BOARD OF STUDIES
NEW SOUTH WALES

2009

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–24

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet for Questions 1–22.

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1 A newspaper states: 'It will most probably rain tomorrow.'

Which of the following best represents the probability of an event that will most probably occur?

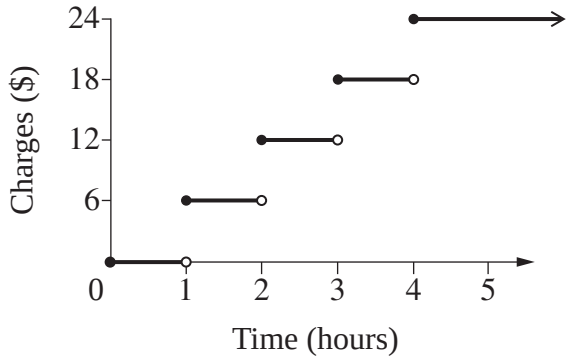
(A) $33\frac{1}{3}\%$

(B) 50%

(C) 80%

(D) 100%

2 The step graph shows the charges for a carpark.



Maria enters the carpark at 10:10 am and exits at 1:30 pm.

How much will she pay in charges?

(A) $6

(B) $12

(C) $18

(D) $24

3 The eye colours of a sample of children were recorded.

When analysing this data, which of the following could be found?

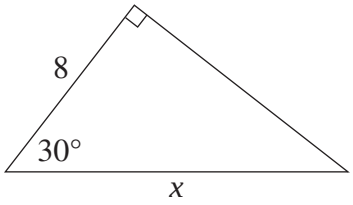
(A) Mean

(B) Median

(C) Mode

(D) Range

4 Which is the correct expression for the value of x in this triangle?



NOT TO SCALE

(A) $\dfrac{8}{\cos 30°}$

(B) $\dfrac{8}{\sin 30°}$

(C) $8 \times \cos 30°$

(D) $8 \times \sin 30°$

5 Jamie wants to know how many songs were downloaded legally from the internet in the last 12 months by people aged 18–25 years. He has decided to conduct a statistical inquiry.

After he collects the data, which of the following shows the best order for the steps he should take with the data to complete his inquiry?

(A) Display, organise, conclude, analyse

(B) Organise, display, conclude, analyse

(C) Display, organise, analyse, conclude

(D) Organise, display, analyse, conclude

6 A house was purchased in 1984 for \$35 000. Assume that the value of the house has increased by 3% per annum since then.

Which expression gives the value of the house in 2009?

(A) $35\,000(1 + 0.03)^{25}$

(B) $35\,000(1 + 3)^{25}$

(C) $35\,000 \times 25 \times 0.03$

(D) $35\,000 \times 25 \times 3$

7 Two people are to be selected from a group of four people to form a committee.

How many different committees can be formed?

(A) 6

(B) 8

(C) 12

(D) 16

8 Some men and women were surveyed at a football game. They were asked which team they supported. The results are shown in the two-way table.

	Team A	*Team B*	*Totals*
Men	125	100	225
Women	75	90	165
Totals	200	190	390

What percentage of the women surveyed supported *Team B*, correct to the nearest percent?

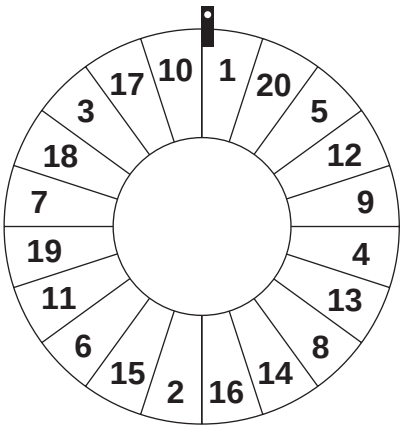
(A) 23%

(B) 45%

(C) 47%

(D) 55%

9 A wheel has the numbers 1 to 20 on it, as shown in the diagram. Each time the wheel is spun, it stops with the marker on one of the numbers.



The wheel is spun 120 times.

How many times would you expect a number less than 6 to be obtained?

(A) 20

(B) 24

(C) 30

(D) 36

10 Billy worked for 35 hours at the normal hourly rate of pay and for five hours at double time. He earned $561.60 in total for this work.

What was the normal hourly rate of pay?

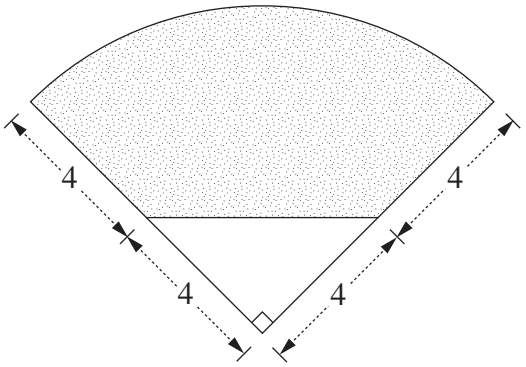
(A) $7.02

(B) $12.48

(C) $14.04

(D) $16.05

11



NOT TO SCALE

All measurements are in centimetres.

