BOARD OF STUDIES
NEW SOUTH WALES

2013

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
  Black pen is preferred
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–12

25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

Section II Pages 13–26

75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

Section I

25 marks
Attempt Questions 1–25
Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

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1 Which of the following events would be LEAST likely to occur?

(A) Tossing a fair coin and obtaining a head

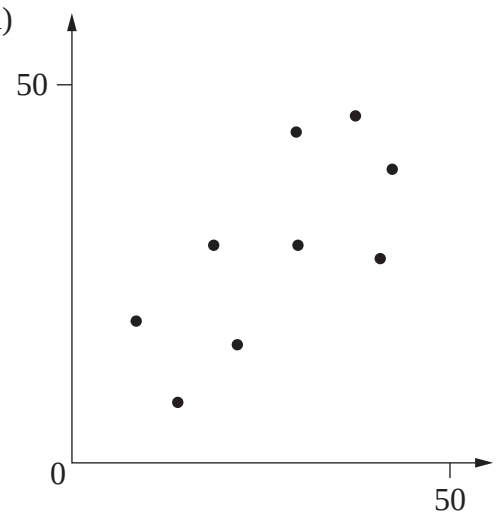
(B) Rolling a standard six-sided die and obtaining a 3

(C) Randomly selecting the letter ‘G’ from the 26 letters of the alphabet

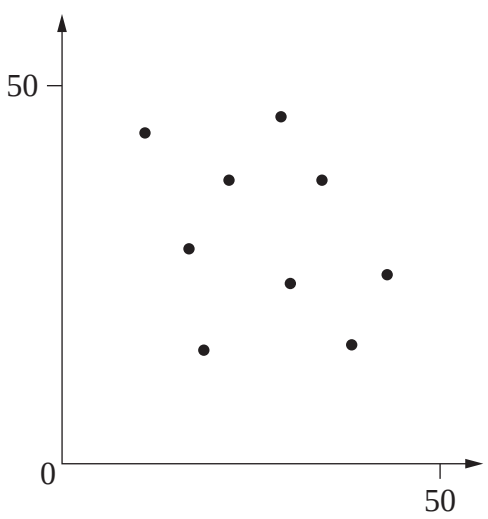
(D) Winning first prize in a raffle of 100 tickets in which you have 4 tickets

2 Which graph best shows data with a correlation closest to 0.3?

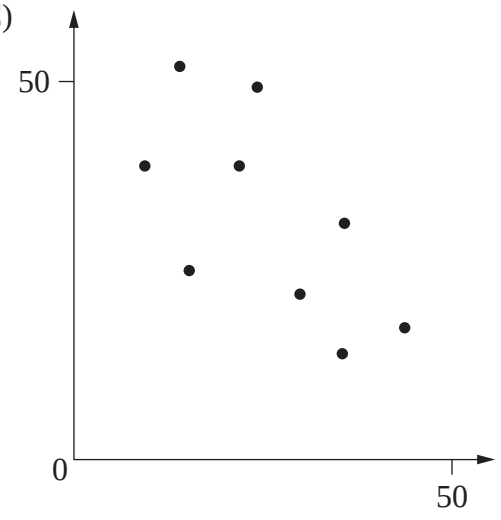
(A)



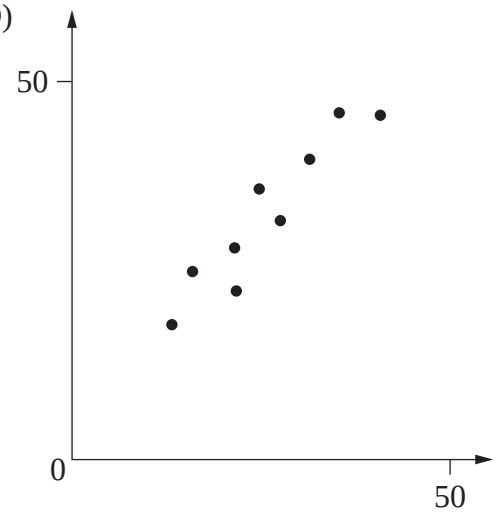
(B)



(C)



(D)



3 Luke's normal rate of pay is \$24.80 per hour. In one week he worked 14 hours at the normal rate, 4 hours at time-and-a-half, and $3\frac{1}{2}$ hours at double time. He was also paid a wet weather allowance of \$50 for the week.

What was his pay for the week?

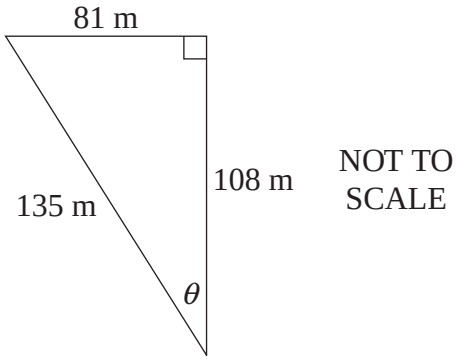
(A) \$583.20

(B) \$620.40

(C) \$669.60

(D) \$719.60

4 What is the value of θ, to the nearest degree?



(A) 31°

(B) 37°

(C) 49°

(D) 53°

5 Which of the following is $\frac{12T^2}{3T \times 2W}$ in its simplest form?

(A) $\frac{2T}{W}$

(B) $2TW$

(C) $\frac{8T}{W}$

(D) $8TW$

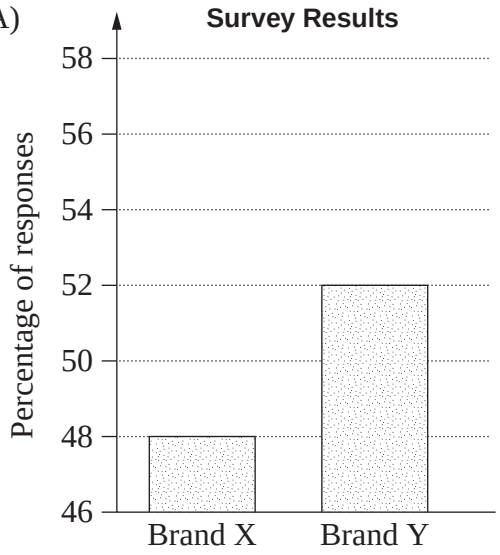
6 A survey was conducted where people were asked which of two brands of smartphones they preferred. The results were:

- 48% preferred Brand X
- 52% preferred Brand Y.

