BOARD OF STUDIES
NEW SOUTH WALES

2007

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–23

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet for Questions 1–22.

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1 What is 0.000 000 326 mm expressed in scientific notation?

(A) 0.326×10^{-6} mm

(B) 3.26×10^{-7} mm

(C) 0.326×10^{6} mm

(D) 3.26×10^{7} mm

2 Each student in a class is given a packet of lollies. The teacher records the number of red lollies in each packet using a frequency table.

Number of red lollies in each packet	*Frequency*
0	2
1	4
2	2
3	7
4	3
5	1

What is the relative frequency of a packet of lollies containing more than three red lollies?

(A) $\frac{4}{19}$

(B) $\frac{4}{15}$

(C) $\frac{11}{19}$

(D) $\frac{11}{15}$

3 Joe is about to go on holidays for four weeks. His weekly salary is \$280 and his holiday loading is $17\frac{1}{2}\%$ of four weeks pay.

What is Joe's total pay for the four weeks holiday?

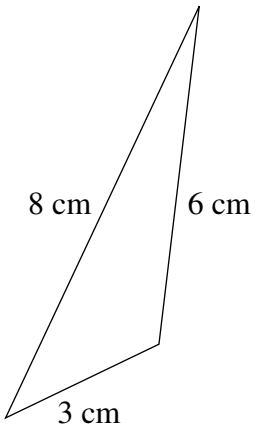
(A) \$196

(B) \$329

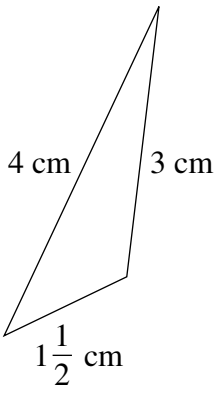
(C) \$1169

(D) \$1316

4 What scale factor has been used to transform Triangle *A* to Triangle *B*?



Triangle *A*



Triangle *B*

NOT TO SCALE

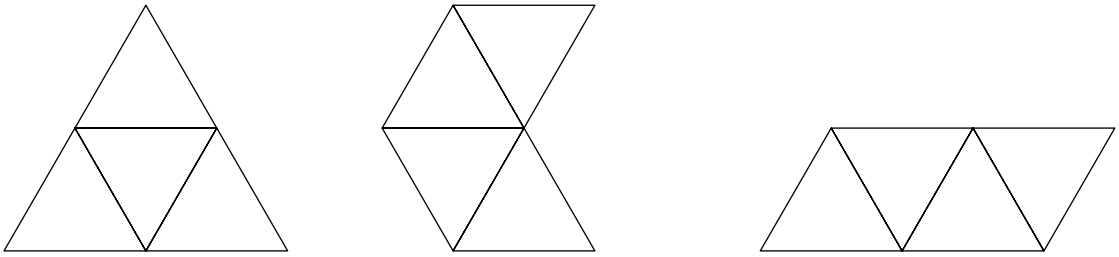
(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) 2

(D) 3

5 Which of the following nets can be folded to form a triangular pyramid?



I II III

(A) I only

(B) I and II

(C) I and III

(D) II and III

6 The price of a CD is $22.00, which includes 10% GST.

What is the amount of GST included in this price?

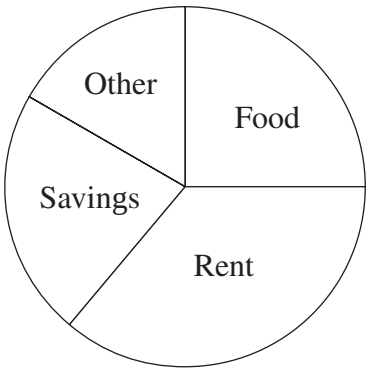
(A) $2.00

(B) $2.20

(C) $19.80

(D) $20.00

7 Margaret has a weekly income of $900 and allocates her money according to the budget shown in the sector graph.



How long will it take Margaret to save $3600?

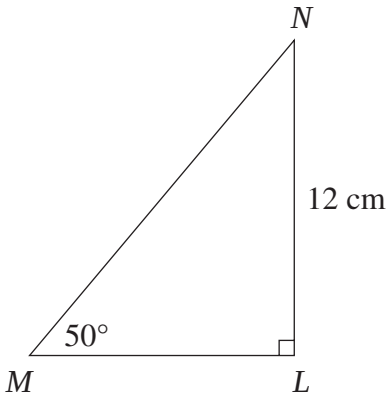
(A) 4 weeks

(B) 5 weeks

(C) 16 weeks

(D) 18 weeks

8 What is the length of the side *MN* in the following triangle, correct to two decimal places?



NOT
TO
SCALE

(A) 9.19 cm

(B) 10.07 cm

(C) 15.66 cm

(D) 18.67 cm

9 Which of the following would be most likely to have a positive correlation?

(A) The population of a town and the number of schools in that town

(B) The price of petrol per litre and the number of litres of petrol sold

(C) The hours training for a marathon and the time taken to complete the marathon

(D) The number of dogs per household and the number of televisions per household

10 Each time she throws a dart, the probability that Mary hits the dartboard is $\frac{2}{7}$.

She throws two darts, one after the other.

