Student Number:_____

St. Catherine's School
Waverley

August 2008

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions
- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120
- Attempt Questions 1 – 10
- All questions are of equal value
- Complete each section in a separate booklet
- Section A (Questions 1-3)
  Section B (Questions 4-6)
  Section C (Questions 7-8)
  Section D (Questions 9-10)

Attempt Questions 1 – 10
All questions are of equal value **Marks**

Begin each question on a new page. Answer Questions 1 to 3 in the same booklet.

Section A

Question 1 (12 marks) Answer on a new page.

a) Evaluate $\frac{\pi+2}{\pi-2}$ correct to two decimal places **2**

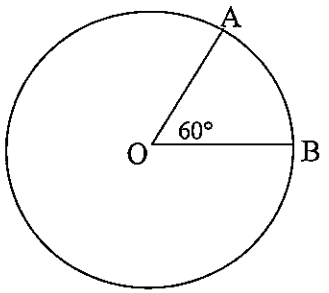
b) Solve $1-2x<7$ **2**

c) Find the exact value of $\log_5 150-(\log_5 2+\log_5 3)$ **2**

d) Show that $2x^2+3x-4=0$ has two solutions. **2**

e) Solve $\frac{2a+7}{3}=2$ **2**

f) **2**



If O is the centre of this circle of radius 4cm find the exact length of the arc AB

End of Question 1

Question 2 (12 marks) Start your answer on a new page. **Marks**

a) Differentiate with respect to x

(i) $2x^3+x^{-3}$ **2**

(ii) $\frac{1}{e^{2x}}-\sin x$ **2**

b) (i) Find $\int \sec^2 x-e^{4x}\ dx$ **2**

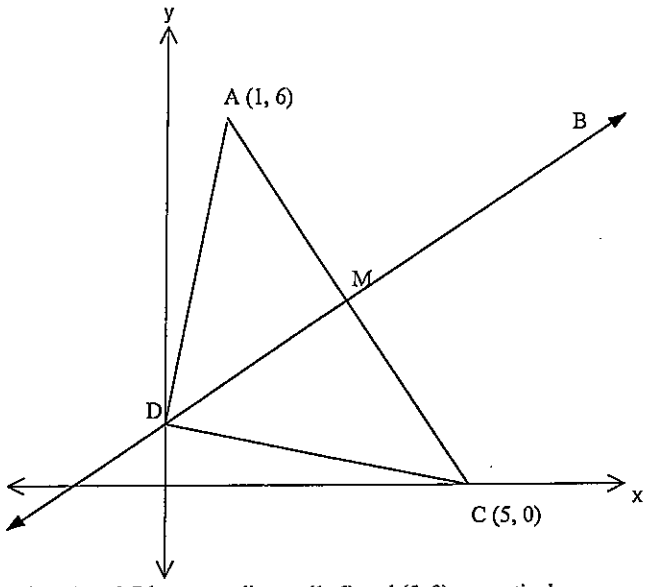
(ii) Evaluate $\int_1^e x^2+\frac{2}{x}dx$ **3**

c) Find the equation of the tangent to the curve y=2 sinx when $x=\frac{\pi}{4}$ **3**

End of Question 2

Question 3 (12 marks) Start your answer on a new page. **Marks**

a)



The points A and C have coordinates (1, 6) and (5, 0) respectively.
The line BD has an equation of $2x-3y+3=0$ and meets the y axis in D.

i) The point M is the midpoint of AC. Show that M has coordinates (3, 3). **1**

ii) Show that M lies on BD. **1**

iii) Find the gradient of the line AC. **1**

iv) Show that BD is perpendicular to AC. **2**

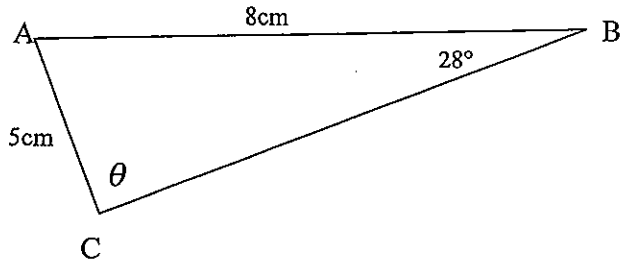
v) Find the distance AC. **1**

vi) Explain why the quadrilateral ABCD is a kite regardless of the position of B. **1**

Question 3 continues on the next page

Question 3 continued **Marks**

b) Solve for x: $(\ln x)^2 - 4\ln x = 0$ 2

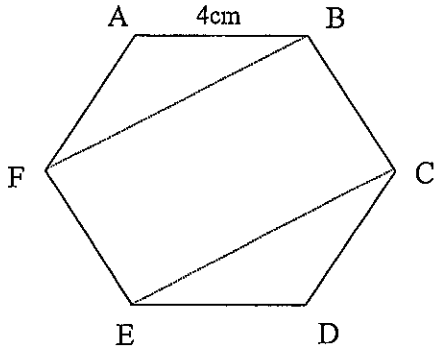
c) 

3

AB=8cm, AC=5cm, $\angle ABC = 28^0$
Given that θ is obtuse find the size of θ to the nearest degree