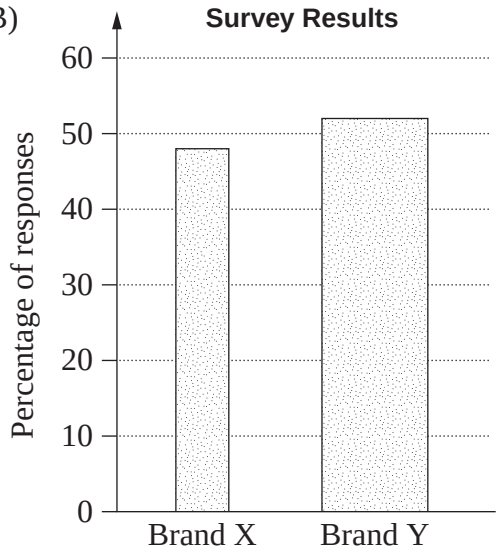
A graph displaying the data is to be included in a magazine article. The editor of the magazine wishes to ensure that the graph is not misleading in any way.

Which graph should the editor choose to include in the article?

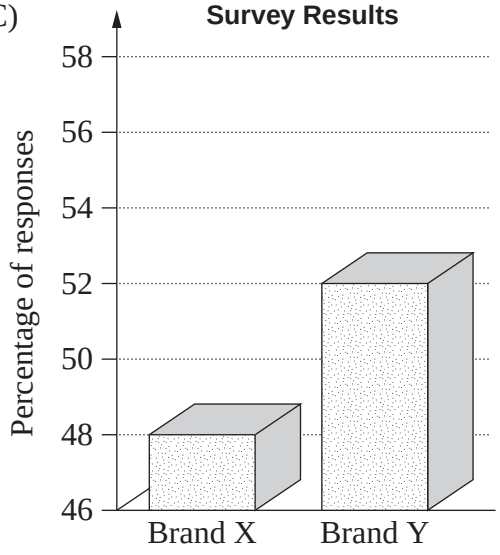
(A)



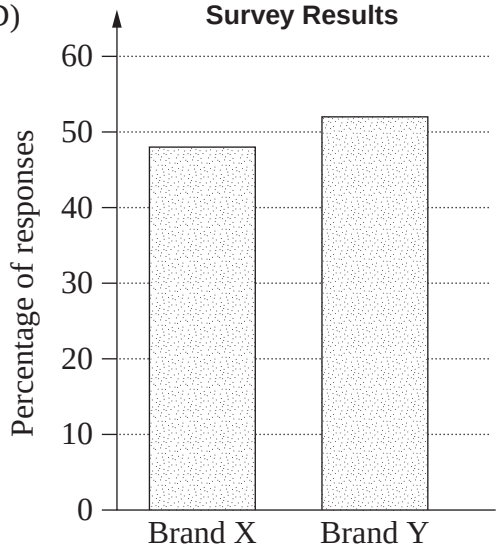
(B)



(C)



(D)



7 In an experiment, a standard six-sided die was rolled 72 times. The results are shown in the table.

Number on die	*Frequency*
1	6
2	12
3	10
4	20
5	9
6	15

Which number on the die was obtained the expected number of times?

(A) 1

(B) 2

(C) 3

(D) 6

8 A high school has 100 students in each year group, Year 7 to Year 12. A survey is to be conducted to determine the average number of text messages sent per month by students at the school.

Which of the following would provide the most representative sample for this survey?

(A) All Year 7 students

(B) All Physics students in Years 11 and 12

(C) 20 students chosen at random from each year group

(D) 120 students chosen at random from the school roll

9 Lynne invests $1000 for a term of 15 months. Interest is paid at a flat rate of 3.75% per annum.

How much will Lynne's investment be worth at the end of the term?

(A) $1046.88

(B) $1047.09

(C) $1296.88

(D) $1468.75

10 Students studying vocational education courses were surveyed about their living arrangements.

	Females	*Males*	*Totals*
Living with parent(s)	46	155	201
Not living with parent(s)	182	122	304
Totals	228	277	505

One of these students is selected at random.

What is the probability that this student is male and living with his parent(s)?

(A) 31%

(B) 40%

(C) 56%

(D) 77%

11 An enterprise agreement has the following annual salary arrangements:

Base Salary	
Step 1	$35 000
Step 2	$40 000
Step 3	$45 000

Leadership Allowance	
Leader 1	$5000
Leader 2	$7500
Leader 3	$10 000

George's employer pays 6% more than the enterprise agreement. He is on Step 3 and receives an allowance for Leader 2.

What is George's gross monthly pay?

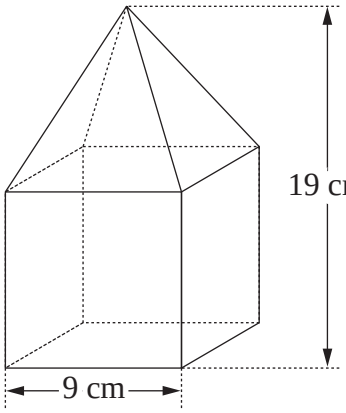
(A) $4375.00

(B) $4412.50

(C) $4600.00

(D) $4637.50

12 A square pyramid fits exactly on top of a cube to form a solid.



NOT TO SCALE

What is the volume of the solid?