What is the probability that she hits the dartboard with both darts?

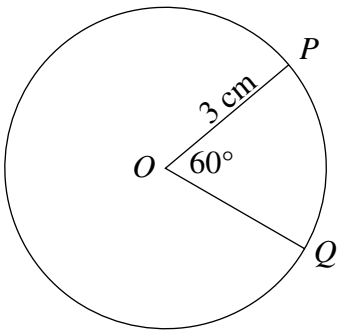
(A) $\frac{1}{21}$

(B) $\frac{4}{49}$

(C) $\frac{2}{7}$

(D) $\frac{4}{7}$

11 P and Q are points on the circumference of a circle with centre O and radius 3 cm.



NOT TO SCALE

What is the length of the arc PQ, in centimetres, correct to three significant figures?

(A) 1.57

(B) 3.14

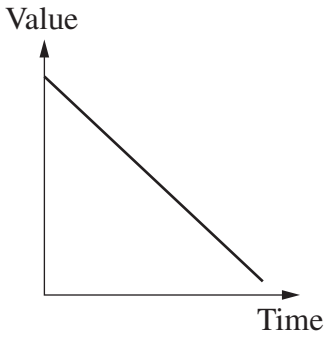
(C) 4.71

(D) 18.8

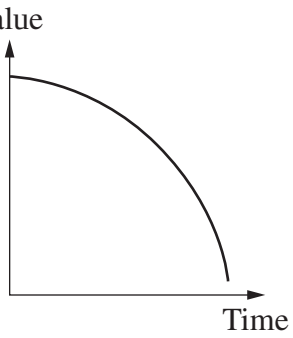
12 The value of a car is depreciated using the declining balance method.

Which graph best illustrates the value of the car over time?

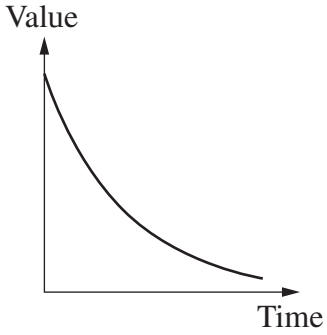
(A)



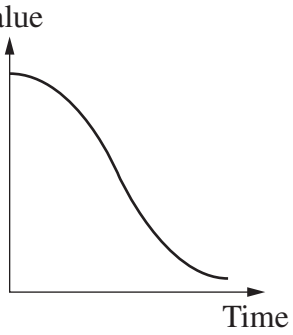
(B)



(C)



(D)



13 The positions of President, Secretary and Treasurer of a club are to be chosen from a committee of 5 people.

In how many ways can the three positions be chosen?

(A) 3

(B) 10

(C) 60

(D) 125

14 Which expression is equivalent to $3x^2(x + 8) + x^2$?

(A) $3x^3 + x^2 + 8$

(B) $3x^3 + 25x^2$

(C) $4x^3 + 32x^2$

(D) $24x^3 + x^2$

15 If pressure (p) varies inversely with volume (V), which formula correctly expresses p in terms of V and k, where k is a constant?

(A) $p = \frac{k}{V}$

(B) $p = \frac{V}{k}$

(C) $p = kV$

(D) $p = k + V$

16 Leanne copied a two-way table into her book.

	Male	*Female*	*Totals*
Full-time work	279	356	635
Part-time work	187	439	716
Totals	466	885	1351

Leanne made an error in copying one of the values in the shaded section of the table.

Which value has been incorrectly copied?

(A) The number of males in full-time work

(B) The number of males in part-time work

(C) The number of females in full-time work

(D) The number of females in part-time work

17 Ms Wigginson decided to survey a sample of 10% of the students at her school.

The school enrolment is shown in the table.

Year	7	8	9	10	11	12	*Total*
Number of students	225	232	233	230	150	130	1200

She surveyed the same number of students in each year group.

How would the numbers of students surveyed in Year 10 and Year 11 have changed if Ms Wigginson had chosen to use a stratified sample based on year groups?