What is the area of the shaded part of this quadrant, to the nearest square centimetre?

(A) 34 cm^2

(B) 42 cm^2

(C) 50 cm^2

(D) 193 cm^2

12 How many square centimetres are in 0.0075 square metres?

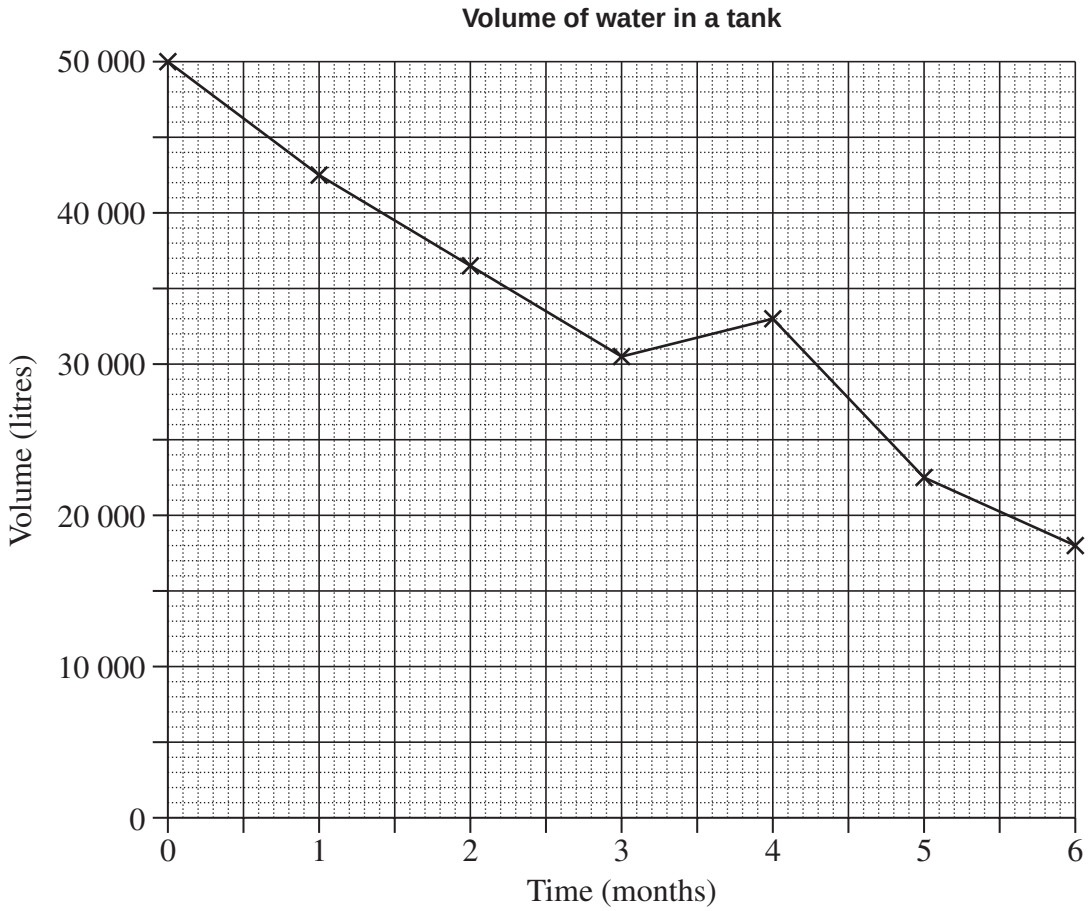
(A) 0.75

(B) 7.5

(C) 75

(D) 7500

13 The volume of water in a tank changes over six months, as shown in the graph.



Consider the overall decrease in the volume of water.

What is the average percentage decrease in the volume of water per month over this time, to the nearest percent?

