BOARD OF STUDIES
NEW SOUTH WALES

2005

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–24

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

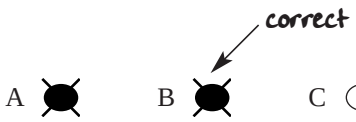
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



---

1 What is the mean of the set of scores?

3, 4, 5, 6, 6, 8, 8, 8, 15

(A) 6

(B) 7

(C) 8

(D) 9

2 What is the value of $\frac{a-b}{4}$, if $a = 240$ and $b = 56$?

(A) 4

(B) 46

(C) 226

(D) 736

3 Four radio stations reported the probability of rain as shown in the table.

Radio station	*Probability of rain*
2AT	0.53
2BW	17%
2CZ	$\frac{13}{25}$
2DL	0.6

Which radio station reported the highest probability of rain?

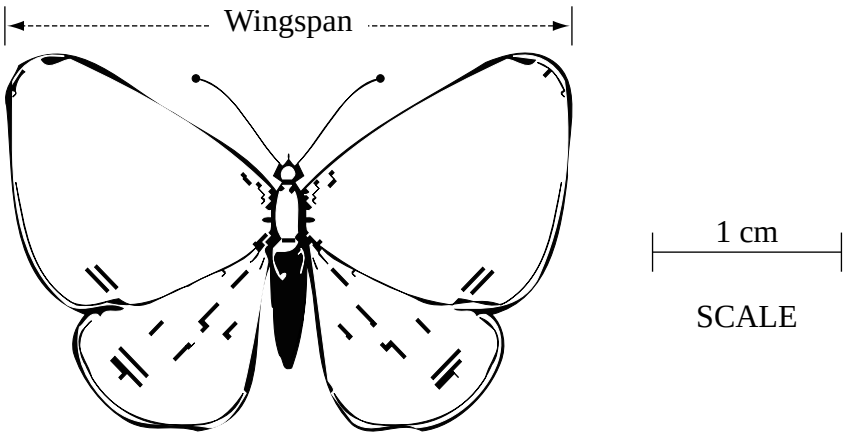
(A) 2AT

(B) 2BW

(C) 2CZ

(D) 2DL

4 The diagram is a scale drawing of a butterfly.



What is the actual wingspan of the butterfly?

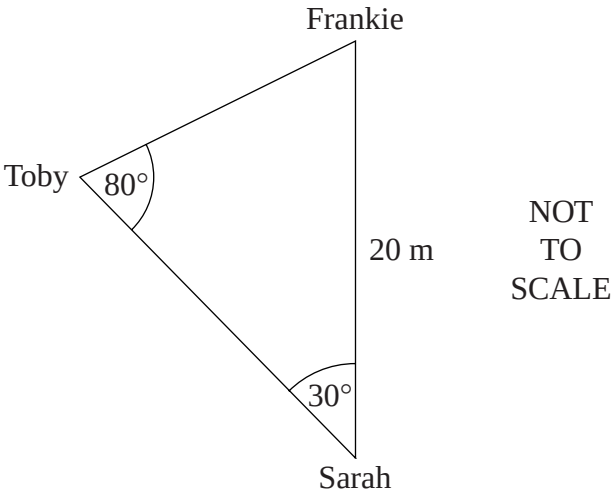
(A) 2.5 cm

(B) 3 cm

(C) 15 cm

(D) 18.75 cm

5



Which formula should be used to calculate the distance between Toby and Frankie?

(A) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

(B) $c^2 = a^2 + b^2$

(C) $A = \dfrac{1}{2}ab\sin C$

(D) $c^2 = a^2 + b^2 - 2ab\cos C$

6 Janet's gross income last year was \$60 000. She had allowable tax deductions of \$5000. Janet paid 1.5% of her taxable income for the Medicare levy.

How much was Janet's Medicare levy?

(A) \$750

(B) \$825

(C) \$900

(D) \$975

7 Simplify $2m^2 \times 3mp^2$.

(A) $5m^2p^2$

(B) $5m^3p^2$

(C) $6m^2p^2$

(D) $6m^3p^2$

8 If $\tan\theta = 85$, what is the value of θ, correct to the nearest minute?

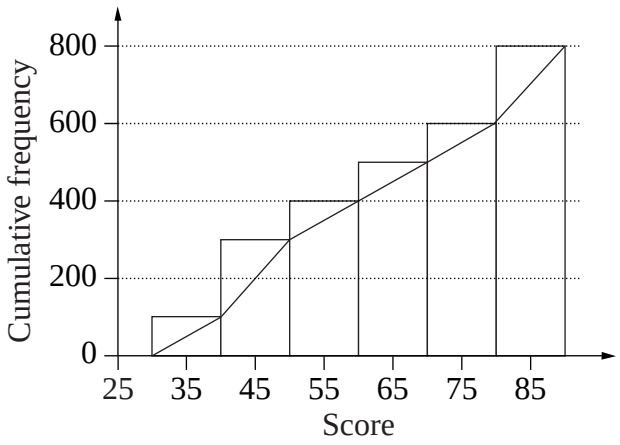
(A) 11° 25'