(A) 513 cm^3

(B) 999 cm^3

(C) 1242 cm^3

(D) 1539 cm^3

13 Polly borrowed $11 000. She repaid the loan in full at the end of two years with a lump sum of $12 000.

What annual simple interest rate was she charged?

(A) 4.17%

(B) 4.55%

(C) 8.33%

(D) 9.09%

14 The July sale prices for properties in a suburb were:

$552 000, $595 000, $607 000, $607 000, $682 000 and $685 000.

On 1 August, another property in the same suburb was sold for over one million dollars.

If this property had been sold in July, what effect would it have had on the mean and median sale prices for July?

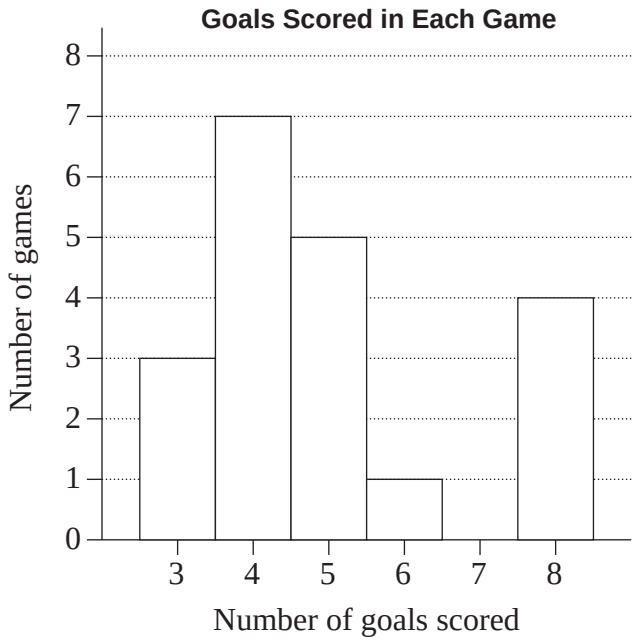
(A) Both the mean and the median would have changed.

(B) Neither the mean nor the median would have changed.

(C) The mean would have changed and the median would have stayed the same.

(D) The mean would have stayed the same and the median would have changed.

15 The frequency histogram shows the number of goals scored by a football team in each game in a season.



What was the mean number of goals scored per game by this team?

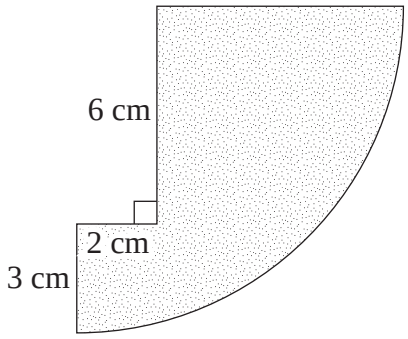
(A) 4

(B) 4.5

(C) 5

(D) 5.5

16 The shaded region shows a quadrant with a rectangle removed.



NOT TO SCALE

What is the area of the shaded region, to the nearest cm^2?

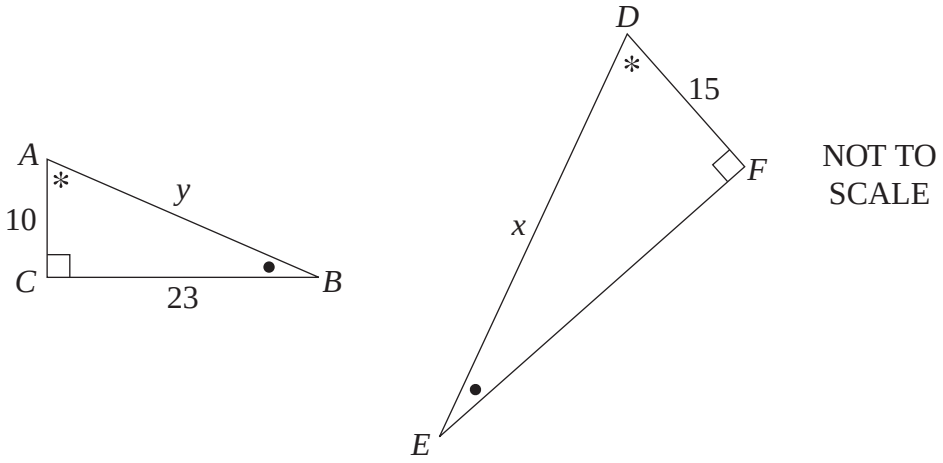
(A) 38 cm^2

(B) 52 cm^2

(C) 61 cm^2

(D) 70 cm^2

17 Triangles *ABC* and *DEF* are similar.



Which expression could be used to find the value of x?

(A) $y \times \frac{10}{15}$

(B) $y \times \frac{10}{23}$

(C) $y \times \frac{15}{10}$

(D) $y \times \frac{23}{15}$

18 Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled.

What is the probability of obtaining a sum of 6?

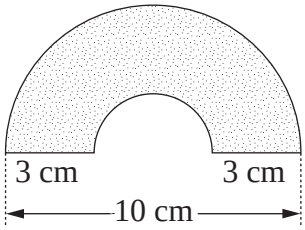
(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{12}$

(D) $\frac{5}{36}$

19 A logo is designed using half of an annulus.



NOT TO SCALE

What is the area of the logo, to the nearest cm^2?

(A) 25 cm^2

(B) 33 cm^2

(C) 132 cm^2

(D) 143 cm^2

20 There are 60 000 students sitting a state-wide examination. If the results form a normal distribution, how many students would be expected to score a result between 1 and 2 standard deviations above the mean?

You may assume for normally distributed data that:

- 68% of scores have *z*-scores between –1 and 1
- 95% of scores have *z*-scores between –2 and 2
- 99.7% of scores have *z*-scores between –3 and 3.