(A) 6%

(B) 11%

(C) 32%

(D) 64%

14 If $A = 6x + 10$, and x is increased by 2, what will be the corresponding increase in A?

(A) $2x$

(B) $6x$

(C) 2

(D) 12

15 Which of the following correctly expresses n as the subject of $v = \frac{3mn^2}{r}$?

(A) $n = \pm \frac{\sqrt{rv}}{3m}$

(B) $n = \pm r\sqrt{\frac{v}{3m}}$

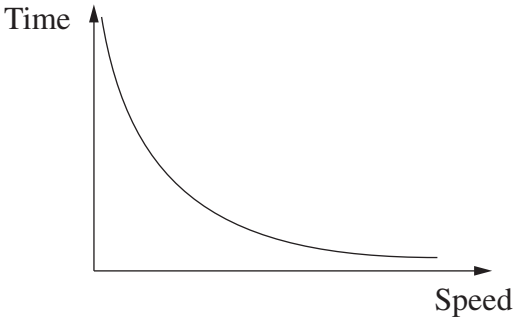
(C) $n = \pm \frac{r\sqrt{v}}{3m}$

(D) $n = \pm \sqrt{\frac{rv}{3m}}$

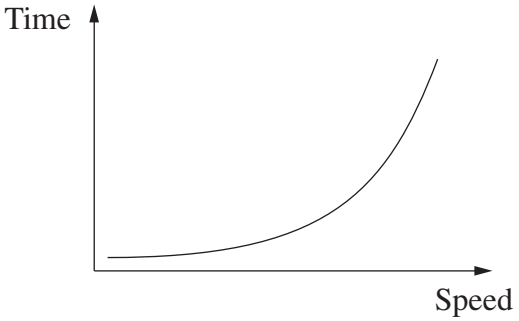
16 The time for a car to travel a certain distance varies inversely with its speed.

Which of the following graphs shows this relationship?

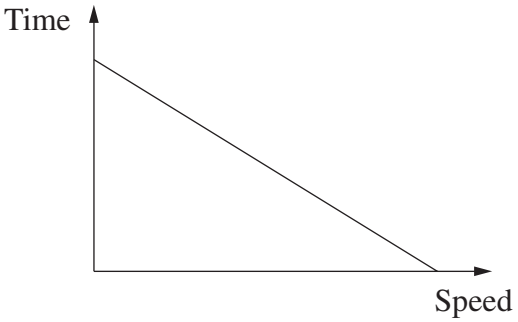
(A)



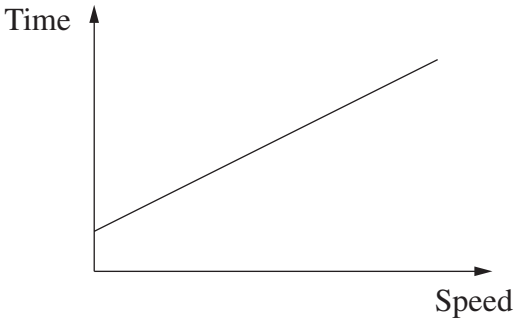
(B)



(C)



(D)



17 Sally decides to put \$100 per week into her superannuation fund. The interest rate quoted is 8% per annum, compounded weekly.

Which expession will calculate the future value of her superannuation at the end of 35 years?

(A) $100\left\{\dfrac{\left(1+\dfrac{0.08}{52}\right)^{35}-1}{\dfrac{0.08}{52}}\right\}$

(B) $100\left\{\dfrac{(1+0.08)^{35}-1}{0.08}\right\}$

(C) $100\left\{\dfrac{\left(1+\dfrac{0.08}{52}\right)^{1820}-1}{\dfrac{0.08}{52}}\right\}$

(D) $100\left\{\dfrac{(1+0.08)^{1820}-1}{0.08}\right\}$

18 Huong used the 'capture–recapture' technique to estimate the number of trout living in a dam.

- She caught, tagged and released 20 trout.
- Later she caught 36 trout at random from the same dam.
- She found that 8 of these 36 trout had been tagged.

What estimate should Huong give for the total number of trout living in this dam, based on her use of the 'capture–recapture' technique?

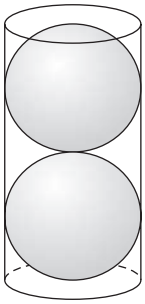
(A) 56

(B) 90

(C) 160

(D) 162

19 Two identical spheres fit exactly inside a cylindrical container, as shown.



The diameter of each sphere is 12 cm.

What is the volume of the cylindrical container, to the nearest cubic centimetre?

(A) 1357 cm^3

(B) 2714 cm^3

(C) 5429 cm^3

(D) 10 857 cm^3

20 Lou bought a plasma TV which was priced at $3499. He paid $1000 deposit and got a loan for the balance that was paid off by 24 monthly instalments of $135.36.

What simple interest rate per annum, to the nearest percent, was charged on his loan?

(A) 11%

(B) 15%

(C) 30%

(D) 46%

21 The mean of a set of ten scores is 14. Another two scores are included and the new mean is 16.

What is the mean of the two additional scores?

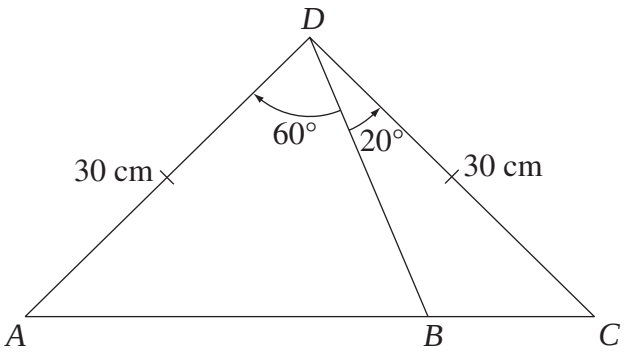
(A) 4

(B) 16

(C) 18

(D) 26

22 In the diagram, *AD* and *DC* are equal to 30 cm.



NOT TO SCALE

What is the length of *AB* to the nearest centimetre?