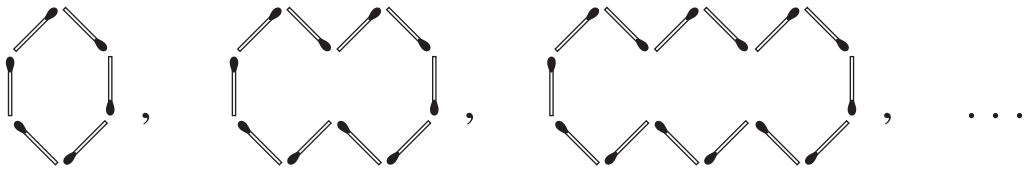
(A) Increased in both Year 10 and Year 11

(B) Decreased in both Year 10 and Year 11

(C) Increased in Year 10 and decreased in Year 11

(D) Decreased in Year 10 and increased in Year 11

18 Chris started to make this pattern of shapes using matchsticks.



Shape 1 Shape 2 Shape 3

If the pattern of shapes is continued, which shape would use exactly 486 matchsticks?

(A) Shape 96

(B) Shape 97

(C) Shape 121

(D) Shape 122

19 Which of the following correctly expresses T as the subject of $B = 2\pi\left(R + \frac{T}{2}\right)$?

(A) $T = \frac{B}{\pi} - 2R$

(B) $T = \frac{B}{\pi} - R$

(C) $T = 2R - \frac{B}{\pi}$

(D) $T = \frac{B}{4\pi} - \frac{R}{2}$

20 Kim lives in Perth (32°S, 115°E). He wants to watch an ice hockey game being played in Toronto (44°N, 80°W) starting at 10.00 pm on Wednesday.

What is the time in Perth when the game starts?

(A) 9.00 am on Wednesday

(B) 7.40 pm on Wednesday

(C) 12.20 am on Thursday

(D) 11.00 am on Thursday

21 This set of data is arranged in order from smallest to largest.

5, 6, 11, x, 13, 18, 25

The range is six less than twice the value of x.

Which one of the following is true?

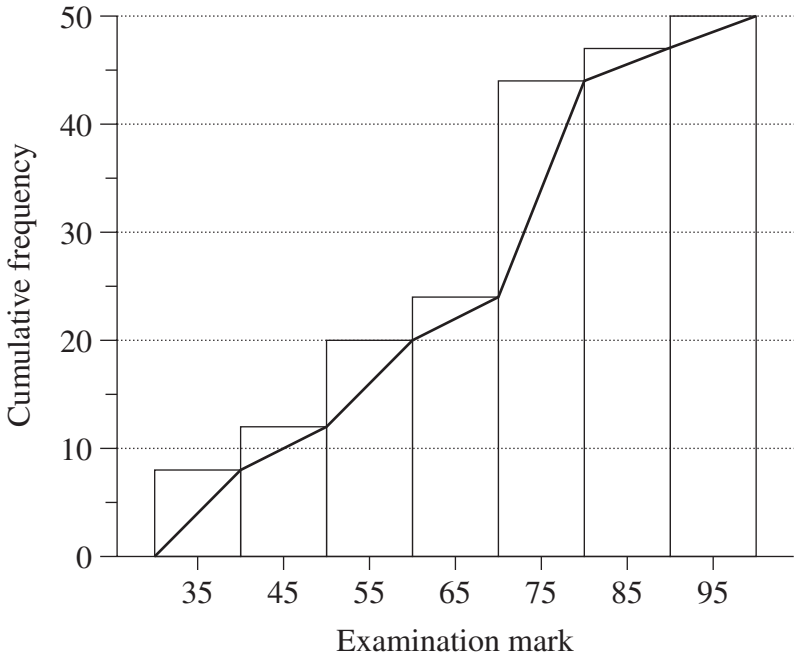
(A) The median is 12 and the interquartile range is 7.

(B) The median is 12 and the interquartile range is 12.

(C) The median is 13 and the interquartile range is 7.

(D) The median is 13 and the interquartile range is 12.

22 A set of examination results is displayed in a cumulative frequency histogram and polygon (ogive).



Sanath knows that his examination mark is in the 4th decile.

Which of the following could have been Sanath's examination mark?