(B) 11° 26'

(C) 89° 19'

(D) 89° 20'

9 A set of data is represented by the cumulative frequency histogram and ogive.



What is the best approximation for the interquartile range for this set of data?

(A) 25

(B) 30

(C) 35

(D) 40

10 The table is used to calculate monthly loan repayments.

Monthly loan repayments (in dollars) per $1000 borrowed

Interest rate % pa	*5 years*	*10 years*	*15 years*	*20 years*
5%	18.87	10.61	7.91	6.60
6%	19.33	11.10	8.44	7.16
7%	19.80	11.61	8.99	7.75
8%	20.28	12.13	9.56	8.36
9%	20.76	12.67	10.14	9.00

Samantha has borrowed $70 000 at 8% per annum for 15 years.

What is her monthly loan repayment?

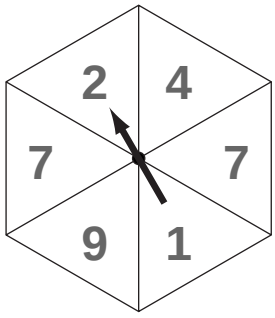
(A) $143.40

(B) $669.20

(C) $8 030.40

(D) $10 038.00

11 The diagram shows a spinner.



The arrow is spun and will stop in one of the six sections.

What is the probability that the arrow will stop in a section containing a number greater than 4?

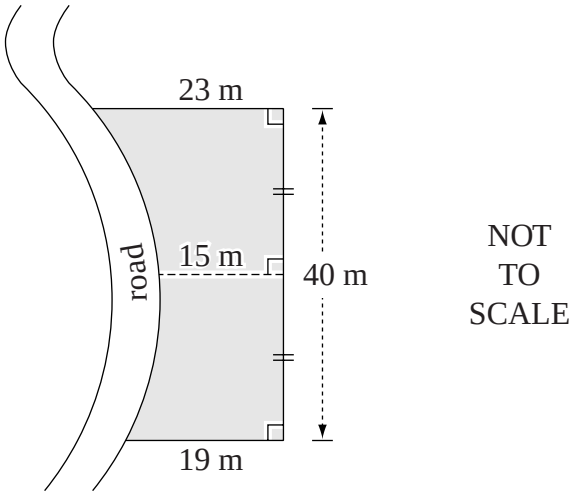
(A) $\frac{2}{5}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

12 The shaded region represents a block of land bounded on one side by a road.



What is the approximate area of the block of land, using Simpson's rule?

(A) 680 m^2

(B) 760 m^2

(C) 840 m^2

(D) 1360 m^2

13 Last year, Helen bought 150 shares at \$2.00 per share. They are now worth \$2.50 per share. Helen receives a dividend of \$0.10 per share.

What is the dividend yield?

(A) 4%

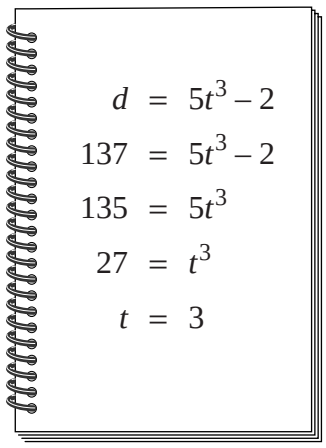
(B) 20%

(C) \$15

(D) \$75

14 Using the formula $d = 5t^3 - 2$, Marcia tried to find the value of t when $d = 137$.

Here is her solution. She has made one mistake.



$d = 5t^3 - 2$

$137 = 5t^3 - 2$ ..... Line *A*

$135 = 5t^3$ ..... Line *B*

$27 = t^3$ ..... Line *C*

$t = 3$ ..... Line *D*

Which line does NOT follow correctly from the previous line?