(A) 28 cm

(B) 31 cm

(C) 34 cm

(D) 39 cm

Section II

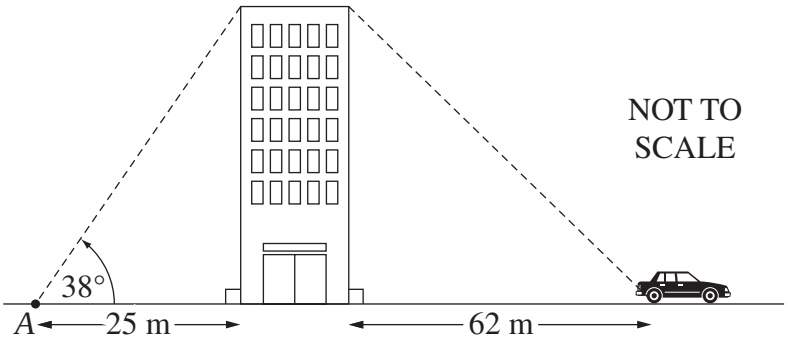
78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 23 (13 marks) Use the Question 23 Writing Booklet.

(a) The point A is 25 m from the base of a building. The angle of elevation from A to the top of the building is 38°.



(i) Show that the height of the building is approximately 19.5 m. **1**

(ii) A car is parked 62 m from the base of the building. **2**

What is the angle of depression from the top of the building to the car? Give your answer to the nearest degree.

(b) A personal identification number (PIN) is made up of four digits. An example of a PIN is $\boxed{0229}$.

(i) When all ten digits are available for use, how many different PINs are possible? **1**

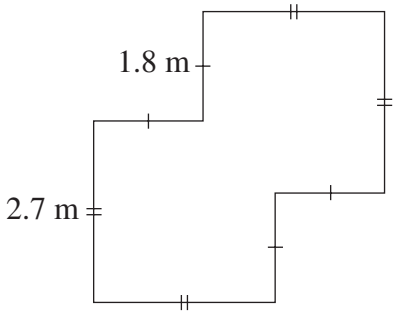
(ii) Rhys has forgotten his four-digit PIN, but knows that the first digit is either 5 or 6. **1**

What is the probability that Rhys will correctly guess his PIN in one attempt?

Question 23 continues on page 13

Question 23 (continued)

(c) The diagram shows the shape and dimensions of a terrace which is to be tiled.



NOT TO SCALE

All angles are right angles.

(i) Find the area of the terrace. **2**

(ii) Tiles are sold in boxes. Each box holds one square metre of tiles and costs \$55. When buying the tiles, 10% more tiles are needed, due to cutting and wastage. **2**

Find the total cost of the boxes of tiles required for the terrace.

(d) The tables below show information about fees for MyBank accounts.

Bank fees for accounts

Types of fees	*Free Access Account*	*Cheap Access Account*
Monthly account fee	\$7	\$4
Withdrawal fees	No	Yes
Each cash withdrawal from other ATM	\$2	\$2

Withdrawal fees

Types of withdrawals	*Fee per withdrawal*
Internet banking	\$0.30
Cash withdrawal from MyBank ATM	\$0.50
EFTPOS purchases	\$0.50

(i) Li has a *Cheap Access Account*. During September, he made **3**

- five withdrawals using internet banking
- two cash withdrawals from a MyBank ATM
- four EFTPOS purchases
- two cash withdrawals at other ATMs.

What was the total amount that Li paid in bank fees for the month of September?

(ii) In October, what is the maximum that Li could pay in withdrawal fees to ensure that a *Cheap Access Account* costs him no more than a *Free Access Account*? **1**

End of Question 23

Question 24 (13 marks) Use the Question 24 Writing Booklet.

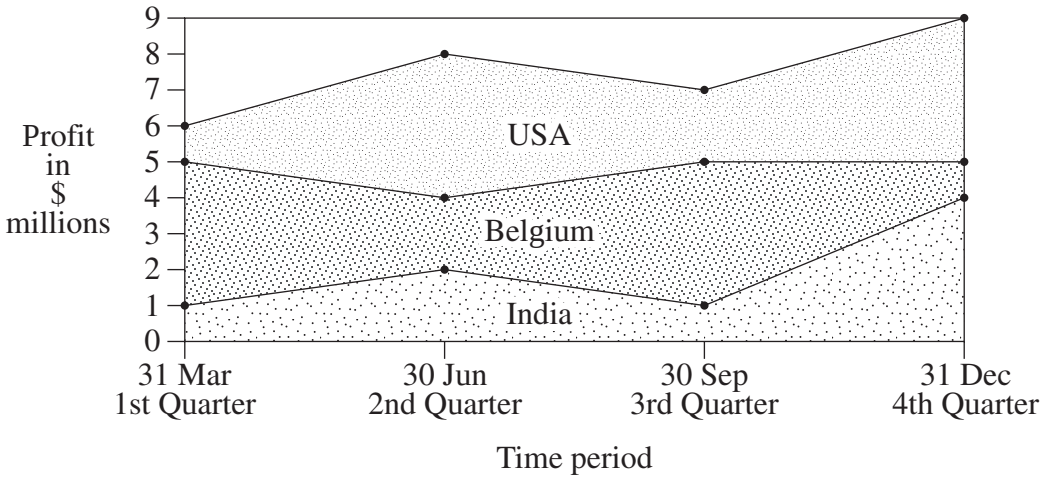
(a) The diagram below shows a stem-and-leaf plot for 22 scores.