End of Question 3

Section B
Use a new booklet for questions 4-6
Question 4 (12 marks) Start your answer on a new page. **Marks**

a) 

ABCDEF is a regular hexagon with side lengths 4cm
Copy this diagram into your exam booklet

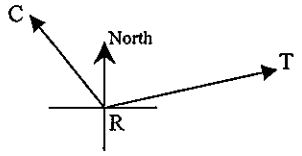
i) Explain why $\angle BAF = 120^0$ 1

ii) Prove that $\angle FBC = 90^0$ 2

iii) Use the cosine rule in ΔABF to show that BF=$4\sqrt{3}$ 2

iv) Find the **exact** area of the hexagon 3

b) Cleo and Tess are pilots of two light planes which leave Resthaven station at the same time. Cleo flies on a bearing of 330° at a speed of 180 km/h and Tess flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



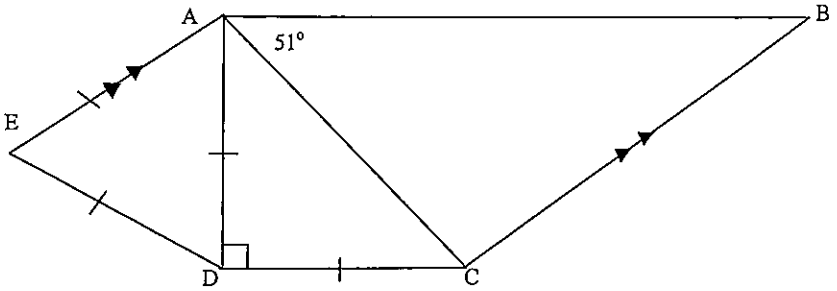
i) How far apart are Cleo and Tess after 2 hours? (to nearest km) 2

ii) What is the bearing of Tess from Cleo after 2 hours? (to nearest degree) 2

End of Question 4

Question 5 (12 marks) Start your answer on a new page. **Marks**

a) In the diagram below AE = ED = AD = DC, $\angle ADC = 90^\circ$ and AE || BC.
$\angle BAC = 51^\circ$



i) Find the size of $\angle EAB$. Give reasons for your answer. **2**

ii) Find the size of $\angle ABC$. Give reasons for your answer. **2**

b) A particle moves in a straight line so that its displacement, in metres, is given by

$x = \dfrac{4t^2 + t + 8}{4t + 1}$ where t is measured in seconds.

i) Find the initial position of the particle. **1**

ii) Show that an expression for the velocity of the particle is $\dfrac{16t^2 + 8t - 31}{(4t+1)^2}$ **1**

iii) Show that the particle is stationary when $t = \dfrac{-1 + 4\sqrt{2}}{4}$ seconds. **2**

iv) Describe the motion of the particle in the first two seconds. **2**

c) Solve the pair of simultaneous equations **2**

$$3x - y = 10$$
$$x = y + 2$$

End of Question 5

Question 6 (12 marks) Start your answer on a new page. **Marks**

a) For the function $y = x^4 - 6x^2$

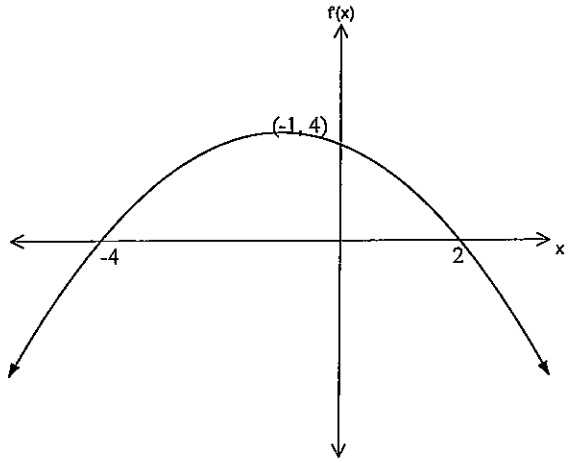
i) Find the x coordinates of the points where the curve crosses the axes. **2**

ii) Find the coordinates of the stationary points and determine their nature. **4**

iii) Find the coordinates of the points of inflexion. **2**

iv) Sketch the graph of $y = x^4 - 6x^2$ indicating clearly the intercepts, stationary points and points of inflexion. **2**

b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima. **2**

End of Question 6

Section C
Use a new booklet for questions 7-8
Question 7 (12 marks) Start your answer on a new page. **Marks**

a) i) For what values of x will the series $1+2x+4x^2+8x^3+....$ have a limiting sum? **2**

ii) If the limiting sum of this series is 2 find the value of x. **2**

b) i) Show that the curves $y=x^2-3x$ and $y=5x-x^2$ intersect at the points (0, 0) and (4, 4). **2**

ii) Find the area enclosed between the two curves. **2**

c) Marella trained for the City to Surf by running each day for 14 consecutive days. Each day she ran 500 metres more than she ran on the previous day.

