BOARD OF STUDIES
NEW SOUTH WALES

2011

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
  Black pen is preferred
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- Write your Centre Number and Student Number on the Question 27 Writing Booklet

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–24

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

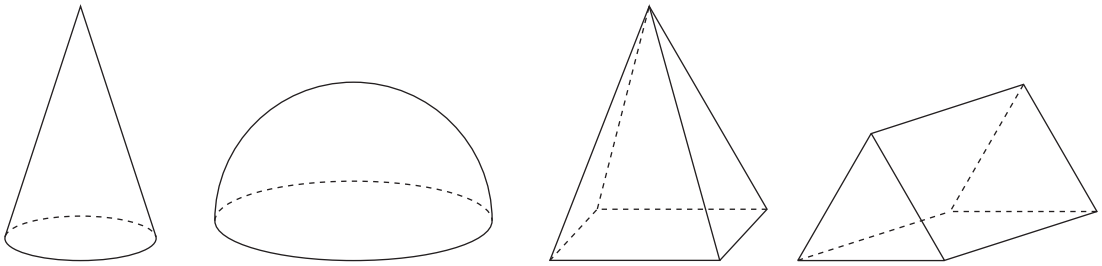
Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet for Questions 1–22.

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1 Which of the solids shown is a prism?



(A) (B) (C) (D)

2 Which of the following could be the probability of an event occurring?

(A) 1

(B) $\frac{6}{5}$

(C) 1.27

(D) 145%

3 Perth in Western Australia is 8 hours ahead of Greenwich in England. Cape Town in South Africa is 2 hours ahead of Greenwich.

What is the time in Cape Town when it is 1 pm in Perth?

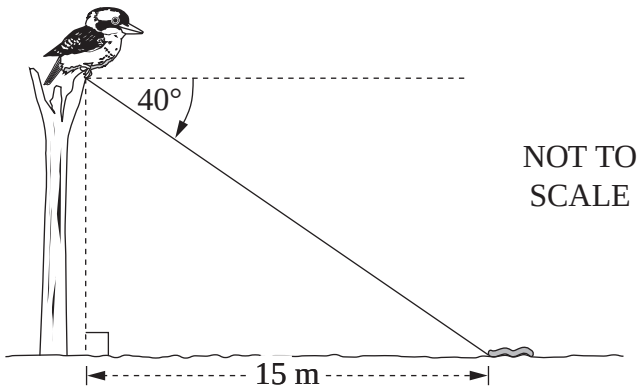
(A) 3 am

(B) 7 am

(C) 7 pm

(D) 11 pm

4 The angle of depression from a kookaburra's feet to a worm on the ground is 40°. The worm is 15 metres from a point on the ground directly below the kookaburra's feet.



How high above the ground are the kookaburra's feet, correct to the nearest metre?

(A) 10 m

(B) 11 m

(C) 13 m

(D) 18 m

5 The letters A, B and C are used to make a three-letter company name. Each letter is used only once.

How many different company names can be made?

(A) 3

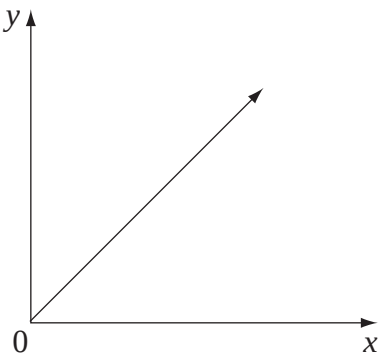
(B) 6

(C) 9

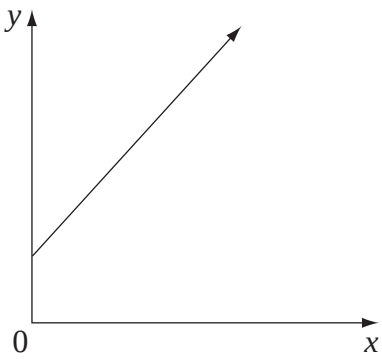
(D) 27

6 Which of the following graphs best represents the equation $y = a^x$, where a is a positive number greater than 1?

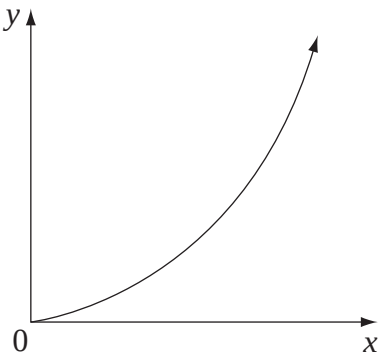
(A)



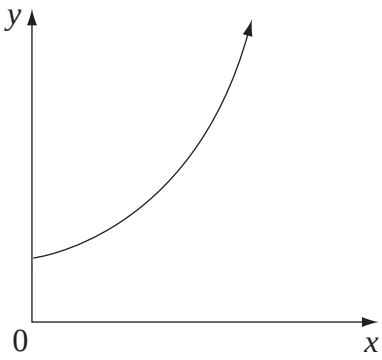
(B)



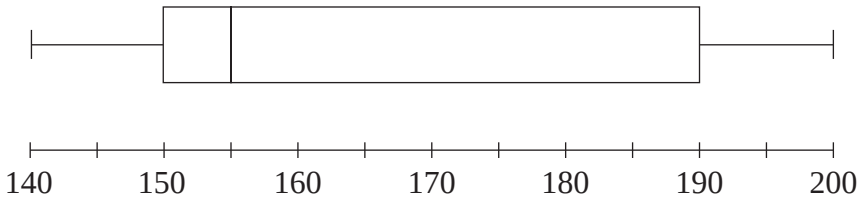
(C)



(D)



7 A set of data is displayed in this box-and-whisker plot.



Which of the following best describes this set of data?