(A) Line *A*

(B) Line *B*

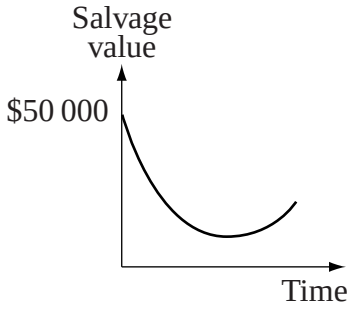
(C) Line *C*

(D) Line *D*

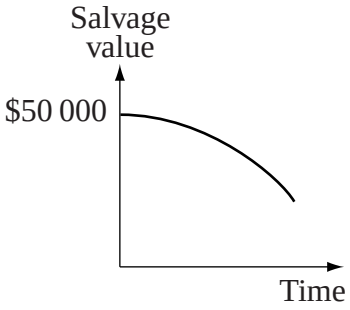
15 A car bought for \$50 000 is depreciated using the declining balance method.

Which graph best represents the salvage value of the car over time?

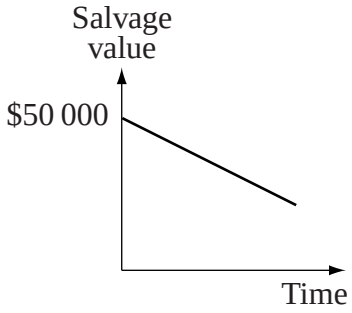
(A)



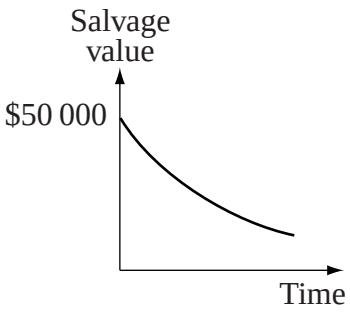
(B)



(C)



(D)



16 On a television game show, viewers voted for their favourite contestant. The results were recorded in the two-way table.

	Male viewers	*Female viewers*
Contestant 1	1372	3915
Contestant 2	2054	3269

One male viewer was selected at random from all of the male viewers.

What is the probability that he voted for Contestant 1?

(A) $\frac{1372}{10\,610}$

(B) $\frac{1372}{5287}$

(C) $\frac{1372}{3426}$

(D) $\frac{1372}{2054}$

17 The total cost, $\$C$, of a school excursion is given by $C = 2n + 5$, where n is the number of students.

If three extra students go on the excursion, by how much does the total cost increase?

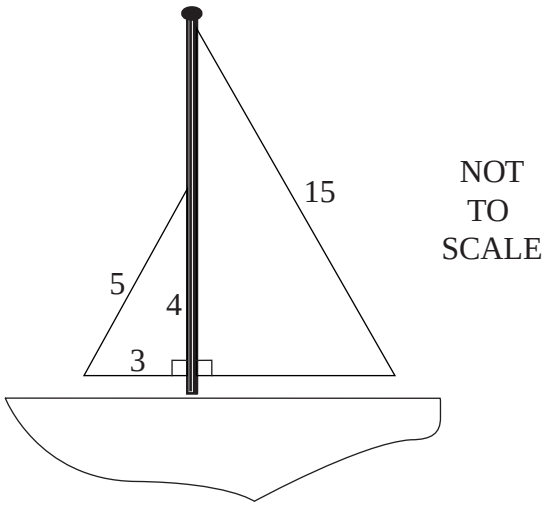
(A) \$6

(B) \$11

(C) \$15

(D) \$16

18 A model yacht has two triangular sails. These triangles are similar to each other. Some dimensions of the sails, in centimetres, are shown on the diagram.



What is the total area of both sails?

(A) 24 cm^2

(B) 27 cm^2

(C) 60 cm^2

(D) 97 cm^2

19 The location of Town *A* is 25°N 45°E. The location of Town *B* is 10°N 105°E.

Which of the following is true? (Ignore time zones.)

(A) Town *A* is four hours behind Town *B*.

(B) Town *A* is four hours ahead of Town *B*.

(C) Town *A* is one hour behind Town *B*.

(D) Town *A* is one hour ahead of Town *B*.

20 Dave's school has computer security codes made up of four digits (eg 0773). Juanita's school has computer security codes made up of five digits (eg 30568).

How many more codes are available at Juanita's school than at Dave's school?

(A) 10

(B) 50

(C) 90 000

(D) 100 000

21 Yousef used the 'capture-recapture' technique to estimate the number of kangaroos living in a particular area.

- He caught, tagged and released 50 kangaroos.
- Later, he caught 200 kangaroos at random from the same area.
- He found that 5 of these 200 kangaroos had been tagged.

What is the correct estimate for the total number of kangaroos living in this area, using the 'capture-recapture' technique?