(A) 8100

(B) 16 200

(C) 20 400

(D) 28 500

21 Which equation correctly shows r as the subject of $S = 800(1 - r)$?

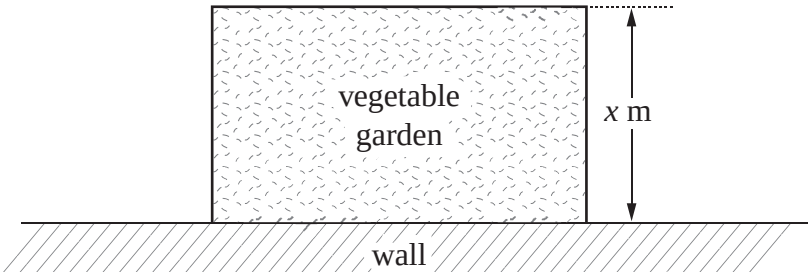
(A) $r = \dfrac{800 - S}{800}$

(B) $r = \dfrac{S - 800}{800}$

(C) $r = 800 - S$

(D) $r = S - 800$

22 Leanne wants to build a rectangular vegetable garden in her backyard. She has 20 metres of fencing and will use a wall as one side of the garden. The plan for her garden is shown, where x metres is the width of her garden.



NOT TO SCALE

Which equation gives the area, A, of the vegetable garden?

(A) $A = 10x - x^2$

(B) $A = 10x - 2x^2$

(C) $A = 20x - x^2$

(D) $A = 20x - 2x^2$

23 Zina opened an account to save for a new car. Six months after opening the account, she made her first deposit of \$1200 and continued depositing \$1200 at the end of each six month period. Interest was paid at 3% per annum, compounded half-yearly.

How much was in Zina's account two years after first opening it?

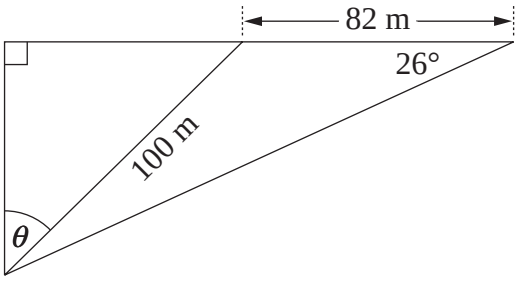
(A) \$4909.08

(B) \$4982.72

(C) \$5018.16

(D) \$5094.55

24 What is the value of θ, to the nearest degree?



NOT TO SCALE

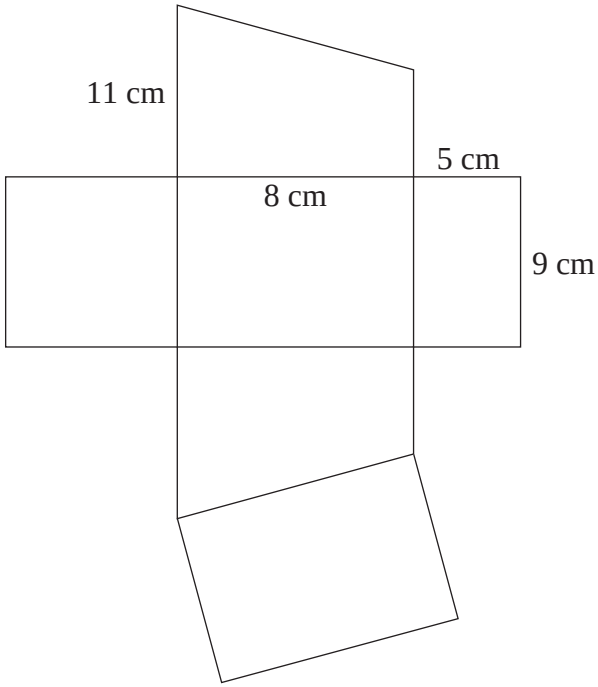
(A) 21°

(B) 32°

(C) 43°

(D) 55°

25 A net is made using four rectangles and two trapeziums. It is folded to form a solid.



NOT TO SCALE

What is the volume of the solid, in cm^3?

(A) 360 cm^3

(B) 434 cm^3

(C) 440 cm^3

(D) 576 cm^3

Section II

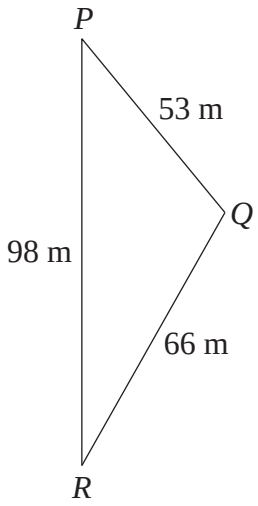
75 marks
Attempt Questions 26–30
Allow about 1 hour and 55 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 26–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks) Use the Question 26 Writing Booklet.

(a) Triangle PQR is shown. **2**



NOT TO SCALE

Find the size of angle Q, to the nearest degree.

(b) Write down a set of six data values that has a range of 12, a mode of 12 and a minimum value of 12. **2**

Question 26 continues on page 14

(c) The probability that Michael will score more than 100 points in a game of bowling is $\frac{31}{40}$.

(i) A commentator states that the probability that Michael will score less than 100 points in a game of bowling is $\frac{9}{40}$. **1**

Is the commentator correct? Give a reason for your answer.

(ii) Michael plays two games of bowling. What is the probability that he scores more than 100 points in the first game and then again in the second game? **1**

(d) A section of Jim's electricity bill is shown.

Energy Used and Costs

METER ID	THIS READING	LAST READING	ENERGY USED	RATE (per kWh)	COST
Peak Energy Charge					
TMV04221/01	531.2	274.8	256.4 kWh	47.7700c	$122.48
Shoulder Energy Charge					
TMV04221/02	***A***	560.9	523.5 kWh	19.4000c	$101.56
Off-peak (Night) Energy Charge					
TMV04221/03	242.5	0.0	242.5 kWh	9.6000c	$23.28

(i) What is the value of ***A***? **1**

(ii) How much will Jim save if he uses 154 kWh of energy at the Off-peak rate rather than at the Peak rate? **2**

Question 26 continues on page 15

(e) Kimberley has invested $3500. **2**

Interest is compounded half-yearly at a rate of 2% per half-year.