(A) Symmetrical

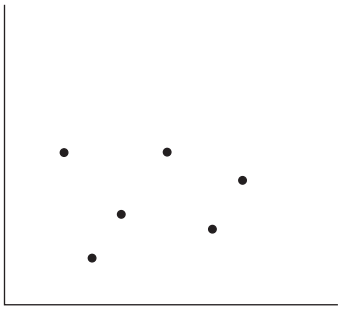
(B) Positively skewed

(C) Negatively skewed

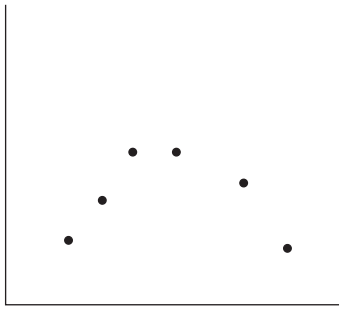
(D) Normally distributed

8 In which graph would the data have a correlation coefficient closest to –0.9?

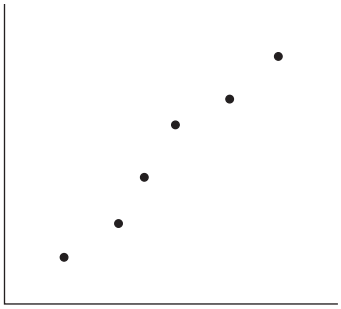
(A)



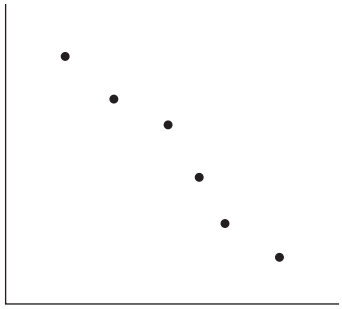
(B)



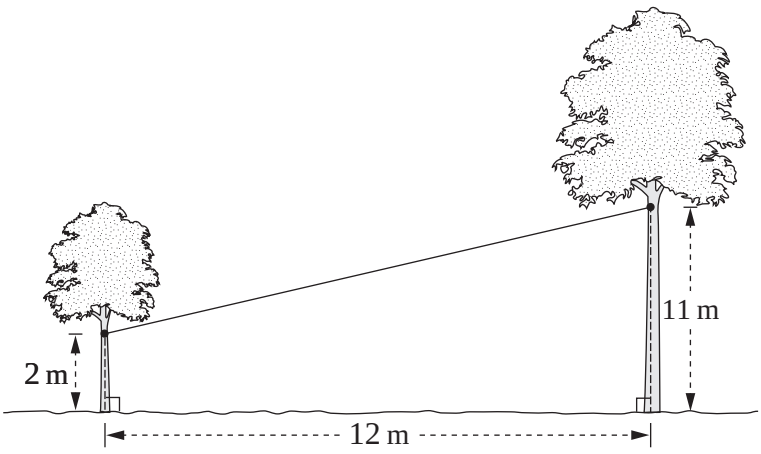
(C)



(D)



9 Two trees on level ground, 12 metres apart, are joined by a cable. It is attached 2 metres above the ground to one tree and 11 metres above the ground to the other.



NOT TO SCALE

What is the length of the cable between the two trees, correct to the nearest metre?

(A) 9 m

(B) 12 m

(C) 15 m

(D) 16 m

10 A television was purchased for $2100 on 12 April 2011 using a credit card. Simple interest was charged at a rate of 19.74% per annum for purchases on this credit card. There were no other purchases on this credit card account.

There was no interest-free period. The period for which interest was charged included the date of purchase and the date of payment.

What amount was paid when the account was paid in full on 20 May 2011?

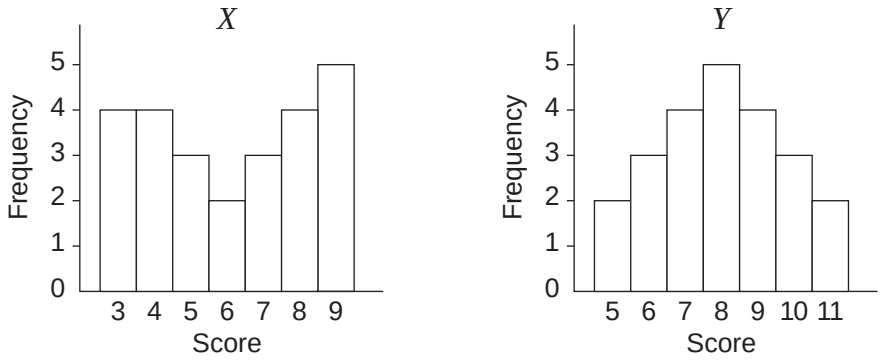
(A) $2143.16

(B) $2143.59

(C) $2144.29

(D) $2144.74

11 The sets of data, *X* and *Y*, are displayed in the histograms.



Which of these statements is true?

(A) *X* has a larger mode and *Y* has a larger range.

(B) *X* has a larger mode and the ranges are the same.

(C) The modes are the same and *Y* has a larger range.

(D) The modes are the same and the ranges are the same.