(A) 37

(B) 57

(C) 67

(D) 77

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

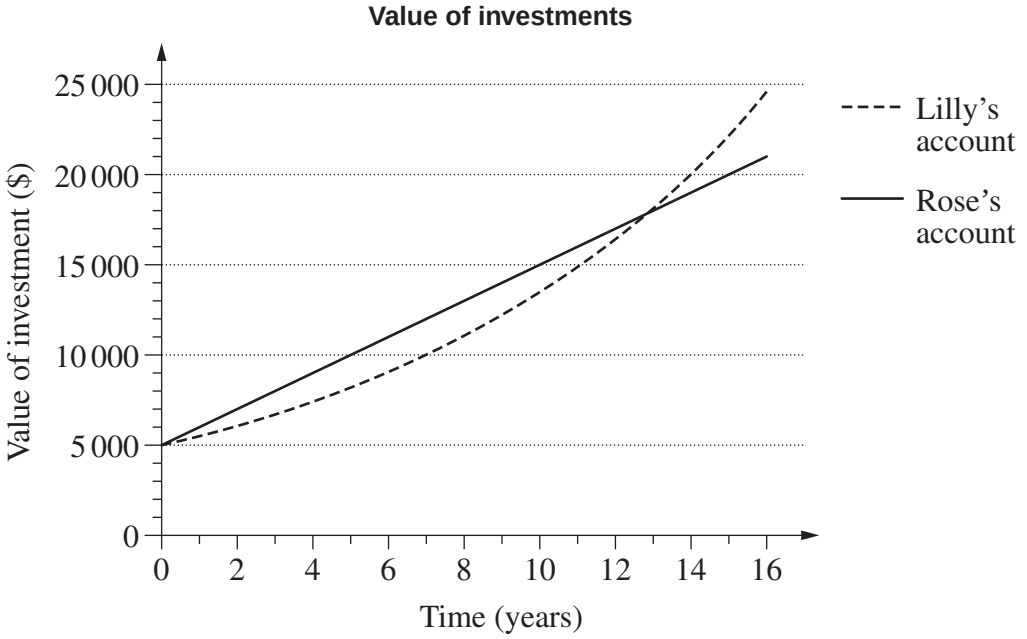
All necessary working should be shown in every question.

Marks

Question 23 (13 marks) Use the Question 23 Writing Booklet.

(a) Lilly and Rose each have money to invest and choose different investment accounts.

The graph shows the values of their investments over time.



(i) How much was Rose's original investment? **1**

(ii) At the end of 6 years, which investment will be worth the most and by how much? **2**

(iii) Lilly's investment will reach a value of $20 000 first. **1**

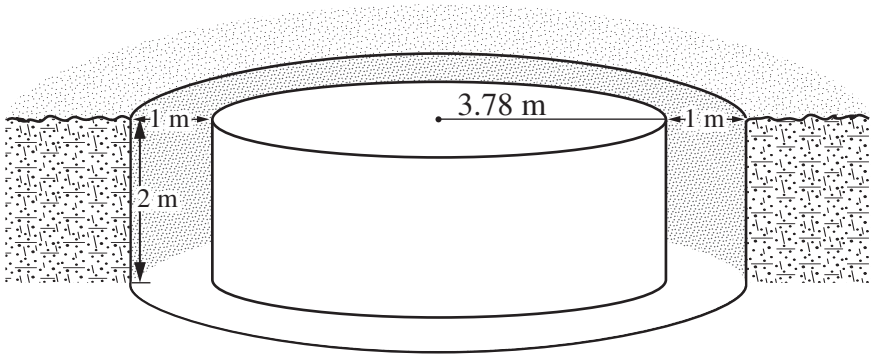
How much longer will it take Rose's investment to reach a value of $20 000?

Question 23 continues on page 13

Question 23 (continued)

(b) A cylindrical water tank, of height 2 m, is placed in the ground at a school.

The radius of the tank is 3.78 metres. The hole is 2 metres deep. When the tank is placed in the hole there is a gap of 1 metre all the way around the side of the tank.



(i) When digging the hole for the water tank, what volume of soil was removed? Give your answer to the nearest cubic metre. **3**

(ii) Sprinklers are used to water the school oval at a rate of 7500 litres per hour. **1**

The water tank holds 90 000 litres when full.

For how many hours can the sprinklers be used before a full tank is emptied?

(iii) Water is to be collected in the tank from the roof of the school hall, which has an area of 400 m^2. During a storm, 20 mm of rain falls on the roof and is collected in the tank. **2**

How many litres of water were collected?

(c) A scientific study uses the 'capture-recapture' technique.

In the first stage of the study, 24 crocodiles were caught, tagged and released.

Later, in the second stage of the study, some crocodiles were captured from the same area. Eighteen of these were found to be tagged, which was 40% of the total captured during the second stage.

(i) How many crocodiles were captured in total during the second stage of the study? **1**

(ii) Calculate the estimate for the total population of crocodiles in this area. **2**

End of Question 23

Marks

Question 24 (13 marks) Use the Question 24 Writing Booklet.