Compounded values of $1

Period	*Interest rate per period*					
	1%	2%	3%	4%	5%	6%
1	1.010	1.02	1.03	1.04	1.05	1.06
2	1.020	1.040	1.061	1.082	1.103	1.124
3	1.030	1.061	1.093	1.125	1.158	1.191
4	1.041	1.082	1.126	1.170	1.216	1.262
5	1.051	1.104	1.159	1.217	1.276	1.338
6	1.062	1.126	1.194	1.265	1.340	1.419
7	1.072	1.149	1.230	1.316	1.407	1.504
8	1.083	1.172	1.267	1.369	1.477	1.594

Use the table to calculate the value of her investment at the end of 4 years.

(f) Jason travels to work by car on all five days of his working week, leaving home at 7 am each day. He compares his travel times using roads without tolls and roads with tolls over a period of 12 working weeks.

He records his travel times (in minutes) in a back-to-back stem-and-leaf plot.

Travel time (minutes)

Without tolls		*With tolls*
9	3	5 8 9 9
9 9 8 7 7 6 5 5 4 4 3 2 0	4	0 1 2 6 7 7 8 8 8 9
9 8 7 5 4 3 3 3 2 2 2 2 1 1 0	5	2 4 4 5 6 8 9
1	6	1 3 5 7
	7	0 2 8
	8	2
	9	0

(i) What is the modal travel time when he uses roads without tolls? **1**

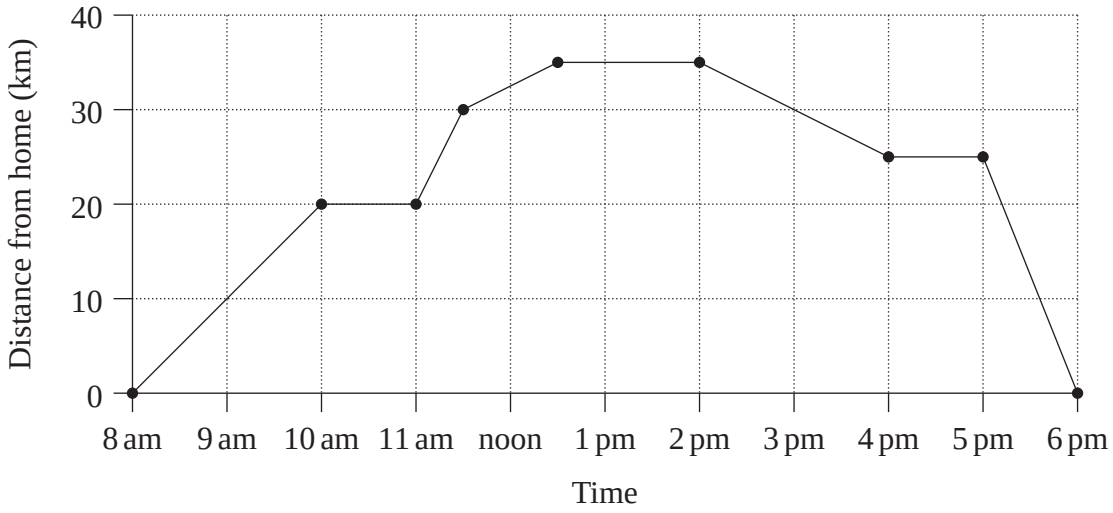
(ii) What is the median travel time when he uses roads without tolls? **1**

(iii) Describe how the two data sets differ in terms of the spread and skewness of their distributions. **2**

End of Question 26

Question 27 (15 marks) Use the Question 27 Writing Booklet.

(a) Lucy went for a bike ride. She left home at 8 am and arrived back at home at 6 pm. A graph representing her journey is shown.



(i) What was the total distance that she rode during the day? **1**

(ii) How much time did Lucy spend riding her bike during the day? **1**

(b) The table shows the tax payable to the Australian Taxation Office for different taxable incomes. **4**

Taxable income	*Tax on this income*
\$0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Acknowledgment: © Australian Taxation Office for the Commonwealth of Australia

Peta has a gross annual salary of \$84 000. She has tax deductions of \$1000 for work-related travel and \$500 for stationery. The Medicare levy that she pays is calculated at 1.5% of her taxable income.