12 Which of the following expresses $\frac{6x^2y}{3} \div \frac{2y}{5}$ in its simplest form?

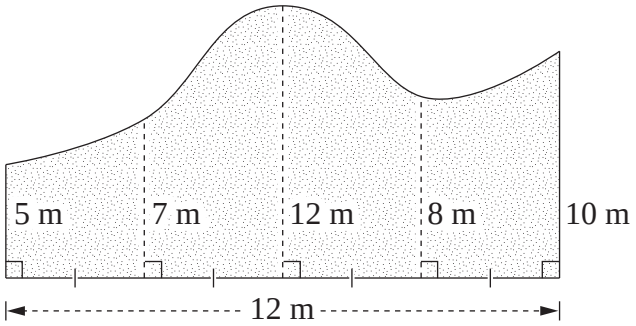
(A) $5x^2$

(B) $30x^2y$

(C) $\frac{1}{5x^2}$

(D) $\frac{5}{4x^2y^2}$

13 The diagram represents a field.



NOT TO SCALE

What is the area of the field, using two applications of Simpson's rule?

(A) 99 m^2

(B) 126 m^2

(C) 198 m^2

(D) 396 m^2

14 A data set of nine scores has a median of 7.

The scores 6, 6, 12 and 17 are added to this data set.

What is the median of the data set now?

(A) 6

(B) 7

(C) 8

(D) 9

15 An unbiased coin is tossed 10 times.

A tail is obtained on each of the first 9 tosses.

What is the probability that a tail is obtained on the 10th toss?

(A) $\frac{1}{2^{10}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{10}$

(D) $\frac{9}{10}$

16 A loan of $25 000 is used to purchase a car. The term of the loan is three years and the interest rate is 9% per annum, compounded fortnightly.

Which equation should be used to calculate the fortnightly repayments, M?

(A) $25\,000 = M\left\{\dfrac{(1+0.09)^3 - 1}{0.09(1+0.09)^3}\right\}$

(B) $25\,000 = M\left\{\dfrac{(1+0.09)^{78} - 1}{0.09(1+0.09)^{78}}\right\}$

(C) $25\,000 = M\left\{\dfrac{\left(1+\dfrac{0.09}{26}\right)^3 - 1}{\dfrac{0.09}{26}\left(1+\dfrac{0.09}{26}\right)^3}\right\}$

(D) $25\,000 = M\left\{\dfrac{\left(1+\dfrac{0.09}{26}\right)^{78} - 1}{\dfrac{0.09}{26}\left(1+\dfrac{0.09}{26}\right)^{78}}\right\}$

17 The heights of the players in a basketball team were recorded as 1.8 m, 1.83 m, 1.84 m, 1.86 m and 1.92 m. When a sixth player joined the team, the average height of the players increased by 1 centimetre.

What was the height of the sixth player?

(A) 1.85 m

(B) 1.86 m

(C) 1.91 m

(D) 1.93 m

18 Which of the following correctly expresses a as the subject of $s = ut + \frac{1}{2}at^2$?

(A) $a = \dfrac{2\left(s - ut\right)}{t^2}$

(B) $a = \dfrac{2s - ut}{t^2}$

(C) $a = \dfrac{\frac{1}{2}\left(s - ut\right)}{t^2}$

(D) $a = \dfrac{\frac{1}{2}s - ut}{t^2}$

19 Simon is a mechanic who receives a normal rate of pay of $22.35 per hour for a 40-hour week.

When he is needed for emergency call-outs he is paid a special allowance of $150 for that week. Additionally, every time he is called out to an emergency he is paid for a minimum of 4 hours at double time.

In the week beginning 2 February, 2011 Simon worked 40 hours normal time and was needed for emergency call-outs. His emergency call-out log book for the week is shown.

Employee: Simon Week: 2/2/11 to 8/2/11	
Date	*Duration of call-out*
3/2/11	5 hours
5/2/11	1.5 hours

What was Simon's total pay for that week?

(A) $1189.28

(B) $1296.30

(C) $1334.55

(D) $1446.30

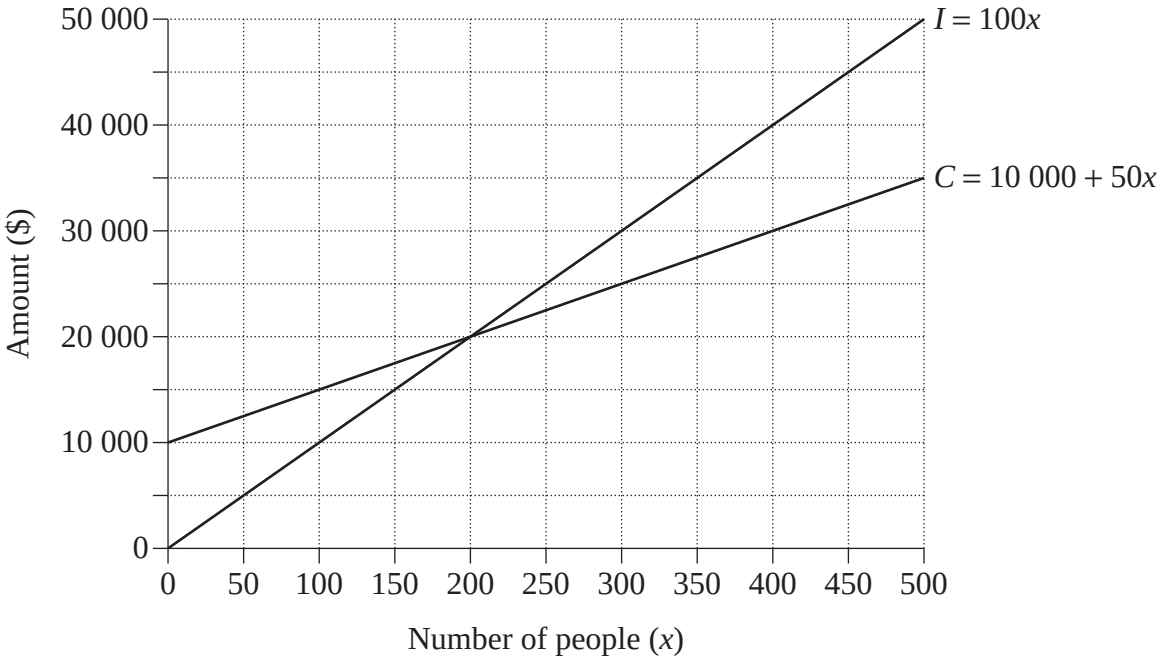
20 A function centre hosts events for up to 500 people. The cost C, in dollars, for the centre to host an event, where x people attend, is given by:

$$C = 10\,000 + 50x$$

The centre charges \$100 per person. Its income I, in dollars, is given by:

$$I = 100x$$

Income and costs for the function centre



How much greater is the income of the function centre when 500 people attend an event, than its income at the breakeven point?