(a) Consider the following set of scores:

3, 5, 5, 6, 8, 8, 9, 10, 10, 50.

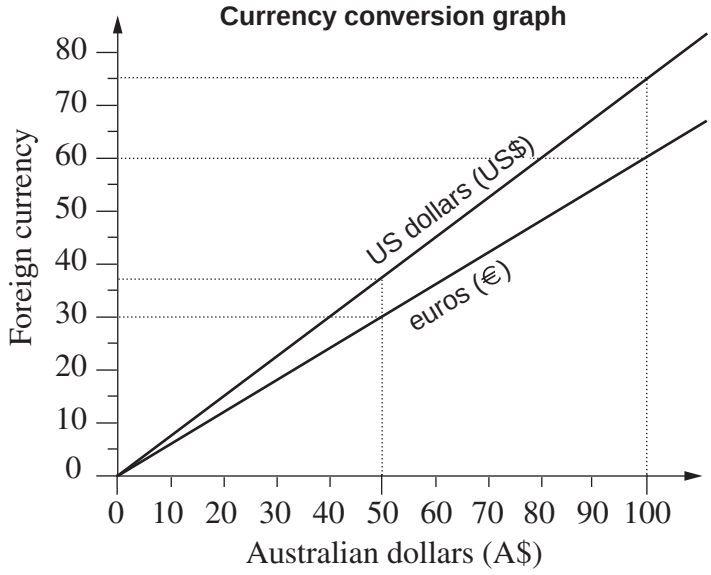
(i) Calculate the mean of the set of scores. **1**

(ii) What is the effect on the mean and on the median of removing the outlier? **2**

(b) The distance in kilometres (D) of an observer from the centre of a thunderstorm can be estimated by counting the number of seconds (t) between seeing the lightning and first hearing the thunder. **1**

Use the formula $D = \frac{t}{3}$ to estimate the number of seconds between seeing the lightning and hearing the thunder if the storm is 1.2 km away.

(c) Sandy travels to Europe via the USA. She uses this graph to calculate her currency conversions.



(i) After leaving the USA she has US$150 to add to the A$600 that she plans to spend in Europe. She converts all of her money to euros. **3**

How many euros does she have to spend in Europe?

(ii) If the value of the euro falls in comparison to the Australian dollar, what will be the effect on the gradient of the line used to convert Australian dollars to euros? **1**

Question 24 continues on page 15

Marks

Question 24 (continued)

(d) Barry constructed a back-to-back stem-and-leaf plot to compare the ages of his students.

Ages of students attending Barry's Ballroom Dancing Studio

Females		*Males*
9	1	1 2 3
7	2	0 2 2 2 4 5
5	3	0 0 1 7
5 2	4	6 7
3 2 0	5	2
4 4 2 1	6	4 4

(i) Write a brief statement that compares the distribution of the ages of males and females from this set of data. **1**

(ii) What is the mode of this set of data? **1**

(iii) Liam decided to use a grouped frequency distribution table to calculate the mean age of the students at Barry's Ballroom Dancing Studio. **2**

For the age group 30–39 years, what is the value of the product of the class centre and the frequency?

(iv) Liam correctly calculated the mean from the grouped frequency distribution table to be 39.5. **1**

Caitlyn correctly used the original data in the back-to-back stem-and-leaf plot and calculated the mean to be 38.2.

What is the reason for the difference in the two answers?

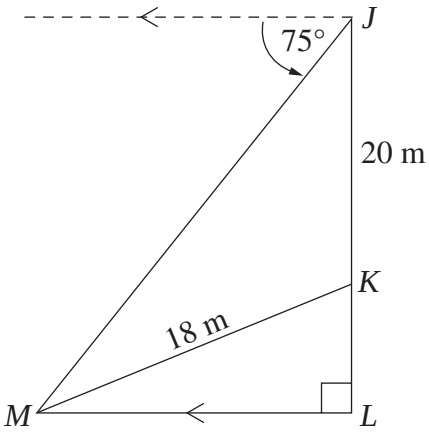
End of Question 24

Marks

Question 25 (13 marks) Use the Question 25 Writing Booklet.

(a) Give an example of an event that has a probability of exactly $\frac{3}{4}$. **1**

(b) The angle of depression from J to M is 75°. The length of JK is 20 m and the length of MK is 18 m. **3**



NOT TO SCALE

Copy or trace this diagram into your writing booklet and calculate the angle of elevation from M to K. Give your answer to the nearest degree.