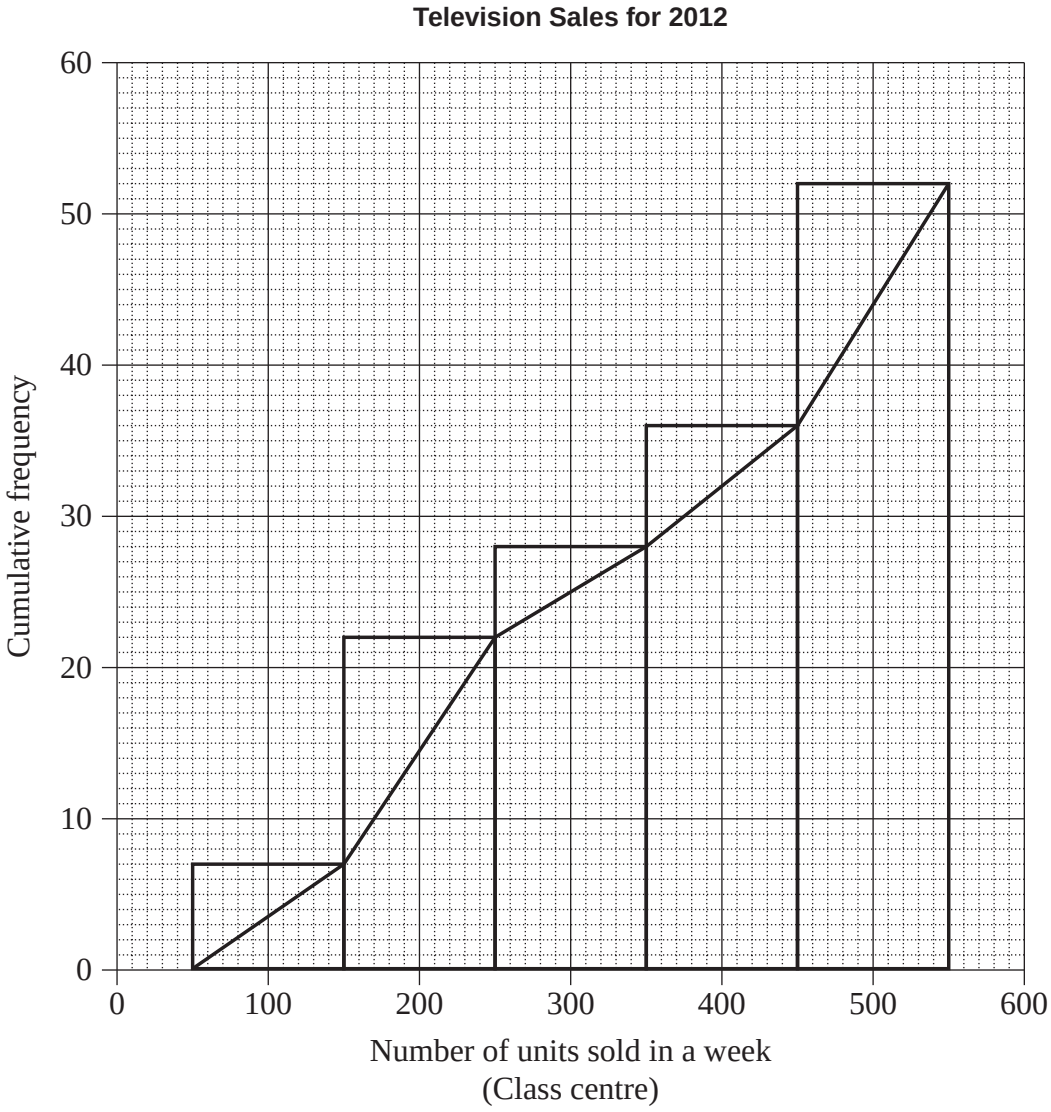
Peta has already paid \$18 500 in tax.

Will Peta receive a tax refund or will she owe money to the Australian Taxation Office? Justify your answer by calculating the refund or amount owed.

Question 27 continues on page 17

(c) A retailer has collected data on the number of televisions that he sold each week in 2012.

He grouped the data into classes and displayed the data using a cumulative frequency histogram and polygon (ogive).



(i) Use the cumulative frequency polygon to determine the interquartile range. **2**

(ii) Oscar said that the retailer sold 300 televisions in 6 of the weeks in 2012. **1**

Is he correct? Give a reason for your answer.

Question 27 continues on page 18

(d) A rectangular wooden chopping board is advertised as being 17 cm by 25 cm, with each side measured to the nearest centimetre.

(i) Calculate the percentage error in the measurement of the longer side. **1**

(ii) Between what lower and upper limits does the actual area of the top of the chopping board lie? **2**

(e) Karin is in Athens, which is two hours ahead of Greenwich Mean Time. Marco is in New York, which is five hours behind Greenwich Mean Time.

(i) Karin is going to ring Marco at 10 pm on Tuesday, Athens time. **1**

What day and time will it be in New York when she rings?

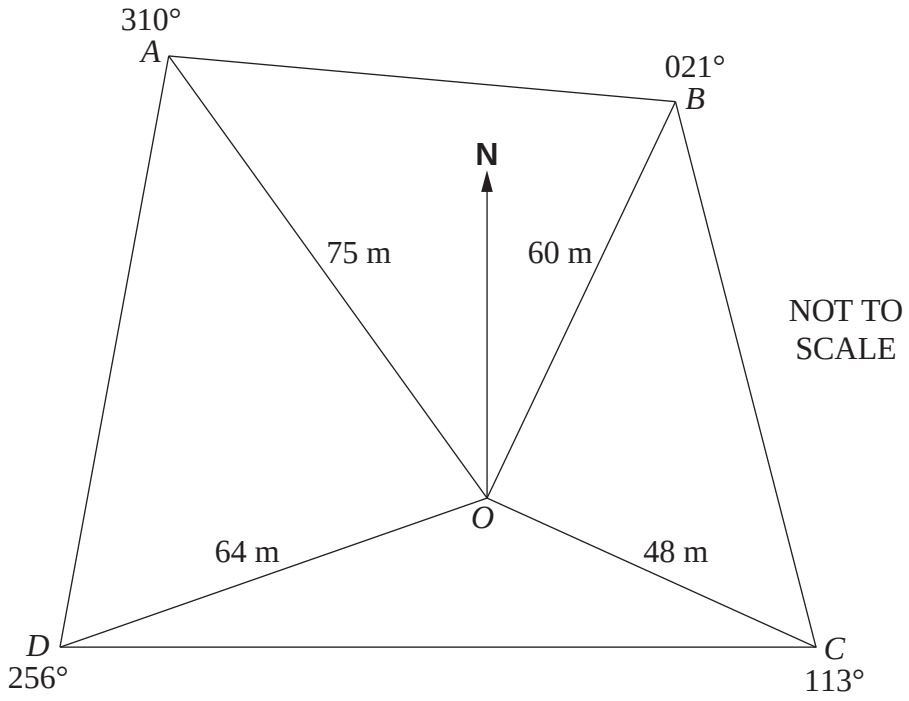
(ii) Marco is going to fly from New York to Athens. His flight will leave on Wednesday at 9 am, New York time, and will take 11 hours. **2**

What day and time will it be in Athens when he arrives?