2	3	5	9		
3	1	4	7	9	
4	2	4	4	5	7
5	1	2	4		
6	2	3	7		
7	5	8	8	8	

(i) What is the mode for this data? **1**

(ii) What is the median for this data? **1**

(b) Tayvan is an international company that reports its profits in the USA, Belgium and India at the end of each quarter. The profits for 2008 are shown in the area chart.



(i) What was the total profit for Tayvan on June 30? **1**

(ii) What was Tayvan's profit in Belgium on March 31? **1**

(c) The Australian Bureau of Statistics provides the NSW government with data on the age of residents living in different areas across the state. After analysing this data, the government makes decisions relating to the provision of services or facilities. **2**

Give an example of a possible decision the government might make and describe how the data might justify this decision.

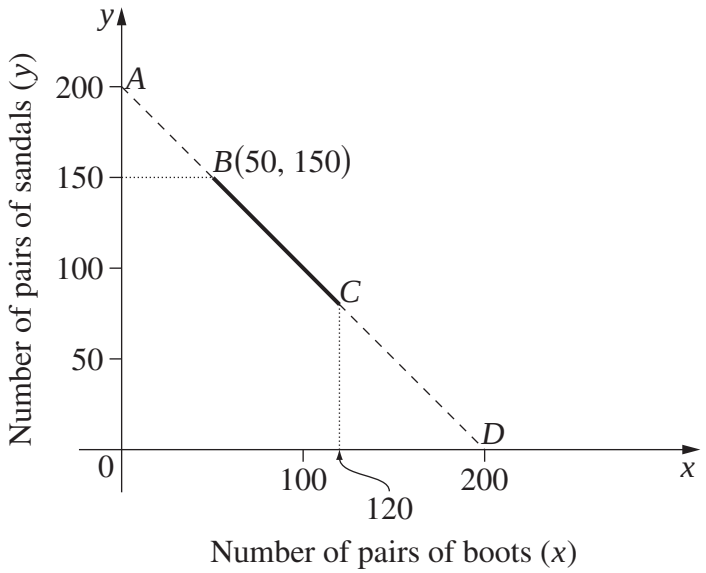
Question 24 continues on page 15

Question 24 (continued)

(d) A factory makes boots and sandals. In any week

- the total number of pairs of boots and sandals that are made is 200
- the maximum number of pairs of boots made is 120
- the maximum number of pairs of sandals made is 150.

The factory manager has drawn a graph to show the numbers of pairs of boots (x) and sandals (y) that can be made.



(i) Find the equation of the line AD. **1**

(ii) Explain why this line is only relevant between B and C for this factory. **1**

(iii) The profit per week, $\$P$, can be found by using the equation **2**

$$P = 24x + 15y.$$

Compare the profits at B and C.

(e) Jay bought a computer for \$3600. His friend Julie said that all computers are worth nothing (i.e. the value is \$0) after 3 years.

(i) Find the amount that the computer would depreciate each year to be worth nothing after 3 years, if the straight line method of depreciation is used. **1**

(ii) Explain why the computer would never be worth nothing if the declining balance method of depreciation is used, with 30% per annum rate of depreciation. Use suitable calculations to support your answer. **2**

End of Question 24

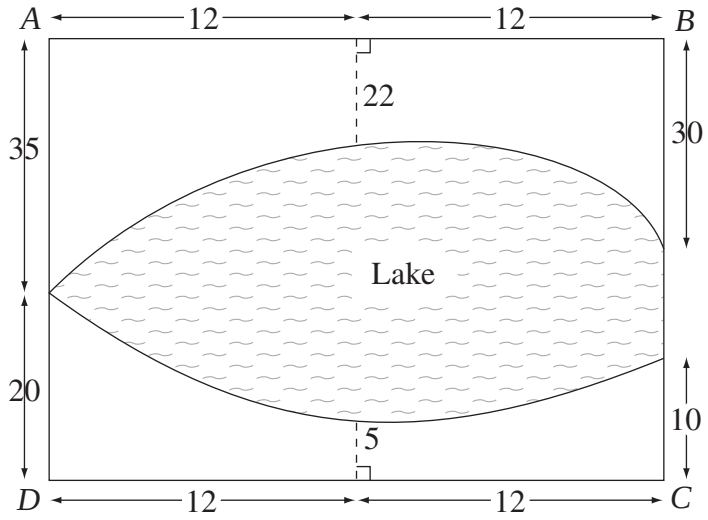
Question 25 (13 marks) Use the Question 25 Writing Booklet.