(A) \$15 000

(B) \$20 000

(C) \$30 000

(D) \$40 000

21 A train departs from Town *A* at 3.00 pm to travel to Town *B*. Its average speed for the journey is 90 km/h, and it arrives at 5.00 pm. A second train departs from Town *A* at 3.10 pm and arrives at Town *B* at 4.30 pm.

What is the average speed of the second train?

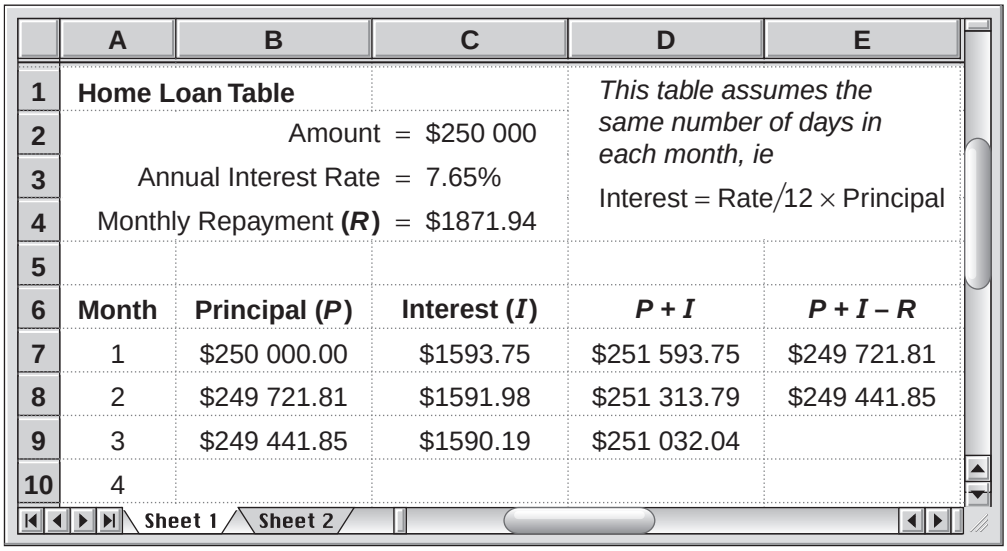
(A) 135 km/h

(B) 150 km/h

(C) 216 km/h

(D) 240 km/h

22 Ying borrowed $250 000 to buy a house. The interest rate and monthly repayment for her loan are shown in the spreadsheet.



	A	B	C	D	E
1	**Home Loan Table**			*This table assumes the same number of days in each month, ie*	
2		Amount	= $250 000		
3		Annual Interest Rate	= 7.65%	Interest = Rate/12 × Principal	
4		Monthly Repayment (***R***)	= $1871.94		
5					
6	**Month**	**Principal (*P*)**	**Interest (*I*)**	***P* + *I***	***P* + *I* – *R***
7	1	$250 000.00	$1593.75	$251 593.75	$249 721.81
8	2	$249 721.81	$1591.98	$251 313.79	$249 441.85
9	3	$249 441.85	$1590.19	$251 032.04	
10	4				

Sheet 1 Sheet 2

What is the total interest charged for the first four months of this loan?

(A) $6364.32

(B) $6366.11

(C) $6369.67

(D) $6376.25

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 23 (13 marks) Use the Question 23 Writing Booklet.

(a) Sri has a gross salary of $56 350. She has tax deductions of $350 for union fees, $2000 in work-related expenses and $250 in donations to charities. **3**

The Medicare levy is 1.5% of her taxable income.

Calculate Sri's Medicare levy.

(b) Sticks were used to create the following pattern.



Figure 1 Figure 2 Figure 3

The number of sticks used is recorded in the table.

Figure (F)	1	2	3
Number of sticks (N)	5	8	11

(i) Draw Figure 4 of this pattern. **1**

(ii) How many sticks would be required for Figure 100? **1**

(iii) Is it possible to create a figure in this pattern using exactly 543 sticks? **2**

Show suitable calculations to support your answer.

Question 23 continues on page 13

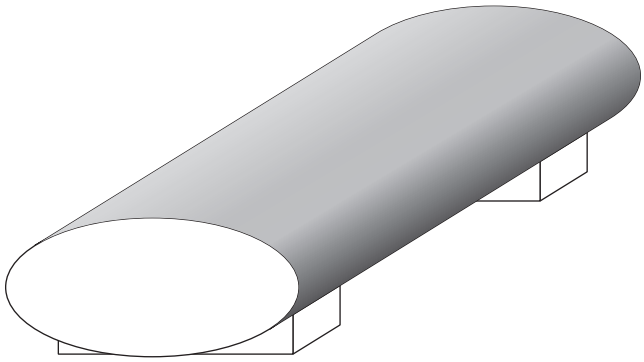
(c) An amount of $5000 is invested at 10% per annum, compounded six-monthly. **2**

Compounded values of $1

Period	Interest rate per period				
	1%	*5%*	*10%*	*15%*	*20%*
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.464	1.750	2.074
5	1.051	1.276	1.611	2.011	2.488
6	1.062	1.340	1.772	2.313	2.986

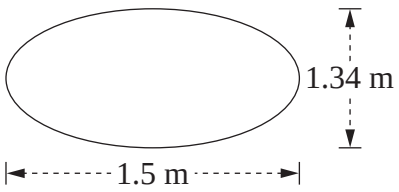
Use the table to find the value of this investment at the end of three years.