(A) 245

(B) 250

(C) 2000

(D) 10 000

22 Two groups of people were surveyed about their weekly wages. The results are shown in the box-and-whisker plots.



Which of the following statements is true for the people surveyed?

(A) The same percentage of people in each group earned more than $325 per week.

(B) Approximately 75% of people under 21 years earned less than $350 per week.

(C) Approximately 75% of people 21 years and older earned more than $350 per week.

(D) Approximately 50% of people in each group earned between $325 and $350 per week.

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

Question 23 (13 marks) Use a SEPARATE writing booklet.

(a) There are 100 tickets sold in a raffle. Justine sold all 100 tickets to five of her friends. The number of tickets she sold to each friend is shown in the table.

Friend	*Number of tickets*
Danielle	45
Khalid	5
Nancy	10
Shani	14
Herman	26
Total	100

(i) Justine claims that each of her friends is equally likely to win first prize. **1**

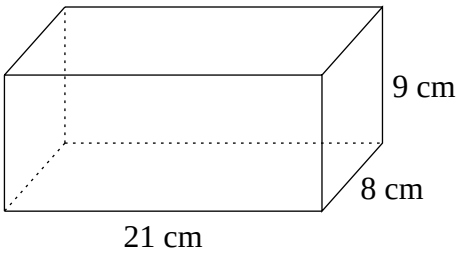
Give a reason why Justine's statement is NOT correct.

(ii) What is the probability that first prize is NOT won by Khalid or Herman? **2**

Question 23 continues on page 13

Question 23 (continued)

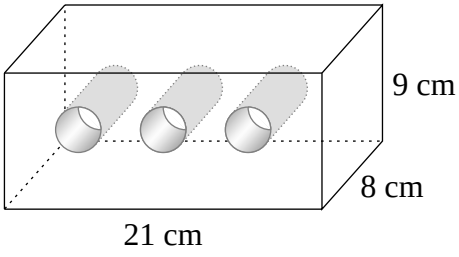
(b) A clay brick is made in the shape of a rectangular prism with dimensions as shown.



NOT TO SCALE

(i) Calculate the volume of the clay brick. **1**

Three identical cylindrical holes are made through the brick as shown. Each hole has a radius of 1.4 cm.



NOT TO SCALE

(ii) What is the volume of clay remaining in the brick after the holes have been made? (Give your answer to the nearest cubic centimetre.) **3**

(iii) What percentage of clay is removed by making the holes through the brick? (Give your answer correct to one decimal place.) **1**

Question 23 continues on page 14

Marks

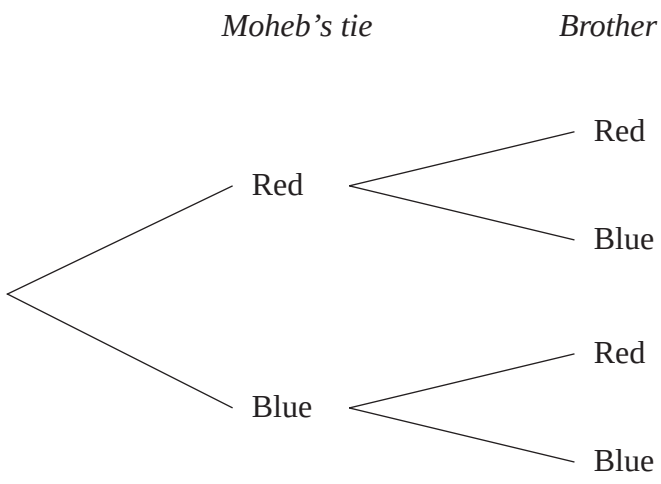
Question 23 (continued)

(c) Moheb owns five red and seven blue ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother.

(i) What is the probability that Moheb chooses a red tie for himself? **1**

(ii) Copy the tree diagram into your writing booklet. **2**

Complete your tree diagram by writing the correct probability on each branch.



(iii) Calculate the probability that both of the ties are the same colour. **2**

End of Question 23

Marks

Question 24 (13 marks) Use a SEPARATE writing booklet.

(a) (i) Draw a stem-and-leaf plot for the following set of scores. **2**

21 45 29 27 19 35 23 58 34 27

(ii) What is the median of the set of scores? **1**

(iii) Comment on the skewness of the set of scores. **1**

(b) The formula $D = \frac{2A}{15}$ is used to calculate the dosage of Hackalot cough medicine to be given to a child.

- D is the dosage of Hackalot cough medicine in millilitres (mL).
- A is the age of the child in months.