(a) Simplify $5 - 2(x + 7)$. **2**

(b) The mass of a sample of microbes is 50 mg. There are approximately 2.5×10^6 microbes in the sample. **2**

In scientific notation, what is the approximate mass in grams of one microbe?

(c) There is a lake inside the rectangular grass picnic area *ABCD*, as shown in the diagram.



(i) Use Simpson's Rule to find the approximate area of the lake's surface. **3**

(ii) The lake is 60 cm deep. Bozo the clown thinks he can empty the lake using a four-litre bucket. **2**

How many times would he have to fill his bucket from the lake in order to empty the lake? (Note that 1 m^3 = 1000 L).

Question 25 continues on page 17

(d) In Broken Hill, the maximum temperature for each day has been recorded. The mean of these maximum temperatures during spring is 25.8°C, and their standard deviation is 4.2°C.

(i) What temperature has a z-score of -1? **1**

(ii) What percentage of spring days in Broken Hill would have maximum temperatures between 21.6°C and 38.4°C? **3**

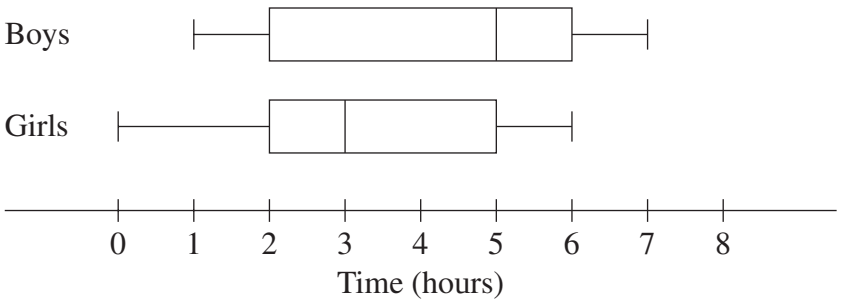
You may assume that these maximum temperatures are normally distributed and that

- 68% of maximum temperatures have z-scores between -1 and 1
- 95% of maximum temperatures have z-scores between -2 and 2
- 99.7% of maximum temperatures have z-scores between -3 and 3.

End of Question 25

Question 26 (13 marks) Use the Question 26 Writing Booklet.

(a) In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend. These results were displayed in box-and-whisker plots, as shown below.



(i) Find the interquartile range for boys. **1**

(ii) What percentage of girls usually spend 5 or less hours on the internet over a weekend? **1**

(iii) Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend. **1**

Under what circumstances would this statement be true?

(b) Osaka is at 34°N, 135°E, and Denver is at 40°N, 105°W.

(i) Show that there is a 16-hour time difference between the two cities. (Ignore time zones.) **2**

(ii) John lives in Denver and wants to ring a friend in Osaka. In Denver it is 9 pm Monday. **1**

What time and day is it in Osaka then?

(iii) John's friend in Osaka sent him a text message which happened to take 14 hours to reach him. It was sent at 10 am Thursday, Osaka time. **2**

What was the time and day in Denver when John received the text?

Question 26 continues on page 19

(c) Margaret borrowed \$300 000 to buy an apartment. The interest rate is 6% per annum, compounded monthly. The repayments were set by the bank at \$2200 per month for 20 years.

The loan balance sheet shows the interest charged and the balance owing for the first month.

Month	*Principal at the start of the month*	*Monthly interest*	*Monthly repayment*	*Balance at end of month*
1	\$300 000	\$1500	\$2200	\$299 300
2	\$299 300	***A***	\$2200	***B***

(i) What is the total amount that is to be paid for this loan over the 20 years? **1**

(ii) Find the values of ***A*** and ***B***. **2**

(iii) Margaret knows that she can check the bank's calculations by using the present value of an annuity formula to calculate the monthly repayment.

(1) Write down the present value of an annuity formula with the correct substitutions for this home loan. **1**

(2) Use this formula to find the calculated monthly repayment. **1**

End of Question 26

Question 27 (13 marks) Use the Question 27 Writing Booklet.

(a) The table shows the future value of a $1 annuity at different interest rates over different numbers of time periods.