(d) Aviation fuel is stored in a tank. The cross-section of the tank is an ellipse.



(i) The tank holds 10 000 litres of fuel. What is the volume of the tank in cubic metres? ($1\ m^3 = 1000$ L) **1**

(ii) The cross-section of the tank has the dimensions shown. **3**



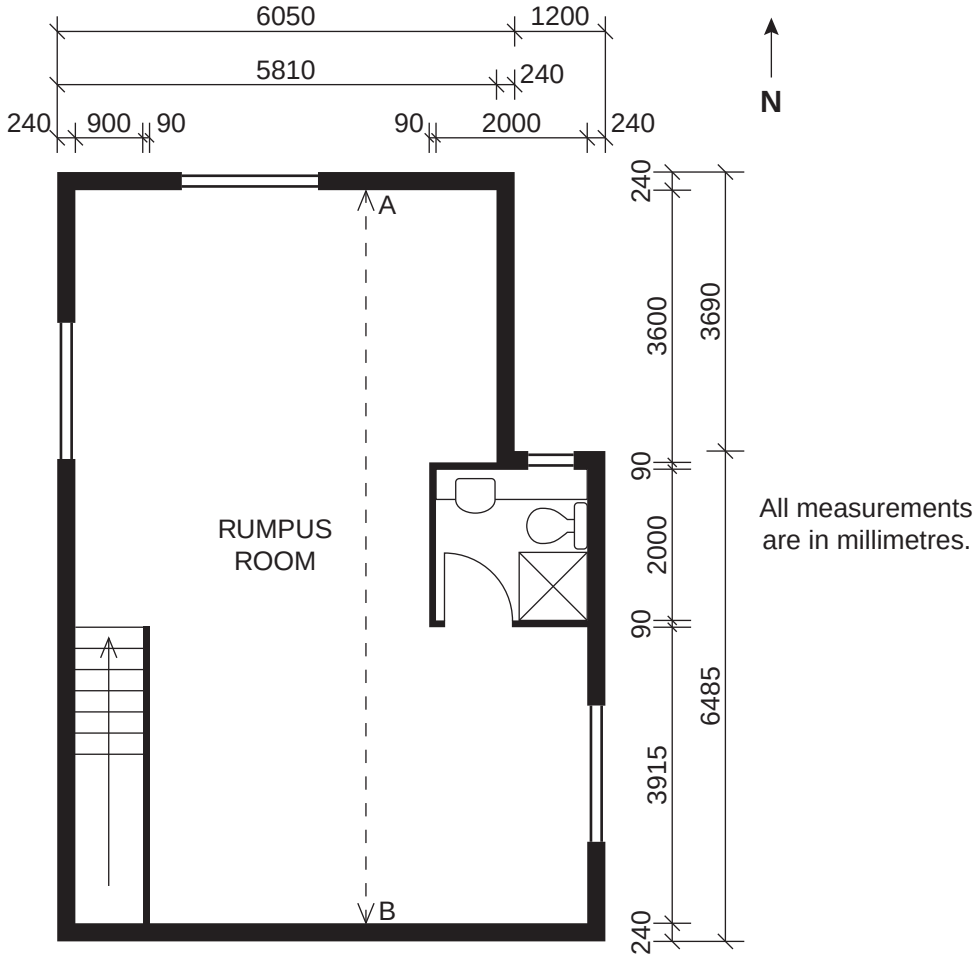
NOT TO SCALE

What is the length of the tank? Give your answer correct to the nearest centimetre.

End of Question 23

Question 24 (13 marks) Use the Question 24 Writing Booklet.

(a) Part of the floor plan of a house is shown. The plan is drawn to scale.



(i) What is the width of the stairwell, in millimetres? **1**

(ii) What are the internal dimensions of the bathroom, in millimetres? **1**

(iii) What is the length AB, the internal length of the rumpus room, in millimetres? **1**

(iv) There are three identical windows to be purchased for this rumpus room. Use the floor plan to determine the width of the windows to be purchased. Give your answer in millimetres. **1**

Question 24 continues on page 15

(b) A die was rolled 72 times. The results for this experiment are shown in the table.

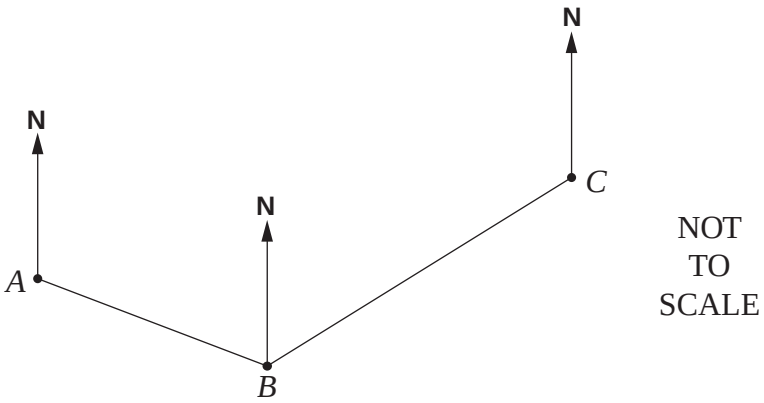
Number obtained	*Frequency*
1	16
2	11
3	***A***
4	8
5	12
6	15

(i) Find the value of ***A***. **1**

(ii) What was the relative frequency of obtaining a 4? **1**

(iii) If the die was unbiased, which number was obtained the expected number of times? **1**

(c) A ship sails 6 km from *A* to *B* on a bearing of 121°. It then sails 9 km to *C*. The size of angle *ABC* is 114°.