Question 25 continues on page 17

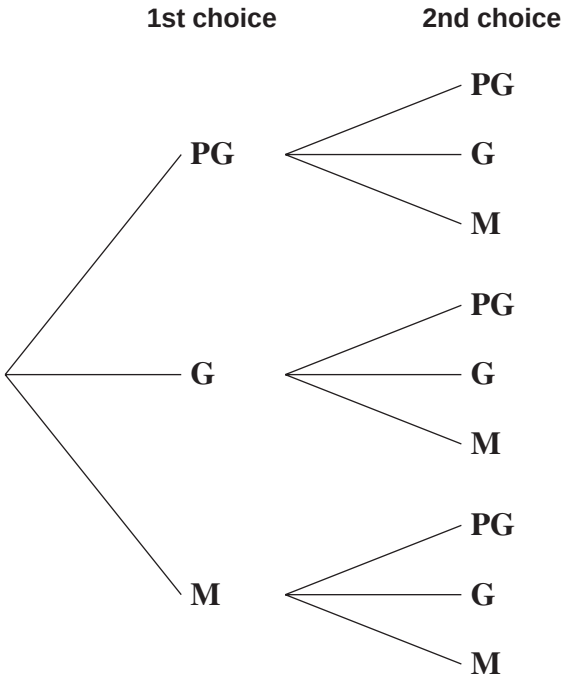
Marks

Question 25 (continued)

(c) In a stack of 10 DVDs, there are 5 rated **PG**, 3 rated **G** and 2 rated **M**.

(i) A DVD is selected at random. What is the probability that it is rated **M**? **1**

(ii) Grant chooses two DVDs at random from the stack. Copy or trace the tree diagram into your writing booklet. Complete your tree diagram by writing the correct probability on each branch. **2**



(iii) Calculate the probability that Grant chooses two DVDs with the same rating. **2**

(d) The results of two class tests are normally distributed. The means and standard deviations of the tests are displayed in the table.

	Test 1	*Test 2*
Mean	60	58
Standard deviation	6.2	6.0

(i) Stuart scored 63 in Test 1 and 62 in Test 2. He thinks that he has performed better in Test 1. Do you agree? Justify your answer using appropriate calculations. **2**

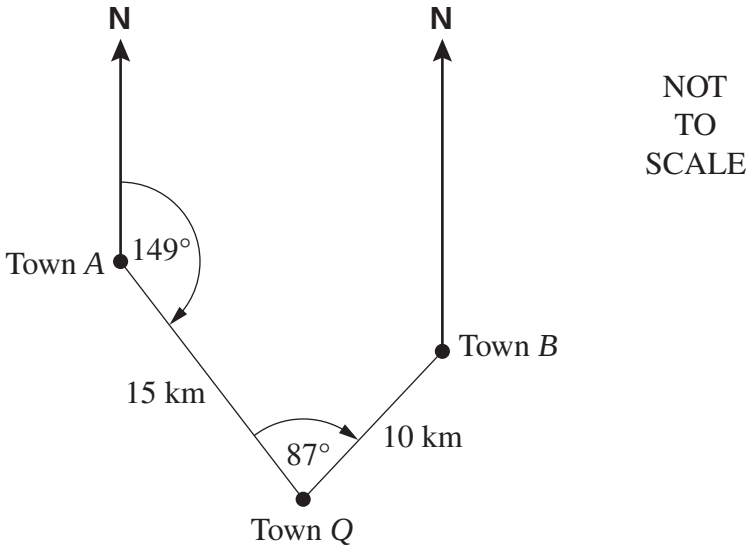
(ii) If 150 students sat for Test 2, how many students would you expect to have scored less than 64? **2**

End of Question 25

Marks

Question 26 (13 marks) Use the Question 26 Writing Booklet.

(a) The diagram shows information about the locations of towns A, B and Q.



(i) It takes Elina 2 hours and 48 minutes to walk directly from Town A to Town Q. Calculate her walking speed correct to the nearest km/h. **1**

(ii) Elina decides, instead, to walk to Town B from Town A and then to Town Q. **2**

Find the distance from Town A to Town B. Give your answer to the nearest km.

(iii) Calculate the bearing of Town Q from Town B. **1**

Question 26 continues on page 19

Marks

Question 26 (continued)

(b) Myles is in his third year as an apprentice film editor.

(i) Myles purchased film-editing equipment for $5000. After 3 years it has depreciated to $3635 using the straight-line method. **2**

Calculate the rate of depreciation per year as a percentage.