Future values of a $1 annuity

Time Period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

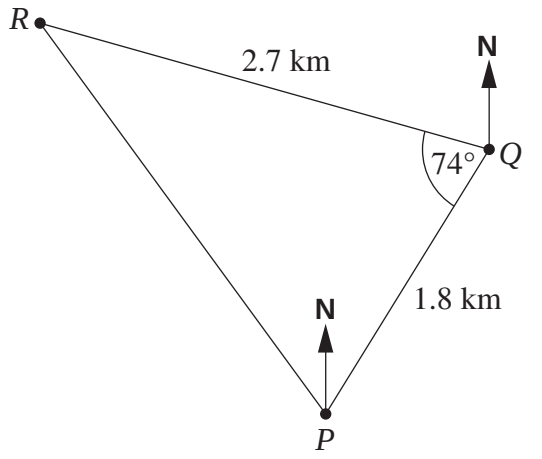
(i) What would be the future value of a $5000 per year annuity at 3% per annum for 6 years, with interest compounding yearly? **1**

(ii) What is the value of an annuity that would provide a future value of $407 100 after 7 years at 5% per annum compound interest? **1**

(iii) An annuity of $1000 per quarter is invested at 4% per annum, compounded quarterly for 2 years. What will be the amount of interest earned? **3**

Question 27 continues on page 21

(b) A yacht race follows the triangular course shown in the diagram. The course from P to Q is 1.8 km on a true bearing of 058°. At Q the course changes direction. The course from Q to R is 2.7 km and $\angle PQR = 74°$.



(i) What is the bearing of R from Q? **1**

(ii) What is the distance from R to P? **2**

(iii) The area inside this triangular course is set as a 'no-go' zone for other boats while the race is on. **1**

What is the area of this 'no-go' zone?

(c) In each of three raffles, 100 tickets are sold and one prize is awarded. **4**

Mary buys two tickets in one raffle. Jane buys one ticket in each of the other two raffles.

Determine who has the better chance of winning at least one prize. Justify your response using probability calculations.

End of Question 27

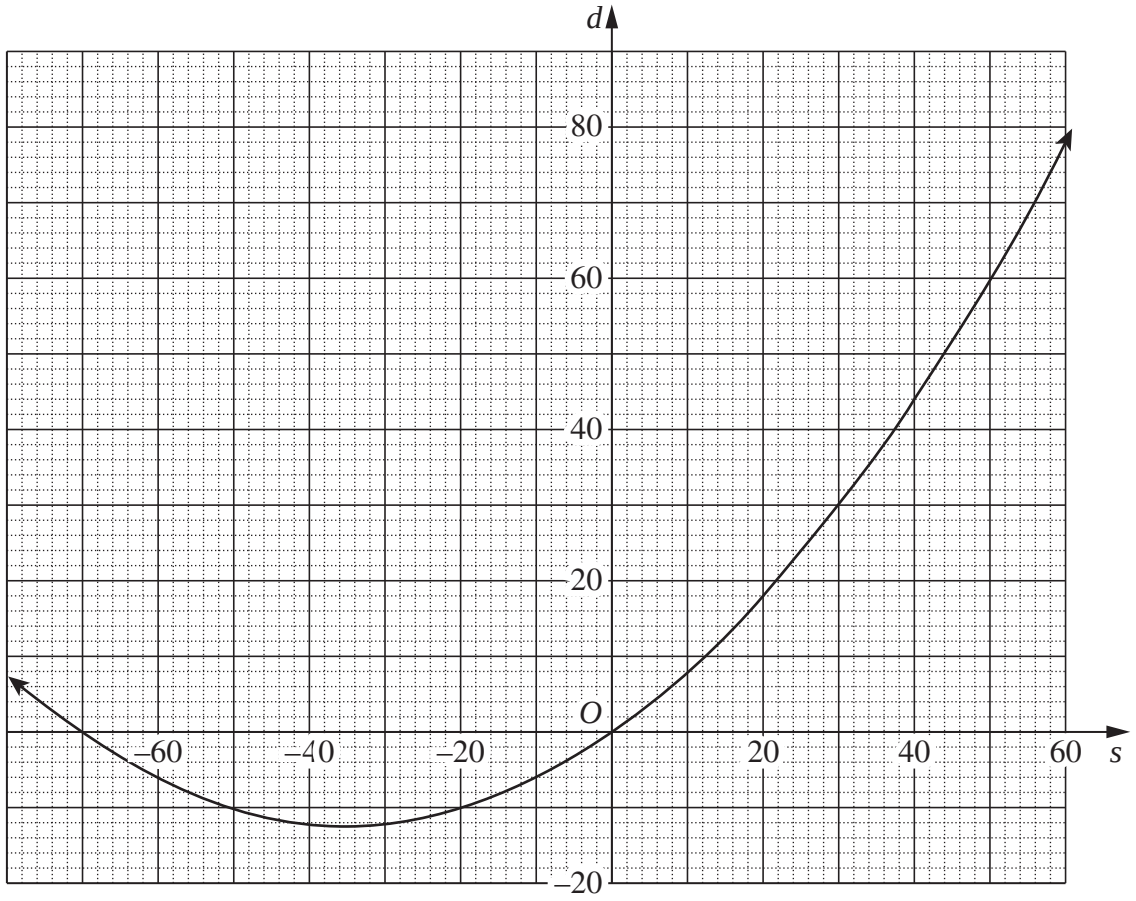
Question 28 (13 marks) Use the Question 28 Writing Booklet.