NOT TO SCALE

Copy the diagram into your writing booklet and show all the information on it.

(i) What is the bearing of *C* from *B*? **1**

(ii) Find the distance *AC*. Give your answer correct to the nearest kilometre. **2**

(iii) What is the bearing of *A* from *C*? Give your answer correct to the nearest degree. **3**

End of Question 24

Question 25 (13 marks) Use the Question 25 Writing Booklet.

(a) A study on the mobile phone usage of NSW high school students is to be conducted. Data is to be gathered using a questionnaire.

The questionnaire begins with the three questions shown.

> Q1: Do you own a mobile phone?
> Yes ☐ No ☐
>
> Q2: Which phone company do you use?
> .....
>
> Q3: Do you use pre-paid or a plan?
> Pre-paid ☐ Plan ☐

(i) Classify the type of data that will be collected in Q2 of the questionnaire. **1**

(ii) Write a suitable question for this questionnaire that would provide quantitative data. **1**

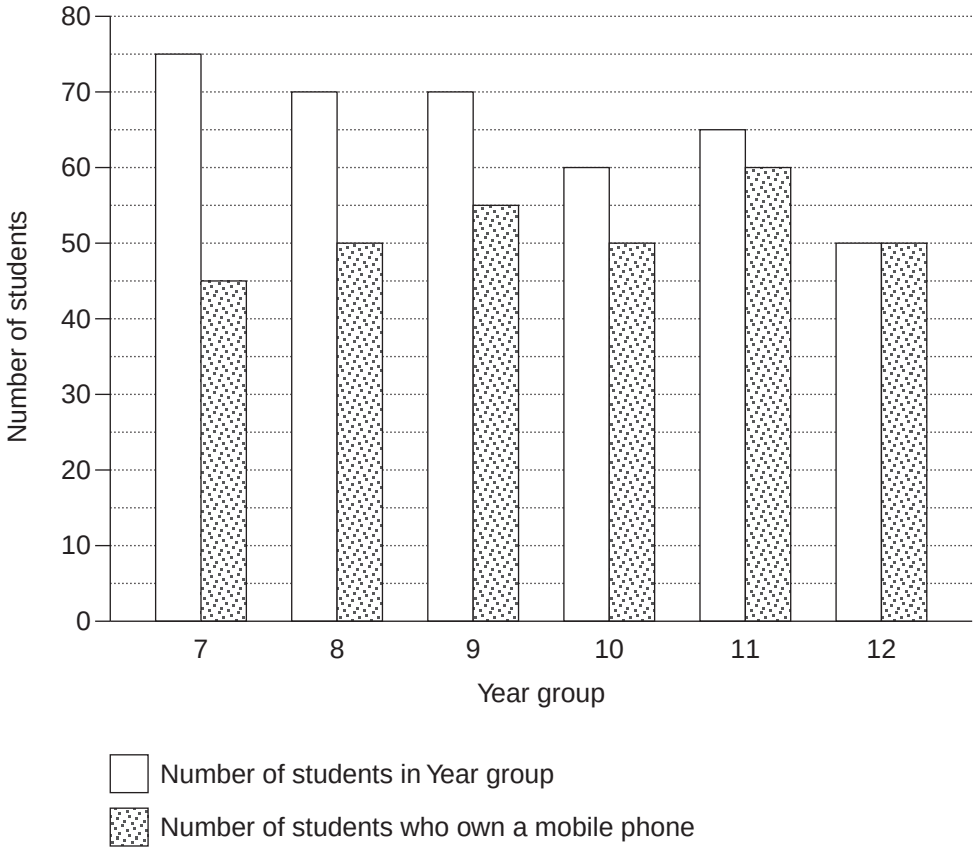
(iii) An initial study is to be conducted using a stratified sample. **1**

Describe a method that could be used to obtain a representative stratified sample.

(iv) Who should be surveyed if it is decided to use a census for the study? **1**

Question 25 continues on page 17

(b) The graph below displays data collected at a school on the number of students in each Year group, who own a mobile phone.



(i) Which Year group has the highest percentage of students with mobile phones? **1**

(ii) Two students are chosen at random, one from Year 9 and one from Year 10. Which student is more likely to own a mobile phone? Justify your answer with suitable calculations. **2**

(iii) Identify a trend in the data shown in the graph. **1**

Question 25 continues on page 18

(c) At another school, students who use mobile phones were surveyed. The set of data is shown in the table.

	Pre-paid	*Plan*	TOTAL
Female students	172	147	319
Male students	158	103	261
TOTAL	330	250	

(i) How many students were surveyed at this school? **1**

(ii) Of the female students surveyed, one is chosen at random. What is the probability that she uses pre-paid? **1**

(iii) Ten new male students are surveyed and all ten are on a plan. The set of data is updated to include this information. What percentage of the male students surveyed are now on a plan? Give your answer to the nearest per cent. **1**

(d) Data was collected from 30 students on the number of text messages they had sent in the previous 24 hours. The set of data collected is displayed.