(i) If George is nine months old, what dosage of Hackalot cough medicine should he be given? **1**

(ii) The correct dosage of Hackalot cough medicine for Sam is 4 mL. **3**

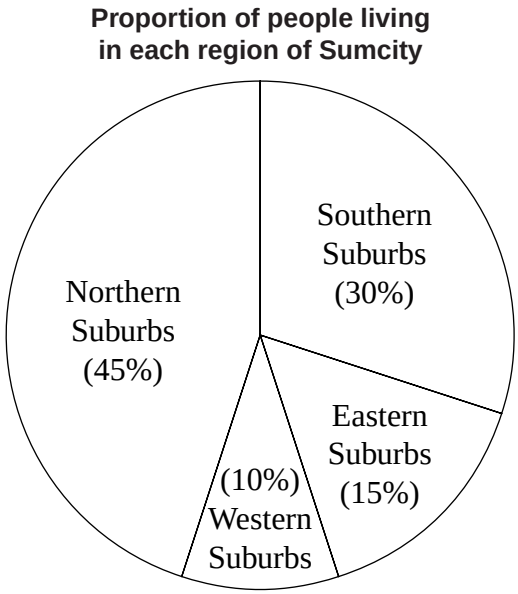
What is the difference in the ages of Sam and George, in months?

(c) Make L the subject of the equation $T = 2\pi L^2$. **2**

Question 24 continues on page 16

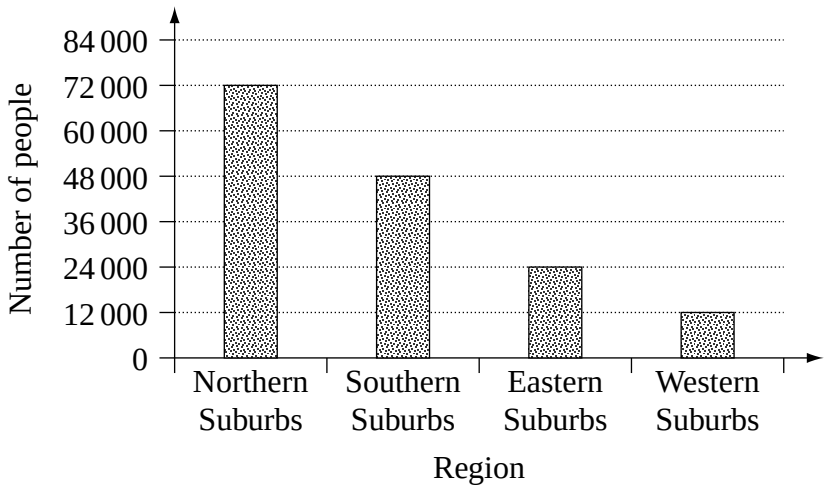
Question 24 (continued)

(d) The sector graph shows the proportion of people, as a percentage, living in each region of Sumcity. There are 24 000 people living in the Eastern Suburbs.



(i) Show that the total number of people living in Sumcity is 160 000. **1**

Jake used the information above to draw a column graph.



(ii) The column graph height is incorrect for one region. **2**

Identify this region and justify your answer.

End of Question 24

Marks

Question 25 (13 marks) Use a SEPARATE writing booklet.

(a) Reece is preparing his annual budget for 2006.

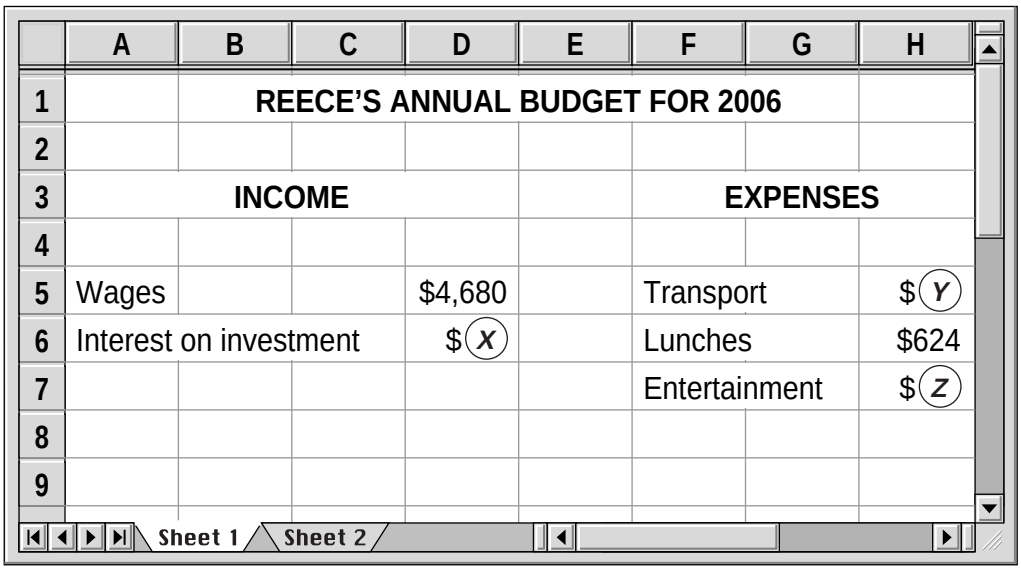
His expected income is:

