant is divided into two parts by the line $y = p$ where $0 < p < 1$.



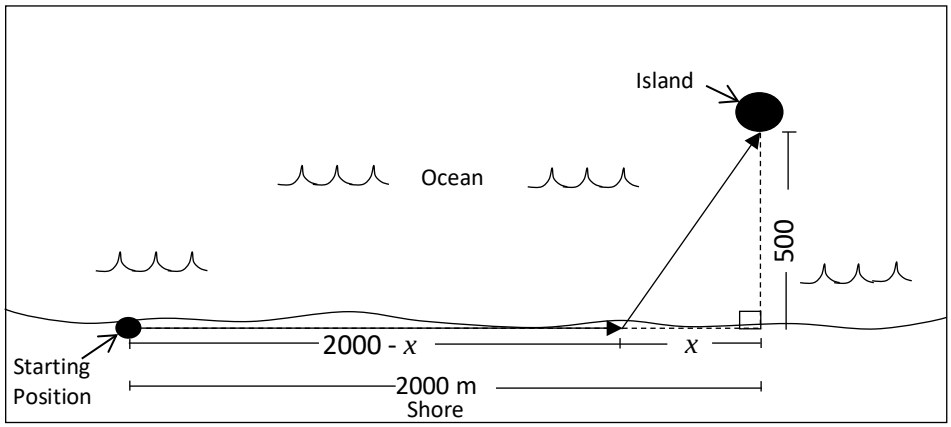
The volumes of the solids of revolutions formed when the two parts are rotated about the y axis are equal.

(i) Show that p satisfies the equation $2p^4 - 8p + 3 = 0$. **2**

(ii) Given that $p_1 = 0.4$ is the first approximation for p, use one application of Newton's method to find a second approximation for p. **1**

Question 13 continued next page

c) A contestant on a TV show is to participate in a race to get to an island off-shore by first running along the shoreline and then swimming to the island. The perpendicular distance of the island to the shoreline is 500 m. The contestant's starting position is on the shoreline, 2000 m from the perpendicular line from the island to the shoreline, as shown in the diagram.



NOT TO SCALE

The contestant is planning her route in order to minimise the time it takes her to get to the island. She knows that she can run five times as fast as she can swim. By using x in the diagram, or otherwise, find how far along the shoreline she should run before swimming to the island in order to get there in the fastest time. **4**

d) Given $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dt} = 2$ find the value of $\frac{dr}{dt}$ when $r = 2$. **2**

Question 13 continued next page

e) A tea enthusiast believes that black tea should be prepared with boiling water and consumed at 70°C. She wants to model the temperature of her tea with respect to time. She pours hot water into her favourite mug. The initial temperature of the water is 100°C and the room temperature is 25°C. After 5 minutes, the temperature of the water has reached 85°C. The temperature, T, of the water after t minutes can be modelled by the equation

$$T = 25 + Ae^{kt}.$$

(i) Find the value of A. **1**

(ii) Find the value of k, leaving your answer in exact form. **1**

(iii) How long will it take for the temperature to reach 70°C? **1**

(iv) If she starts drinking the tea when it reaches 70°C and takes 7 minutes to drink her tea, what will the temperature of the tea be when she finishes drinking it? (Assume the temperature continues to follow the same model.) **1**

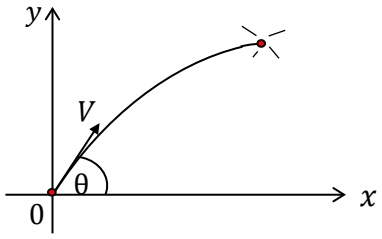
End of Question 13

Question 14 (15 Marks) Start a new writing booklet. **Marks**

a) At time t the displacement, x, of a particle satisfies $t = ln\, x - ln\,(x + 1)$. Find the acceleration of the particle as a function of x. **3**

b) A firework is launched from a point $(0, 0)$ with a velocity of V ms^{-1} at an angle of θ to the horizontal. This firework explodes when it reaches its maximum height. Use the axes as shown, and assume that there is no air resistance and that the position of the firework t seconds after being launched is given by the equations

$$x = Vt\cos\theta \text{ and } y = Vt\sin\theta - \frac{1}{2}gt^2 \text{ (Do NOT prove these.)}$$



(i) Show that the maximum height where the firework explodes, is given by **2**

$$y = \frac{V^2\sin^2\theta}{2g}$$

(ii) A second firework is launched from the point $(0, 0)$ with a velocity of $\frac{7V}{10}$ ms^{-1} at an angle of 2θ to the horizontal and it also explodes when it reaches its maximum height. Given that the two fireworks reach the same maximum height,

α) show that $\cos\theta = \frac{5}{7}$ and **2**

β) if $V^2 = 98g$ find the maximum height reached by the two fireworks. **2**

Question 14 continued next page

c) A supply chain management service is testing a new delivery system by unmanned aerial vehicle (UAV). In one particular test, the operator is flying the UAV at a height of 220 metres above the ground with a horizontal velocity of $40\ m/s$ when it releases its parcel.

(i) If a parachute activates when the parcel reaches a speed of $50\ m/s$, find the horizontal distance the parcel has travelled when the parachute activates. (Use $g = 10m/s^2$.) **3**

(ii) Once the parachute activates, its horizontal velocity reduces to $5\ m/s$ and its vertical acceleration changes to $-2\ m/s^2$. Find the total horizontal distance travelled from the time the UAV releases the parcel to the time it hits the ground. **3**

End of Paper

Year 12 Mathematics Extension Section I – Multiple Choice Answer Sheet

Student Name _____

Class 12MTX _____

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

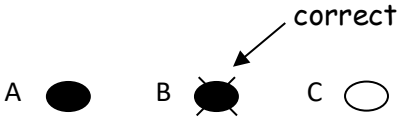
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



A ● B ● C ○ D ○

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1. **A** ○ **B** ○ **C** ○ **D** ○

2. **A** ○ **B** ○ **C** ○ **D** ○

3. **A** ○ **B** ○ **C** ○ **D** ○

4. **A** ○ **B** ○ **C** ○ **D** ○

5. **A** ○ **B** ○ **C** ○ **D** ○

6. **A** ○ **B** ○ **C** ○ **D** ○

7. **A** ○ **B** ○ **C** ○ **D** ○

8. **A** ○ **B** ○ **C** ○ **D** ○

9. **A** ○ **B** ○ **C** ○ **D** ○

10. **A** ○ **B** ○ **C** ○ **D** ○

Question		Marks
MC	1-7	**/7**
	8-10	**/3**
Q11		**/15**
Q12		**/15**
Q13	a, b, c	**/9**
	d, e	**/6**
Q14		**/15**
Total		**/70**