
Student Number

Knox Grammar School

2010
Trial HSC Examination

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time - 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Subject Teachers
Mr I. Mulray
Ms F. Yamaner
Mr D Sedgman
Mr R Maxwell

This paper MUST NOT be removed from the examination room

Number of Students in Course: 72

Number of Writing Booklets Per Student (Four Page) 10

Total Marks – 120
- Attempt Questions 1 – 10
- Answer each question in a separate writing booklet
- All questions are of equal value

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Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE Writing Booklet. Extra Writing Booklets are available.

Question 1 (12 marks) Use a SEPARATE Writing Booklet.

(a) Solve the equation $\log_e x = 5$. Give your answer correct to four significant figures. **2**

(b) Factorise $3x^2 + x - 2$. **2**

(c) Simplify $\frac{2x-1}{3} + \frac{1}{4}$. **2**

(d) Sketch the graph of $3y - x = 6$, showing the intercepts on both axes. **2**

(e) Rationalise the denominator of $\frac{4}{\sqrt{3}-2}$. **2**

(f) Graph the solution of $|x-1| \leq 8$ on a number line. **2**

Question 2 (12 marks) Use a SEPARATE Writing Booklet.

(a) Differentiate with respect to x:

(i) $(3+\sin x)^{12}$ **2**

(ii) $\dfrac{x}{3e^x}$. **2**

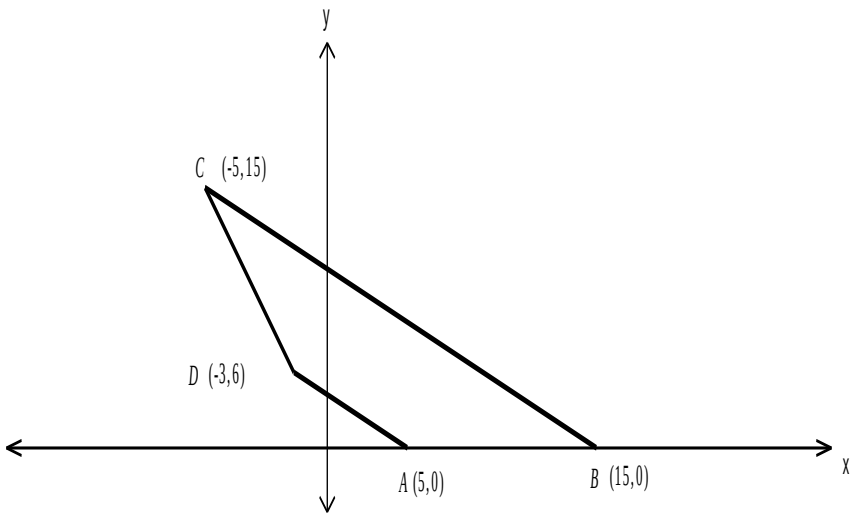
(b) (i) Find $\displaystyle\int \frac{x}{x^2-8}\,dx$. **2**

(ii) Evaluate $\displaystyle\int_4^9 \frac{1}{x^2}+\sqrt{x}\,dx$. **3**

(c) Find the equation of the tangent to $y=\cos x$ at the point $x=\dfrac{\pi}{3}$. **3**

Question 3 (12 marks) Use a SEPARATE Writing Booklet.

(a)



(i) Show that *ABCD* is a trapezium, by showing that *AD* is parallel to *BC*. **2**

(ii) Show that the equation of the line *BC* is $3x + 4y - 45 = 0$. **1**

(iii) Find the length of *BC*. **1**

(iv) Find the perpendicular distance of the point D from the line *BC*. **2**

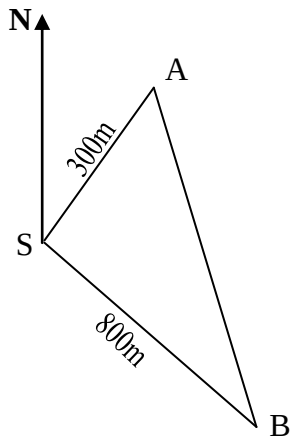
(v) Hence, or otherwise, find the area of the quadrilateral *ABCD*. **2**

(b) An arithmetic sequence has 21 terms. The first terms is 3 and the last term is 53. Find the third term in the sequence. **2**

(c) Shade the region in the plane defined by $y \leq 0$ and $y \geq 9 - x^2$. **2**

Question 4 (12 marks) Use a SEPARATE Writing Booklet.

(a) From a ship, *S*, two lighthouses, *A* and *B* are at a distance of 300 m and 800 m respectively. Lighthouse *A* is on a bearing of 34° from *S*, and lighthouse *B* is on a bearing of 142° from *S*.



(i) Copy and complete this diagram, and give the size of $\angle ASB$. **1**

(ii) Find the distance between the lighthouses, *A* and *B*. **2**

(iii) Find the bearing of *A* from *B*. **2**

(b) A simple game is played, by rolling the following three dice at the same time.