Male		*Female*
9 9 8 7 6 5 5 4 2 1	0	8 9
1 1 0 0	1	1 1 2 5 6 8 8 8
0	2	0 1 7
	3	4
	4	
	5	
	6	
1	7	

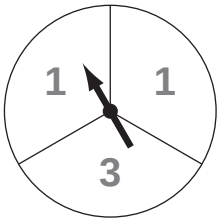
(i) What is the outlier for this set of data? **1**

(ii) What is the interquartile range of the data collected from the female students? **1**

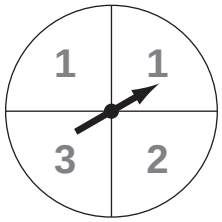
End of Question 25

Question 26 (13 marks) Use the Question 26 Writing Booklet.

(a) The two spinners shown are used in a game.



Spinner A



Spinner B

Each arrow is spun once. The score is the total of the two numbers shown by the arrows.

A table is drawn up to show all scores that can be obtained in this game.

		Spinner B			
		1	**1**	**2**	**3**
Spinner A	**1**	2	2	3	4
	1	2	2	3	4
	3	4	4	X	6

(i) What is the value of X in the table? **1**

(ii) What is the probability of obtaining a score less than 4? **1**

(iii) On Spinner B, a 2 is obtained. What is the probability of obtaining a score of 3? **1**

(iv) Elise plays a game using the spinners with the following financial outcomes. **3**

- Win \$12 for a score of 4
- Win nothing for a score of less than 4
- Lose \$3 for a score of more than 4

It costs \$5 to play this game. Will Elise expect a gain or a loss and how much will it be? Justify your answer with suitable calculations.

Question 26 continues on page 20

(b) Jack needs to find the number of years, t, it will take for a population of bats to first exceed 18 000.

He uses a 'guess-and-check' method to estimate t in the following equation

$$5 \times 3^t = 18\,000.$$

Here is his working:

> Try $t = 9$
>
> $5 \times 3^9 = 98\,415$
>
> Conclusion: $t = 9$ is too big.

(i) Jack's next guess is $t = 6$. Show Jack's correct working for this guess, including the calculation and conclusion. **1**

(ii) Continue using the 'guess-and-check' method to find the number of years, t, it will take for the population to first exceed 18 000, if t is a whole number. Include the calculations and conclusions. **2**

(c) Furniture priced at \$20 000 is purchased. A deposit of 15% is paid. **4**

The balance is borrowed using a flat-rate loan at 19% per annum interest, to be repaid in equal monthly instalments over five years.

What will be the amount of each monthly instalment? Justify your answer with suitable calculations.

End of Question 26

Question 27 (13 marks) Use the Question 27 Writing Booklet.

(a) A company sells handbags in Paris, New York and Florence. **2**

Use the data in the table to complete the area chart on page 1 of the Question 27 Writing Booklet.

	Winter 2010	*Spring 2010*	*Summer 2010*	*Autumn 2010*
Number of handbags sold in Paris	60 000	70 000	50 000	70 000

(b) Pontianak has a longitude of 109°E, and Jarvis Island has a longitude of 160°W. Both places lie on the Equator.

(i) Find the shortest distance between these two places, to the nearest kilometre. You may assume that the radius of the Earth is 6400 km. **2**

(ii) The position of Rabaul is 4° to the south and 48° to the west of Jarvis Island. What is the latitude and longitude of Rabaul? **2**

Question 27 continues on page 22

(c) Two brands of light bulbs are being compared. For each brand, the life of the light bulbs is normally distributed.

Life of light bulbs (in hours)

	Mean	*Standard deviation*
Brand A	450	25
Brand B	500	50

(i) One of the Brand B light bulbs has a life of 400 hours. **1**

What is the z-score of the life of this light bulb?

(ii) A light bulb is considered defective if it lasts less than 400 hours. The following claim is made: **2**

'Brand A light bulbs are more likely
to be defective than Brand B light bulbs.'

Is this claim correct? Justify your answer, with reference to z-scores or standard deviations or the normal distribution.

(d) Josephine invests \$50 000 for 15 years, at an interest rate of 6% per annum, compounded annually. **4**

Emma invests \$500 at the end of each month for 15 years, at an interest rate of 6% per annum, compounded monthly.

Financial gain is defined as the difference between the final value of an investment and the total contributions.

Who will have the better financial gain after 15 years? Justify your answer with suitable calculations. You must show the correct values substituted into appropriate formulas.

End of Question 27

Question 28 (13 marks) Use the Question 28 Writing Booklet.

(a) The air pressure, P, in a bubble varies inversely with the volume, V, of the bubble.

(i) Write an equation relating P, V and a, where a is a constant. **1**

(ii) It is known that $P = 3$ when $V = 2$. **2**

By finding the value of the constant, a, find the value of P when $V = 4$.

(iii) Sketch a graph to show how P varies for different values of V. **2**

Use the horizontal axis to represent volume and the vertical axis to represent air pressure.

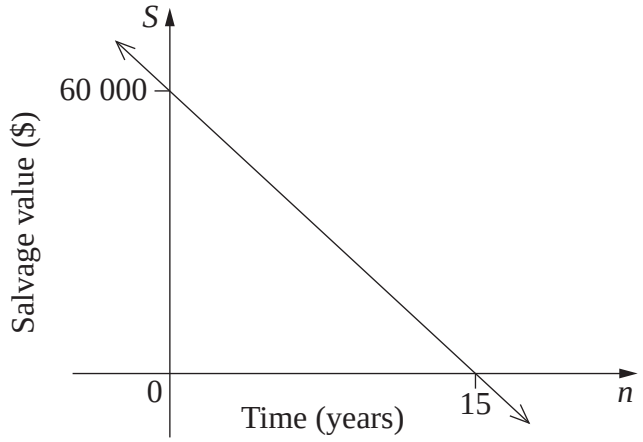
Question 28 continues on page 24

(b) Norman and Pat each bought the same type of tractor for $60 000 at the same time. The value of their tractors depreciated over time.