i) On the final day Marella ran $14\frac{1}{2}$ Km. How far did she run on the first day? **2**

ii) Find the total distance Marella ran in the 14 days. **2**

End of Question 7

Question 8 (12 marks) Start your answer on a new page **Marks**

a) A city has a population which is growing at a rate that is proportional to the current population. The population at time t years after the start of 2006 is given by $P=Ae^{kt}$

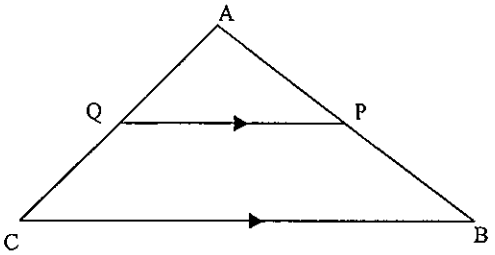
i) Show that $P=Ae^{kt}$ satisfies the equation $\frac{dP}{dt}=kP$. **1**

ii) If the population at the start of 2006 was 147 200 and at the start of 2007 was 154 800, find the values of A and k. **2**

iii) Find the population at the start of 2009 (to nearest hundred) **1**

iv) Find during which year the population will first exceed 200 000. **1**

b) In the diagram below, P is the midpoint of the side AB of the ΔABC. PQ is drawn parallel to BC.



i) Prove that $\Delta ABC \;|||\; \Delta APQ$. **2**

ii) Explain why Q is the midpoint of AC. **2**

c) Find an approximation for $\int_1^3 g(x)dx$ by using Simpson's Rule with the values in the table below. **2**

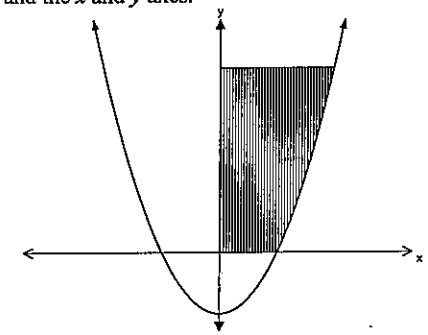
x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

d) Evaluate $\sum_{n=2}^{5} n^2-1$ **1**

End of Question 8

Section D
Use a new booklet for questions 9-10
Question 9 (12 marks) Start your answer on a new page. **Marks**

a) The diagram shows the region bounded by the curve $y=2x^2-2$ the line $y=6$ and the x and y axes. **3**

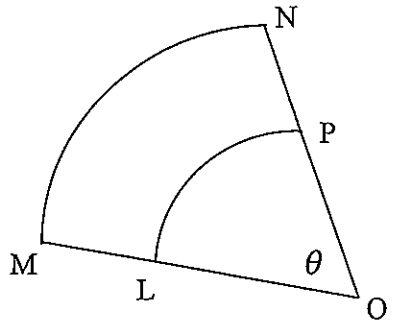


Find the volume of the solid of revolution formed when the region is rotated about the y axis.

b) Solve $2\sin\theta\cot\theta=1$ *for* $0\le\theta\le2\pi$ **3**

c) If $|a-3|=|b-3|$ find the two possible relationships between a and b **2**

d)



MN and LP are arcs of a circle with centre O. OL=3cm and LM=2cm

Find i) the ratio arc LP: arc MN **2**

ii) the ratio area sector OLP: area sector OMN **2**

End of Question 9

Question 10 (12 marks) Start your answer on a new page. **Marks**

a) A plant nursery has a watering system which repeatedly fills a storage tank then empties its contents to water different sections of the nursery.
The volume of water (in cubic metres) in the tank at a time t is given by the equation

$V=2-\sqrt{3}\cos t-\sin t$ where t is measured in minutes.

i) Give an equation for $\frac{dV}{dt}$, the rate of change of the volume at a time t. **1**

ii) Initially is the tank filling or emptying? (give reasons for your answer) **1**

iii) Find the time at which the tank first become completely full. **2**

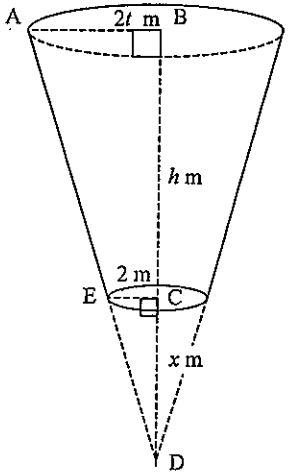
iv) Show that its capacity then is 4 cubic metres. **1**

Question 10 continues on the next page

Question 10 continued

Marks

b) This truncated cone is formed by having the section below EC removed. It is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The top radius is to be t times greater than the bottom radius which is 2 metres.



AB = $2t$ metres
BC = h metres
EC = 2 metres
CD = x metres

i) If x is the height of the removed section of the original cone, show using similar triangles that $x = \dfrac{h}{t-1}$ 2

ii) Show that the volume of the truncated cone is given by $V = \left(\dfrac{4\pi h}{3}\right)\left(t^2 + t + 1\right)$ 2

iii) If the upper radius plus the lower radius plus the height of the truncated cone must total 12 metres find the value of t which will maximise the volume and calculate the maximum volume of the hopper. 3

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$