6-sided dice



8-sided dice



20 sided-dice

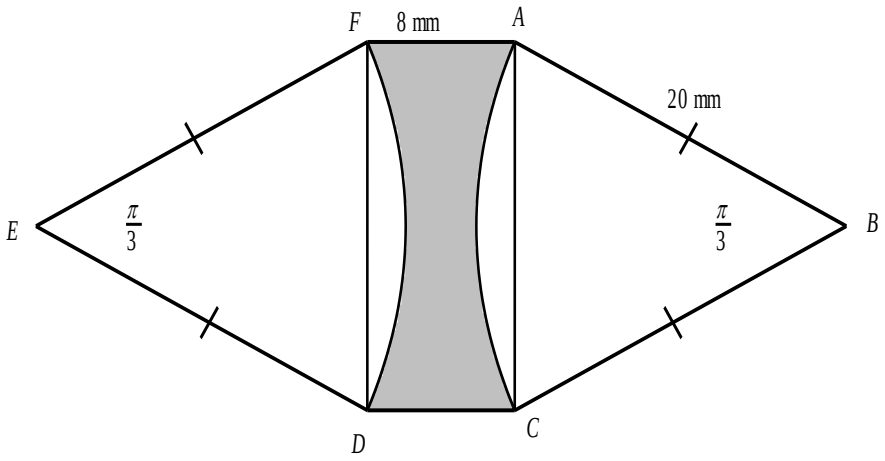
(i) What is the probability that on any of the three dice a 1 will not be the number on top? **1**

(ii) What is the probability that a 3 will appear on at least one of these three dice? **1**

(c) Myopia is a defect in the eye which causes near-sightedness. The cross-section of a lens made to correct this problem is shown below.

(i) Explain why ΔABC is equilateral. **1**

(ii) Calculate the shaded area of the cross-section. **2**



(d) Evaluate: $\lim_{x \to 4} \frac{x^2 - 16}{2x - 8}$ **2**

Question 5 (12 marks) Use a SEPARATE Writing Booklet.

(a) If α and β are the roots of the equation $9x^2 + 3x + 2 = 0$, find the values of:

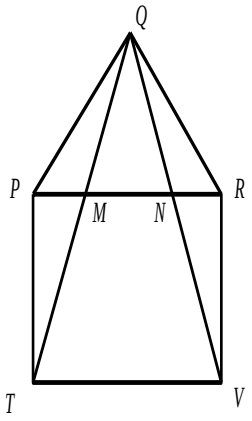
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) $|\alpha - \beta|$. **2**

(b)



In the diagram, ΔPQR is an equilateral triangle and $PRVT$ is a square. P, Q, R, V, T are all in the same plane.

Prove:

(i) $\Delta PQT \equiv \Delta RQV$ **3**

(ii) $\angle QVT = \angle QTV$. **1**

(c) If $y = e^{kx}$ satisfies the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$, find the possible values of k. **2**

Question 6 (12 marks) Use a SEPARATE Writing Booklet.

(a) One model for the number of individuals connected to the internet worldwide is the exponential growth model.

$$N = Ae^{kt}$$

where N is the estimate for the number of individuals connected to the internet (in millions), and t is the time in years after 1 January 2011.

It is estimated that at the start of 2012, when $t = 1$, there will be 1800 million individuals connected to the internet, while at the start of 2013, when $t = 2$, there will be 2400 million individuals connected to the internet.

(i) Show that $A = 1350$ and $k = \ln\left(\frac{4}{3}\right)$. **2**

(ii) According to the model, during which month and year will the number of individuals connected to the internet first exceed 4000 million. **2**

(iii) At what rate will the number of individuals be increasing in 2015? **2**

(b) Use Simpson's Rule with five function values to estimate the area under the curve $y = e^{x^2}$ and the x-axis between $x = 1$ and $x = 3$. **3**

(c) Solve: $\sec^2(2x) = 1$ for $-\pi \le x \le \pi$. **3**

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Question 7 (12 marks) Use a SEPARATE Writing Booklet.

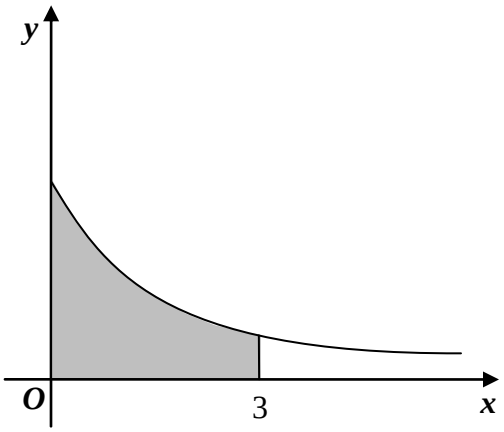
(a) Find the equation of the locus of $P(x, y)$, if P is equidistant from the point $A(4, 2)$ and the line $x = 6$. **2**

(b) The quadratic function $x^2 - 8mx + 3n$ is positive definite.

(i) By considering the discriminant, show that $16m^2 < 3n$. **2**

(ii) Find the range of values of m, if the sum of the roots is twice the product of the roots. **3**

(c) **3**



The shaded area in the diagram is bounded by the curve $y = \frac{1}{4x+3}$, the x-axis and the lines $x = 0$ and $x = 3$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis.