The salvage value S, in dollars, of each tractor, is its depreciated value after n years.

Norman drew a graph to represent the salvage value of his tractor.

Norman's graph of the salvage value of his tractor



(i) Find the gradient of the line shown in the graph. **1**

(ii) What does the value of the gradient represent in this situation? **1**

(iii) Write down the equation of the line shown in the graph. **1**

(iv) Find all the values of n that are not suitable for Norman to use when calculating the salvage value of his tractor. Explain why these values are not suitable. **2**

Pat used the declining balance formula for calculating the salvage value of her tractor. The depreciation rate that she used was 20% per annum.

(v) What did Pat calculate the salvage value of her tractor to be after 14 years? **2**

(vi) Using Pat's method for depreciation, describe what happens to the salvage value of her tractor for all values of n greater than 15. **1**

End of paper

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FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle
r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis
b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements
d_f = first measurement
d_m = middle measurement
d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius
h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius
h = perpendicular height
A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

BOARD OF STUDIES
NEW SOUTH WALES

Centre Number

Student Number

2011

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

Writing Booklet

Question Number
27

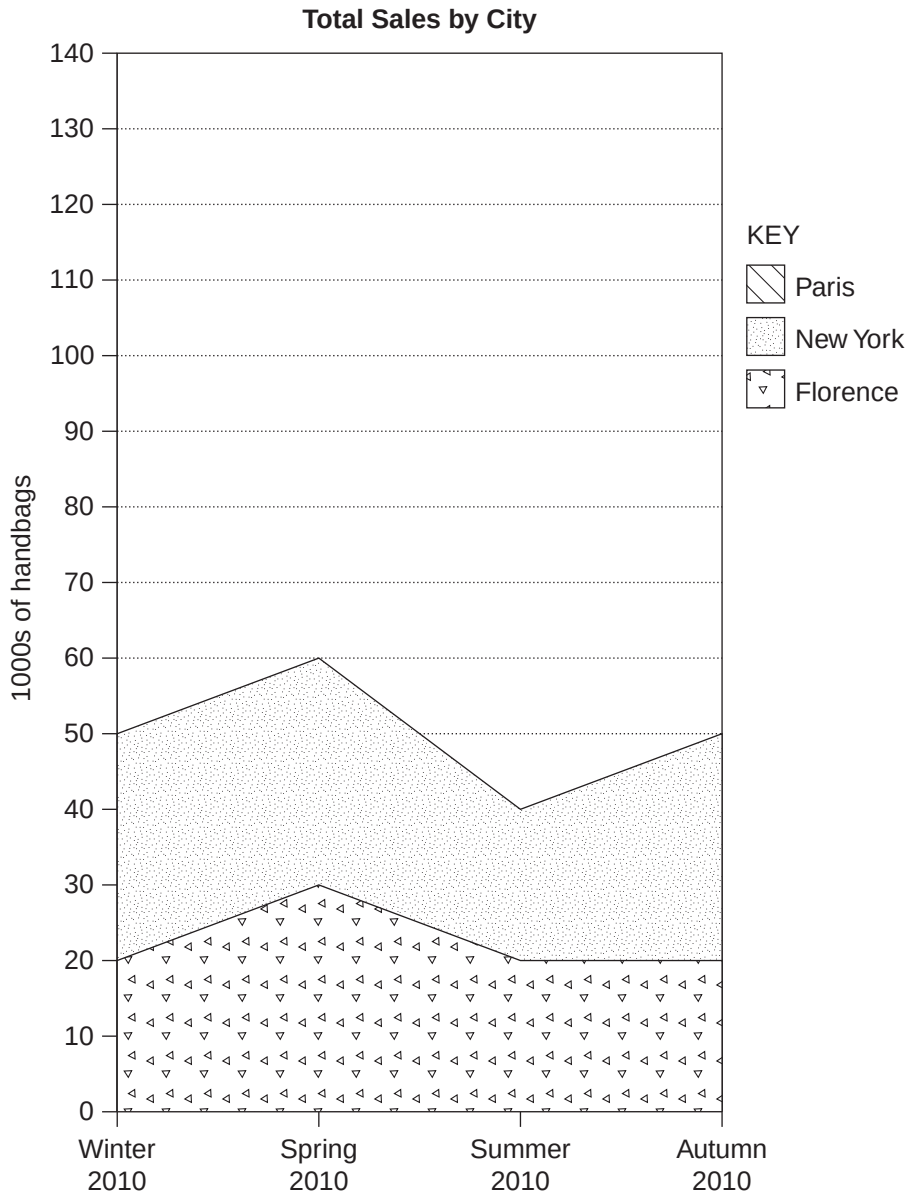
Instructions

- Use this Writing Booklet to answer Question 27.
- Write your Centre Number and Student Number at the top of this page.
- If you have not attempted the question, you must still hand in the Writing Booklet, with the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each question part inside the margin at the beginning of each answer.
- Write using black or blue pen. Black pen is preferred.
- You may ask for an extra Writing Booklet if you need more space.
- **You may NOT take any Writing Booklets, used or unused, from the examination room.**

Office Use Only – Do NOT write anything, or make any marks below this line.

Start here.

Question 27 (a)



Additional writing space on back page.

You may ask for an extra Writing Booklet if you need more space.

Office Use Only – Do NOT write anything, or make any marks below this line.