End of Question 27

Question 28 (15 marks) Use the Question 28 Writing Booklet.

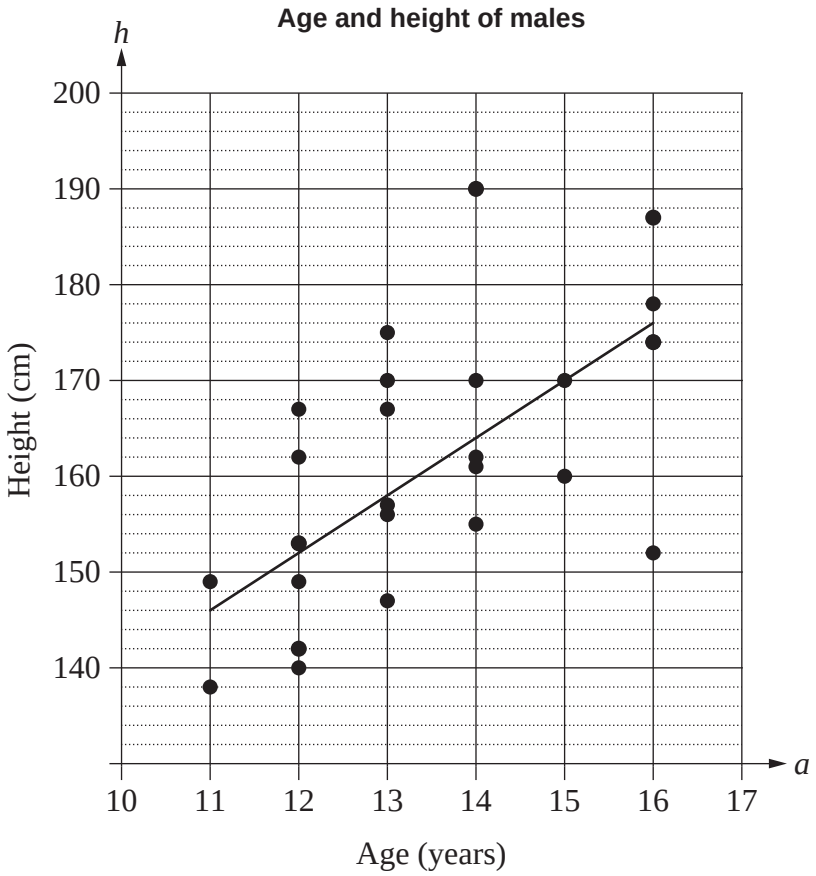
(a) A compass radial survey of the field *ABCD* has been conducted from *O*. **2**



Find the area of the section *ABO*, to the nearest square metre.

Question 28 continues on page 20

(b) Ahmed collected data on the age (a) and height (h) of males aged 11 to 16 years. He created a scatterplot of the data and constructed a line of best fit to model the relationship between the age and height of males.



(i) Determine the gradient of the line of best fit shown on the graph. **1**

(ii) Explain the meaning of the gradient in the context of the data. **1**

(iii) Determine the equation of the line of best fit shown on the graph. **2**

(iv) Use the line of best fit to predict the height of a typical 17-year-old male. **1**

(v) Why would this model not be useful for predicting the height of a typical 45-year-old male? **1**

Question 28 continues on page 21

(c) A ship sails due South from Channel-Port-aux-Basques, Canada, 47°N 59°W to Barbados, 13°N 59°W. **2**

How far did the ship sail, to the nearest kilometre? Assume that the radius of Earth is 6400 km.

(d) Adhele has 2000 shares. The current share price is $1.50 per share. Adhele is paid a dividend of $0.30 per share.

(i) What is the current value of her shares? **1**

(ii) Calculate the dividend yield. **1**

(e) Zheng has purchased a computer for $5000 for his company. He wants to compare two different methods of depreciation over two years for the computer. **3**

Method 1: Straight-line with $1250 depreciation per annum.

Method 2: Declining balance with 35% depreciation per annum.

Which method gives the greatest depreciation over the two years? Justify your answer with suitable calculations.

End of Question 28

Question 29 (15 marks) Use the Question 29 Writing Booklet.

(a) Sarah tried to solve this equation and made a mistake in Line 2. **2**

$$\frac{W+4}{3} - \frac{2W-1}{5} = 1 \quad \text{..... Line 1}$$

$$5W + 20 - 6W - 3 = 15 \text{ ..... Line 2}$$

$$17 - W = 15 \text{ ..... Line 3}$$

$$W = 2 \quad \text{..... Line 4}$$

Copy the equation in Line 1 into your writing booklet and continue your solution to solve this equation for W. Show all lines of working.

(b) Ali's class sits two Geography tests. The results of her class on the first Geography test are shown.

58, 74, 65, 66, 73, 71, 72, 74, 62, 70

The mean was 68.5 for the first test.

(i) Calculate the standard deviation for the first test. Give your answer correct to one decimal place. **1**

(ii) On the second Geography test, the mean for the class was 74.4 and the standard deviation was 12.4. **3**

Ali scored 62 on the first test. Calculate the mark that she needed to obtain in the second test to ensure that her performance relative to the class was maintained.

Question 29 continues on page 23

(c) Mary is designing a website that requires unique logins to be generated. **2**

She plans to generate the logins using two capital letters from the alphabet followed by a series of numerals from 0 to 9 inclusive. All logins will have the same number of numerals. Repetition of letters and numerals is allowed.

What is the minimum number of numerals required for each login so that Mary can generate at least 3 million logins? Justify your answer with suitable calculations.

(d) Jane plays a game which involves two coins being tossed. The amounts to be won for the different possible outcomes are shown in the table. **3**

Win	\$6	for two heads
Win	\$1	for one head and one tail
Win	\$2	for two tails

It costs \$4 to play one game. Will Jane expect a gain or a loss, and how much will it be? Justify your answer with suitable calculations.