- $90 every week as a swimming coach
- Interest earned from an investment of $5000 at a rate of 4% per annum.

His planned expenses are:

- $30 every week on transport
- $12 every week on lunches
- $48 every month on entertainment.

Reece will save his remaining income. He uses the spreadsheet below for his budget.



	A	B	C	D	E	F	G	H
1		REECE'S ANNUAL BUDGET FOR 2006						
2								
3		INCOME				EXPENSES		
4								
5	Wages			$4,680		Transport		$(Y)
6	Interest on investment			$(X)		Lunches		$624
7						Entertainment		$(Z)
8								
9								

Sheet 1 Sheet 2

(i) Determine the values of (X), (Y) and (Z). (Assume there are exactly 52 weeks in a year.) **3**

(ii) At the beginning of 2006, Reece starts saving. **2**

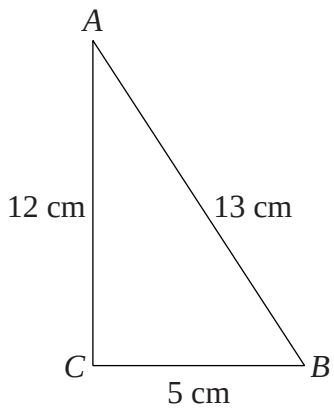
Will Reece have saved enough money during 2006 for a deposit of $2100 on a car if he keeps to his budget? Justify your answer with suitable calculations.

Question 25 continues on page 18

Marks

Question 25 (continued)

(b)



NOT TO SCALE

(i) Use Pythagoras' theorem to show that ΔABC is a right-angled triangle. **1**

(ii) Calculate the size of $\angle ABC$ to the nearest degree. **2**

(c) Robyn plays a game in which she randomly chooses one of these five cards. She plays the game 60 times, replacing the card after each game.



(i) How many times would she expect to win \$4? **1**

(ii) What is the financial expectation of the game? **2**

(iii) Another card is added to the game with '**Win nothing \$0**' written on it. Robyn claims that the financial expectation will not change. **2**

Do you agree? Justify your answer with suitable calculations.

End of Question 25

Marks

Question 26 (13 marks) Use a SEPARATE writing booklet.

(a) A printing machine worth \$150 000 is bought in December 2005. **2**

In December each year, beginning in 2006, the value of the printing machine is depreciated by 10% using the declining balance method of depreciation.

In which year will the depreciated value first fall below \$120 000?

(b) Rod is saving for a holiday. He deposits \$3600 into an account at the end of every year for four years. The account pays 5% per annum interest, compounding annually.

The table shows future values of an annuity of \$1.

Future values of an annuity of \$1

End of year	*Interest rate*				
	1%	**2%**	**3%**	**4%**	**5%**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(i) Use the table to find the value of Rod's investment at the end of four years. **2**

(ii) How much interest does Rod earn on his investment over the four years? **2**

(c) The weights of boxes of Brekky Bicks are normally distributed. The mean is 754 grams and the standard deviation is 2 grams.

(i) What is the z-score of a box of Brekky Bicks with a weight of 754 g? **1**

(ii) What is the weight of a box that has a z-score of –1? **1**

(iii) Brekky Bicks boxes are labelled as having a weight of 750 g. **2**

What percentage of boxes will have a weight less than 750 g?

Question 26 continues on page 20

Question 26 (continued)

(d) Peta borrows \$28 000 from a credit union at 6% per annum compounding monthly. She will repay the money over nine years.

Peta uses the formula:

$$28\,000 = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

to calculate her monthly repayment, M.

