BOARD OF STUDIES
NEW SOUTH WALES

2003

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–23

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



A ⊗ B ⊗ C ○ D ○

1 A number of men and women were surveyed at a railway station. They were asked whether or not they were travelling to work. The table shows the results.

	Going to work	*Not going to work*
Men	64	24
Women	60	42

How many men were surveyed?

(A) 64

(B) 88

(C) 124

(D) 190

2 Simplify $3y^3 \div 12y^2$.

(A) $4y$

(B) $\frac{4}{y}$

(C) $\frac{1}{4y}$

(D) $\frac{y}{4}$

3 Dora works for \$9.60 per hour for eight hours each day on Thursday and Friday. On Saturday she works for six hours at time-and-a-half.

How much does Dora earn in total for Thursday, Friday and Saturday?

(A) \$192.00

(B) \$211.20

(C) \$240.00

(D) \$316.80

4 If $d = \sqrt{\frac{h}{5}}$, what is the value of d, correct to one decimal place, when $h = 28$?

(A) 1.1

(B) 2.4

(C) 2.8

(D) 5.6

5 Jim bought a new car at the beginning of 2001 for $40 000. At the end of 2001 the value of the car had depreciated by 30%. In 2002 the value of the car depreciated by 25% of the value it had at the end of 2001.

What was the value of the car at the end of 2002?

(A) $18 000

(B) $19 600

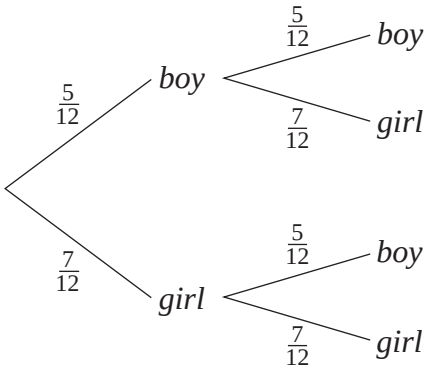
(C) $21 000

(D) $22 000

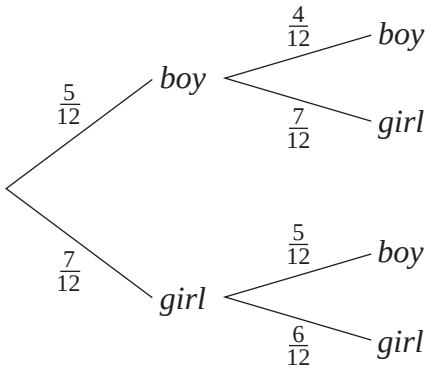
6 From 5 boys and 7 girls, two children will be chosen at random to work together on a project.

Which of the following probability trees could be used to determine the probability of choosing a boy and a girl?

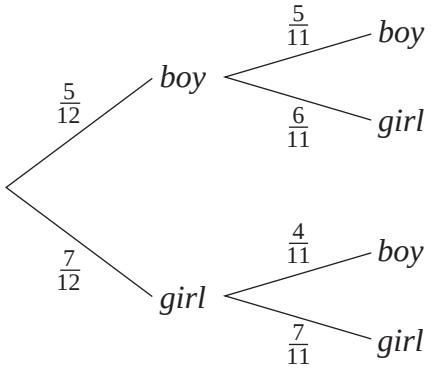
(A)



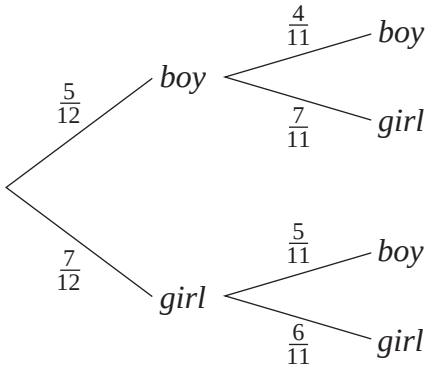
(B)



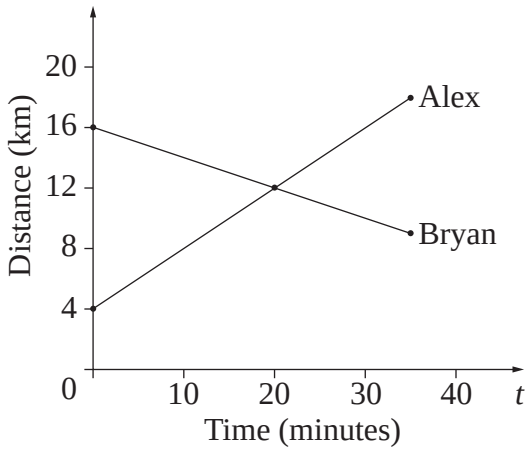
(C)



(D)



7 At the same time, Alex and Bryan start riding towards each other along a road. The graph shows their distances (in kilometres) from town after t minutes.



How many kilometres has Alex travelled when they meet?