(e) Jack borrowed \$300 000 to buy a house. Interest is charged at the rate of 6% per annum, compounded monthly. His minimum monthly repayment is \$1798.65 . **4**

Jack decided to pay an extra \$250 each month from the start of the loan period.

Will he be able to pay off the loan in 20 years? Justify your answer with suitable calculations.

End of Question 29

Question 30 (15 marks) Use the Question 30 Writing Booklet.

(a) Wind turbines, such as those shown, are used to generate power.



Acknowledgement: Stock Photo–Wind Turbines – Image ID: 1051412 © Miguel Saavedra

In theory, the power that could be generated by a wind turbine is modelled using the equation

$$T = 20\,000w^3$$

where T is the theoretical power generated, in watts
w is the speed of the wind, in metres per second.

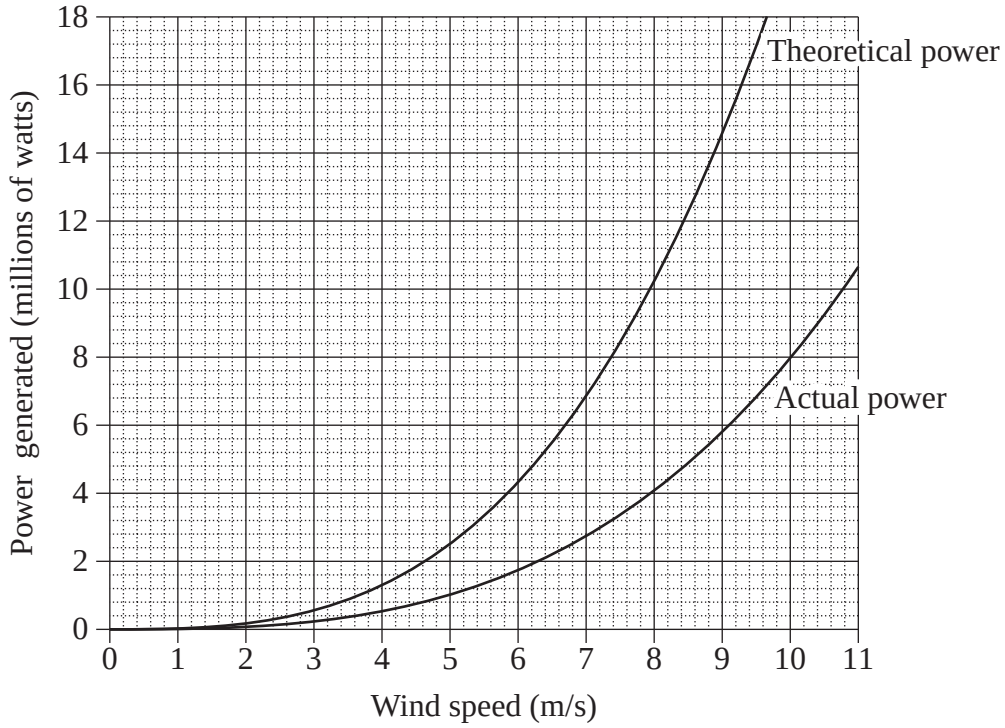
(i) Using this equation, what is the theoretical power generated by a wind turbine if the wind speed is 7.3 m/s? **1**

(ii) In practice, the actual power generated by a wind turbine is only 40% of the theoretical power. **1**

If A is the actual power generated, in watts, write an equation for A in terms of w.

Question 30 continues on page 25

The graph shows both the theoretical power generated and the actual power generated by a particular wind turbine.



(iii) Using the graph, or otherwise, find the difference between the theoretical power and the actual power generated when the wind speed is 9 m/s. **1**

(iv) A particular farm requires at least 4.4 million watts of actual power in order to be self-sufficient. **1**

What is the minimum wind speed required for the farm to be self-sufficient?

(v) A more accurate formula to calculate the power (P) generated by a wind turbine is **3**

$$P = 0.61 \times \pi \times r^2 \times w^3$$

where r is the length of each blade, in metres
w is the speed of the wind, in metres per second.

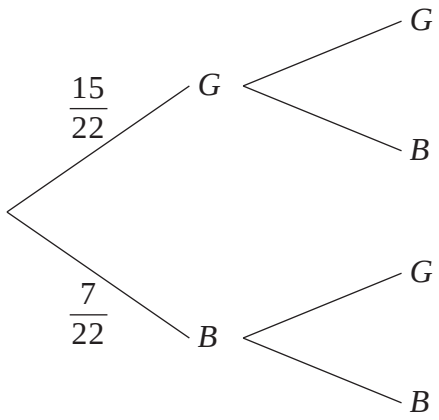
Each blade of a particular wind turbine has a length of 43 metres. The turbine operates at a wind speed of 8 m/s.

Using the formula above, if the wind speed increased by 10%, what would be the percentage increase in the power generated by this wind turbine?

Question 30 continues on page 26

(b) In a class there are 15 girls (G) and 7 boys (B). Two students are chosen at random to be class representatives.

(i) Copy and complete the tree diagram in your answer booklet. **2**



(ii) What is the probability that the two students chosen are of the same gender? **2**

(c) Joel mixes petrol and oil in the ratio 40 : 1 to make fuel for his leaf blower.

(i) Joel pours 5 litres of petrol into an empty container to make fuel for his leaf blower. **1**

How much oil should he add to the petrol to ensure that the fuel is in the correct ratio?

(ii) Joel has 4.1 litres of fuel left in his container after filling his leaf blower. **3**

He wishes to use this fuel in his lawnmower. However, his lawnmower requires the petrol and oil to be mixed in the ratio 25 : 1.

How much oil should he add to the container so that the fuel is in the correct ratio for his lawnmower?

End of paper

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FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle
r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis
b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements
d_f = first measurement
d_m = middle measurement
d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius
h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius
h = perpendicular height
A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$