(i) Rewrite the formula, showing the correct substitutions for r and n. **2**

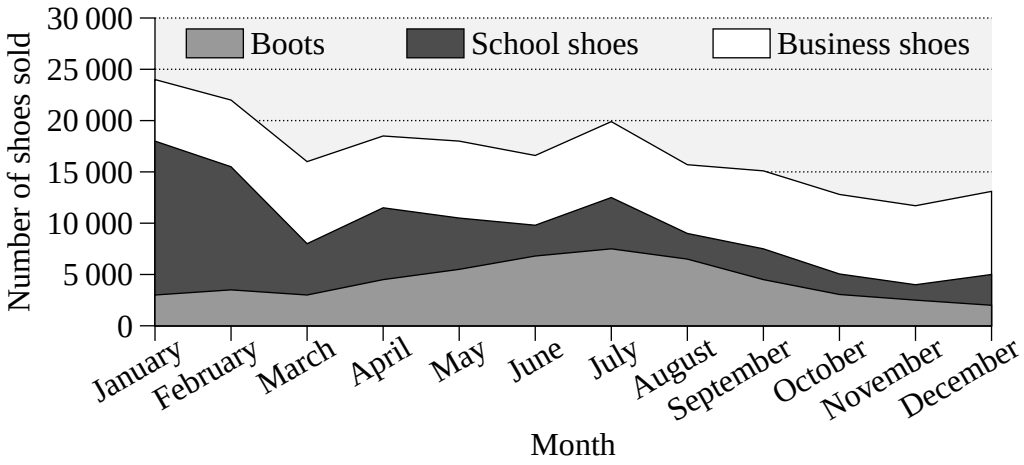
(ii) Calculate Peta's monthly repayment. **1**

End of Question 26

Marks

Question 27 (13 marks) Use a SEPARATE writing booklet.

(a) The area graph shows sales figures for Shoey's shoe store.

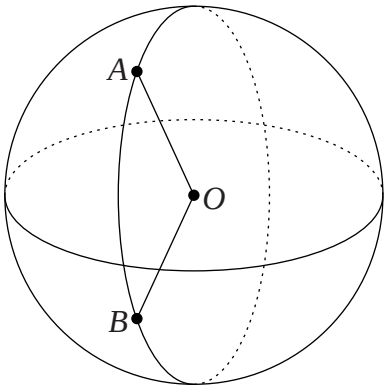


(i) Approximately how many school shoes were sold in January? **1**

(ii) For which month does the graph indicate that the same number of school shoes and business shoes was sold? **1**

(iii) Identify ONE trend in this graph, and suggest a valid reason for this trend. **2**

(b) This diagram represents Earth. O is at the centre, and A and B are points on the surface. **2**



A: 35°N 20°E

B: 8°S 20°E

Calculate the distance from A to B along the great circle through A and B. Give your answer in nautical miles.

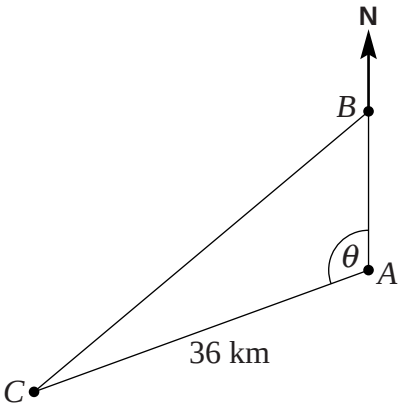
(Radius of Earth is 6400 km. 1.852 km = 1 nautical mile)

Question 27 continues on page 22

Marks

Question 27 (continued)

(c)



NOT
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SCALE

The bearing of C from A is 250° and the distance of C from A is 36 km.

(i) Explain why θ is 110°. **1**

(ii) If B is 15 km due north of A, calculate the distance of C from B, correct to the nearest kilometre. **3**

(d) Nine students were selected at random from a school, and their ages were recorded.

Ages		
12	11	16
14	16	15
14	15	14

(i) What is the sample standard deviation, correct to two decimal places? **2**

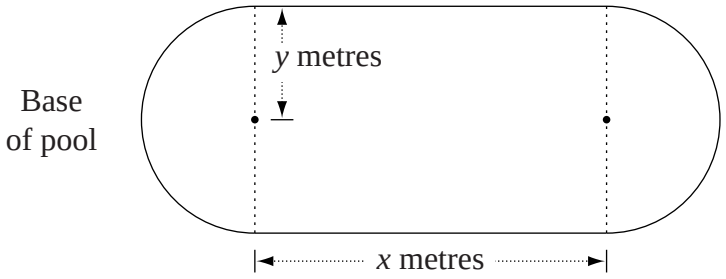
(ii) Briefly explain what is meant by the term *standard deviation*. **1**