(a) Anjali is investigating stopping distances for a car travelling at different speeds. To model this she uses the equation

$$d = 0.01s^2 + 0.7s,$$

where d is the stopping distance in metres and s is the car's speed in km/h.

The graph of this equation is drawn below.



(i) Anjali knows that only part of this curve applies to her model for stopping distances. **1**

In your writing booklet, using a set of axes, sketch the part of this curve that applies for stopping distances.

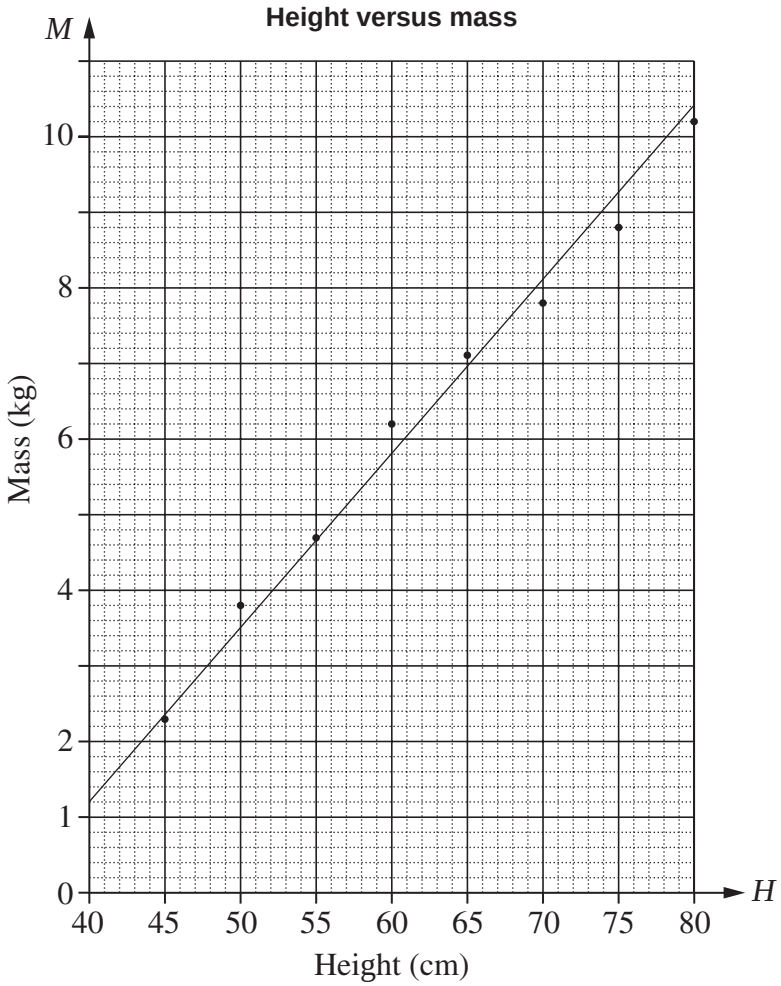
(ii) What is the difference between the stopping distances in a school zone when travelling at a speed of 40 km/h and when travelling at a speed of 70 km/h? **2**

Question 28 continues on page 23

(b) The height and mass of a child are measured and recorded over its first two years.

Height (cm), H	45	50	55	60	65	70	75	80
Mass (kg), M	2.3	3.8	4.7	6.2	7.1	7.8	8.8	10.2

This information is displayed in a scatter graph.



(i) Describe the correlation between the height and mass of this child, as shown in the graph. **1**

(ii) A line of best fit has been drawn on the graph. **2**

Find the equation of this line.

Question 28 continues on page 24

(c) The height above the ground, in metres, of a person's eyes varies directly with the square of the distance, in kilometres, that the person can see to the horizon. **3**

A person whose eyes are 1.6 m above the ground can see 4.5 km out to sea.

How high above the ground, in metres, would a person's eyes need to be to see an island that is 15 km out to sea? Give your answer correct to one decimal place.

(d) In an experiment, two unbiased dice, with faces numbered 1, 2, 3, 4, 5, 6, are rolled 18 times. The difference between the numbers on their uppermost faces is recorded each time. Juan performs this experiment twice and his results are shown in the tables. **4**

Experiment 1

Difference	*Frequency*
0	3
1	3
2	2
3	4
4	3
5	3

Experiment 2

Difference	*Frequency*
0	4
1	4
2	3
3	3
4	2
5	2

Juan states that Experiment 2 has given results that are closer to what he expected than the results given by Experiment 1.

Is he correct? Explain your answer by finding the sample space for the dice differences and using theoretical probability.

End of paper

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General Mathematics

FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$