(ii) Myles earns $800 per week. Calculate his taxable income for this year if the only allowable deduction is the amount of depreciation of his film-editing equipment in the third year of use. **1**

(iii) Use this tax table to calculate Myles's tax payable. **2**

Taxable income ($)	*Tax payable*
$0 – $10 000	Nil
$10 001 – $28 000	Nil plus 25 cents for each $1 over $10 000
$28 001 – $50 000	$4500 plus 30 cents for each $1 over $28 000
$50 001 – $100 000	$11 100 plus 40 cents for each $1 over $50 000
over $100 000	$31 100 plus 60 cents for each $1 over $100 000

(c) When Mina was born, and on every birthday after that, her grandparents deposited $100 into an investment account. The interest rate on the account is fixed at 6% per annum, compounded annually. **4**

Write an expression for the value of the investment immediately after her grandparents deposit $100 on her 21st birthday and calculate the total interest earned on this investment.

End of Question 26

Marks

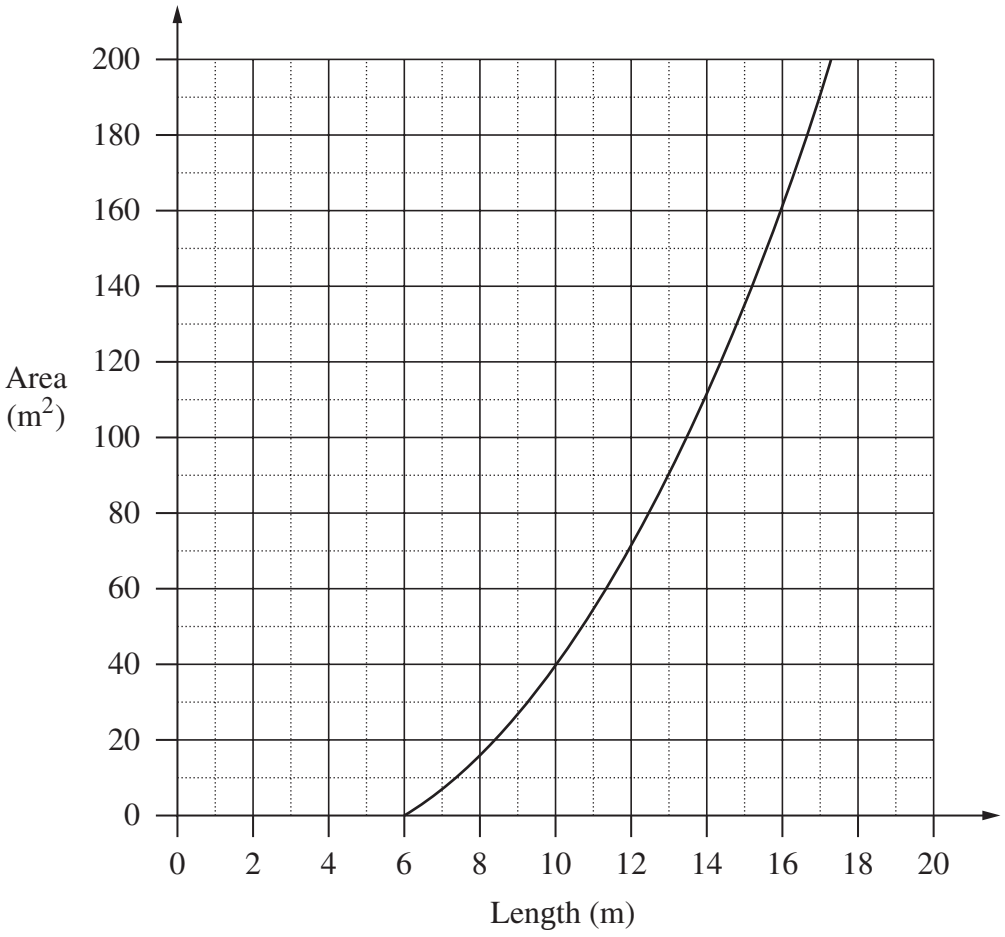
Question 27 (13 marks) Use the Question 27 Writing Booklet.

(a) A rectangular playing surface is to be constructed so that the length is 6 metres more than the width.

(i) Give an example of a length and width that would be possible for this playing surface. **1**

(ii) Write an equation for the area (A) of the playing surface in terms of its length (l). **1**

A graph comparing the area of the playing surface to its length is shown.



(iii) Why are lengths of 0 metres to 6 metres impossible? **1**

(iv) What would be the dimensions of the playing surface if it had an area of 135 m^2? **2**

Question 27 continues on page 21

Question 27 (continued)

Company *A* constructs playing surfaces.