End of Question 27

Marks

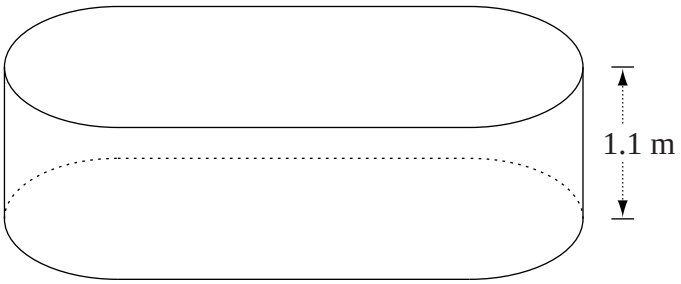
Question 28 (13 marks) Use a SEPARATE writing booklet.

(a) The Mitchell family has moved to a new house which has an empty swimming pool. The base of the pool is in the shape of a rectangle, with a semicircle on each end.



(i) Explain why the expression for the area of the base of the pool is $2xy + \pi y^2$. **1**

(ii) **4**



The pool is 1.1 metres deep.

The sides and base of the pool are covered in tiles. If $x = 6$ and $y = 2.5$, find the total area covered by tiles. (Give your answer correct to the nearest square metre.)

(iii) Before filling the pool, the Mitchells need to install a new shower head, which saves 6 litres of water per minute. **2**

The shower is used 5 times every day, for 3 minutes each time.

If the charge for water is \$1.013 per kilolitre, how much money would be saved in one year by using this shower head? (Assume there are 365 days in a year.)

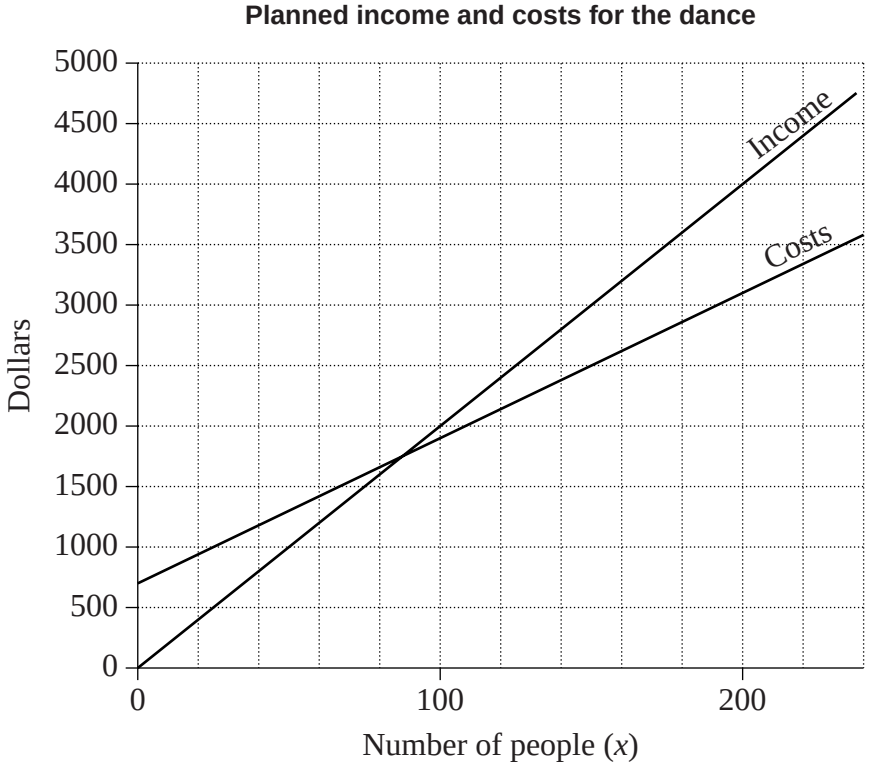
Question 28 continues on page 24

Question 28 (continued)

(b) Sue and Mikey are planning a fund-raising dance. They can hire a hall for $400 and a band for $300. Refreshments will cost them $12 per person.

(i) Write a formula for the cost ($\$C$) of running the dance for x people. **1**

The graph shows planned income and costs when the ticket price is $20.



(ii) Estimate the minimum number of people needed at the dance to cover the costs. **1**

(iii) How much profit will be made if 150 people attend the dance? **1**

(iv) Sue and Mikey plan to sell 200 tickets. They want to make a profit of $1500. **3**

What should be the price of a ticket, assuming all 200 tickets will be sold?

End of paper

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FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$I = Prn$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$A = P(1+r)^n$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$S = V_0 - Dn$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$S = V_0(1-r)^n$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$y = mx + b$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$