(d) Comment on the validity of the following statement: **2**

"The first term of a geometric progression is 5 *and its sum to infinity is* 2*".*

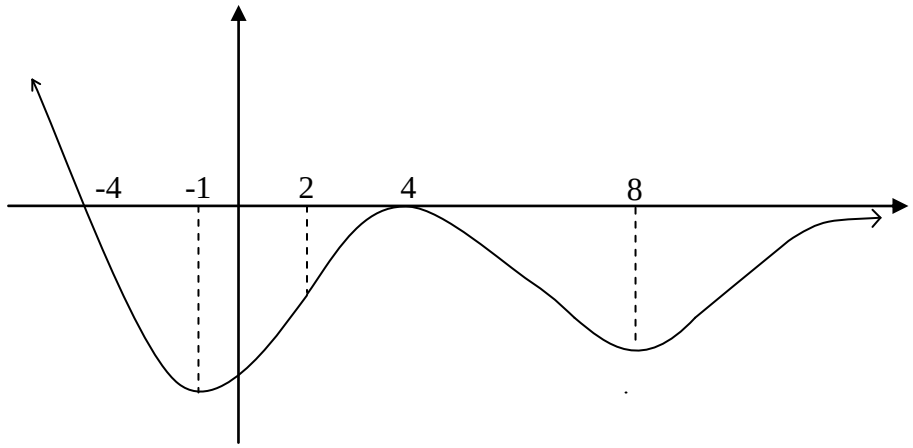
Question 8 (12 marks) Use a SEPARATE Writing Booklet.

(a) Beth deposits \$240 into a superannuation fund at the beginning of each month. The money is invested at 9% p.a. compounded monthly.

(i) After 40 years, what will be the value of the first \$240 invested? **2**

(ii) By writing a geometric series for the value of all Beth's investments, calculate the final value of her investment at the end of the 40 years. **3**

(b) The graph of $y = f'(x)$ is shown below.



(i) There are two stationary points on $y = f(x)$. Determine their x-coordinates and their nature. **2**

(ii) What happens to $f(x)$ for large values of x? **2**

(c) Let $f(x) = \frac{x^3}{3} + x + 1$

(i) Show that the graph of $y = f(x)$ has no stationary points. **2**

(ii) For what values of x is the graph of $y = f(x)$ concave up? **1**

Question 9 (12 marks) Use a SEPARATE Writing Booklet.

The velocity, v, of a particle moving in a straight line is given by $v = 2\cos(2t) + 1$, where t is the time in seconds. The particle is initially at $x = 3$ m.

(a) Find the initial velocity of the particle. **1**

(b) Find the first time after $t = 0$ that the acceleration is zero. **2**

(c) What is the maximum acceleration of the particle? **1**

(d) Sketch the velocity-time graph for $0 \le t \le 2\pi$, showing all important information. **3**

(e) Find the maximum velocity of the particle. **1**

(f) Find the time when the particle first changes direction. **1**

(g) Find the distance travelled by the particle between $t = 0$ and $t = \frac{\pi}{2}$. **3**

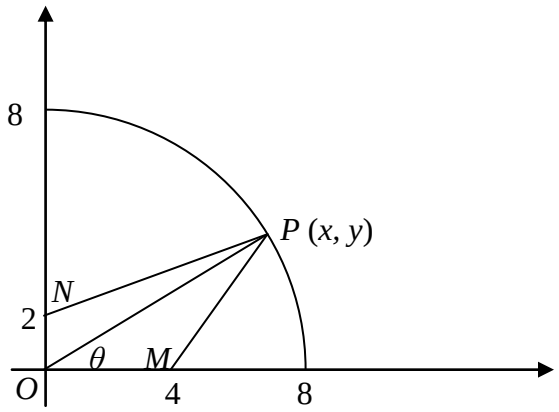
Question 10 (12 marks) Use a SEPARATE Writing Booklet.

(a) (i) Find the stationary points for the curve $y = xe^x$ and determine their nature. **2**

(ii) Sketch the curve $y = xe^x$, showing stationary point(s), intercepts and asymptotes (there is no need to find points of inflexion) **2**

(iii) Hence, or otherwise, find the value(s) of k for which the equation $xe^x = k$ has 2 solutions. **1**

(b)



The diagram shows the part of the circle $x^2 + y^2 = 64$ that lies in the first quadrant. The point $P(x, y)$ is on the circle. O is the origin, M is on the x-axis at $x = 4$ and N is on the y-axis at $y = 2$. The size of $\angle MOP$ is θ radians.

(i) Show that the Area, A, of the quadrilateral $OMPN$ is given by: **2**

$$A = 16\sin\theta + 8\cos\theta$$

(ii) Find the value of $\tan\theta$ for which A is a maximum. **3**

(iii) Hence determine, in surd form, the coordinates of P, such that $OMPN$ has the largest possible area. **2**

END OF PAPER

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$