Company *A* charges

Size of playing surface	*Charges*
Up to and including 150 m^2	\$50 000
Greater than 150 m^2	\$50 000 plus a rate of \$300 per square metre for the area in excess of 150 m^2

(v) Draw a graph to represent the cost of using Company A to construct all playing surface sizes up to and including 200 m^2. **2**

Use the horizontal axis to represent the area and the vertical axis to represent the cost.

(vi) Company *B* charges a rate of \$360 per square metre regardless of size. **1**

Which company would charge less to construct a playing surface with an area of 135 m^2? Justify your answer with suitable calculations.

(b) A clubhouse uses four long-life light globes for five hours every night of the year. The purchase price of each light globe is \$6.00 and they each cost \$$d$ per hour to run.

(i) Write an equation for the total cost (*\$c)* of purchasing and running these four light globes for one year in terms of d. **2**

(ii) Find the value of d (correct to three decimal places) if the total cost of running these four light globes for one year is \$250. **1**

(iii) If the use of the light globes increases to ten hours per night every night of the year, does the total cost double? Justify your answer with appropriate calculations. **1**

(iv) The manufacturer's specifications state that the expected life of the light globes is normally distributed with a standard deviation of 170 hours. **1**

What is the mean life, in hours, of these light globes if 97.5% will last up to 5000 hours?

End of Question 27

Marks

Question 28 (13 marks) Use the Question 28 Writing Booklet.

(a) Two unbiased dice, *A* and *B*, with faces numbered 1, 2, 3, 4, 5 and 6 are rolled. The numbers on the uppermost faces are noted. This table shows all the possible outcomes.

		Die *A*					
		1	*2*	*3*	*4*	*5*	*6*
Die *B*	*1*	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

A game is played where the difference between the highest number showing and the lowest number showing on the uppermost faces is calculated.

(i) What is the probability that the difference between the numbers showing on the uppermost faces of the two dice is one? **1**

In the game, the following applies.

Difference	*Result*
0	Win \$3.50
1	Lose \$5
2, 3, 4 or 5	Win \$2.80

(ii) What is the financial expectation of the game? **3**

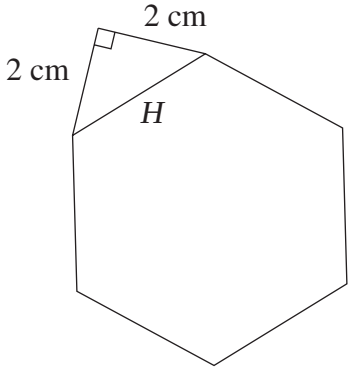
(iii) If Jack pays \$1 to play the game, does he expect a gain or a loss, and how much will it be? **1**

Question 28 continues on page 23

Marks

Question 28 (continued)

(b) This shape is made up of a right-angled triangle and a regular hexagon. **3**

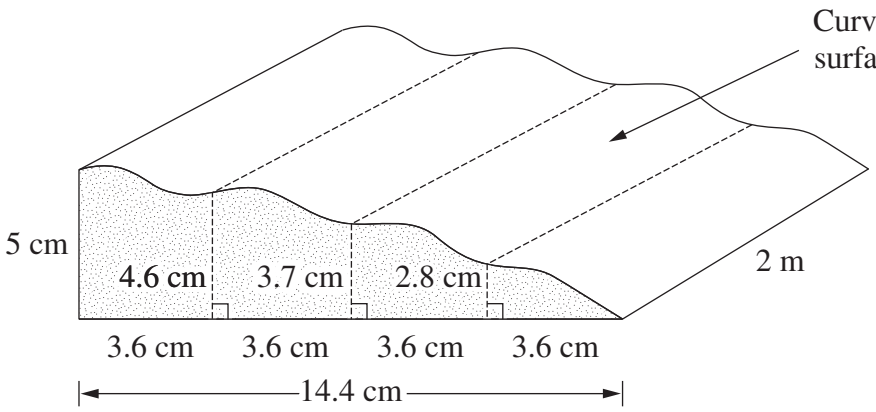


NOT TO SCALE

The area of a regular hexagon can be estimated using the formula $A = 2.598H^2$ where H is the side-length.

Calculate the total area of the shape using this formula.

(c) Apiece of plaster has a uniform cross-section, which has been shaded, and has dimensions as shown.



NOT TO SCALE

(i) Use two applications of Simpson's rule to approximate the area of the cross-section. **3**

(ii) The total surface area of the piece of plaster is 7480.8 cm^2. **2**

Calculate the area of the curved surface as shown on the diagram.

End of paper

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General Mathematics

FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–inte cept form of a straight line

$$y = mx + b$$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$