(A) 4

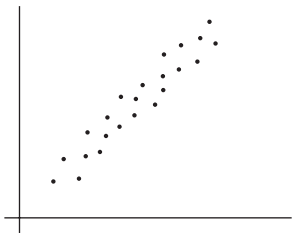
(B) 8

(C) 12

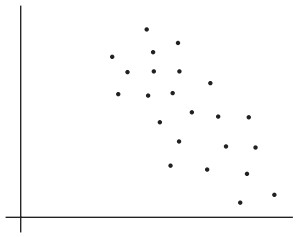
(D) 20

8 Which scatterplot shows a low (weak) positive correlation?

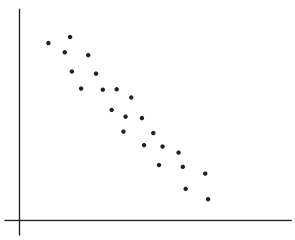
(A)



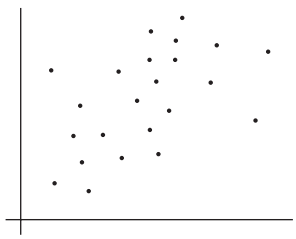
(B)



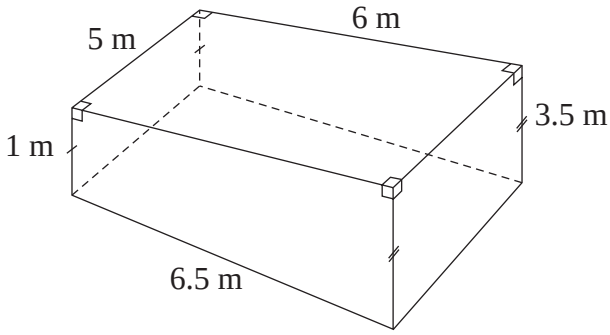
(C)



(D)



9 A swimming pool has a length of 6 m and a width of 5 m. The depth of the pool is 1 m at one end and 3.5 m at the other end, as shown in the diagram.



NOT TO SCALE

What is the volume of this pool in cubic metres?

(A) 67.5

(B) 105

(C) 109.375

(D) 113.75

10 Kathmandu is 30° west of Perth. Using the longitude difference, what is the time in Kathmandu when it is noon in Perth?

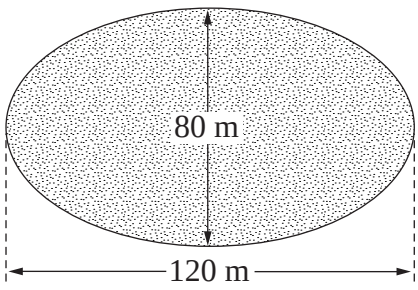
(A) 10:00 am

(B) 11:30 am

(C) 12:30 pm

(D) 2:00 pm

11 The council wants to put new grass on a park which is in the shape of an ellipse. If grass costs $7.50 per square metre, what is the total cost to the nearest dollar?



NOT TO SCALE

(A) $7 540

(B) $30 159

(C) $56 549

(D) $226 195

Use the following information to answer Questions 12 and 13.

Joy asked the students in her class how many brothers they had. The answers were recorded in a frequency table as follows.

Number of brothers	*Frequency*
0	5
1	10
2	3
3	1
4	1

12 What is the mean number of brothers?

(A) 1.15

(B) 2

(C) 2.3

(D) 4

13 One of the students is chosen at random. What is the probability that this student has at least two brothers?

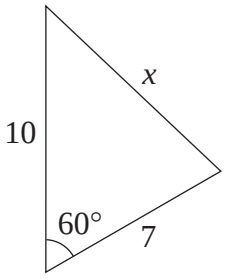
(A) 0.10

(B) 0.15

(C) 0.25

(D) 0.75

14



NOT TO SCALE

Which equation should be used to obtain the value of x in this triangle?

(A) $\dfrac{x}{\sin 60°} = \dfrac{7}{\sin 10°}$

(B) $x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \cos 60°$

(C) $\cos 60° = \dfrac{x^2 + 10^2 - 7^2}{2 \times 10 \times 7}$

(D) $x^2 = 10^2 - 7^2$

15 Kylie and Danny work in a music store. The weekly wage $\$W$ of an employee at the store is given by

$$W = 0.75\,n + 50,$$

where n is the number of CDs the employee sells.

If Kylie sells two more CDs than Danny in one week, how much more will she earn?

(A) \$0.75

(B) \$1.50

(C) \$50.75

(D) \$51.50

16 Pauline calculates the present value (N) of an annuity. The interest rate is 4% per annum, compounded monthly. In five years the future value will be \$100 000.

Which calculation will result in the correct answer?

(A) $N = \dfrac{100\,000}{(1+0.04)^5}$

(B) $N = \dfrac{100\,000}{(1+0.04\div 12)^5}$

(C) $N = \dfrac{100\,000}{(1+0.04)^{60}}$

(D) $N = \dfrac{100\,000}{(1+0.04\div 12)^{60}}$

17 If an electrical current varies inversely with resistance, what is the effect on the current when the resistance is doubled?

(A) The current is doubled.

(B) The current is exactly the same.

(C) The current is halved.

(D) The current is squared.

18 George measures the breadth and length of a rectangle to the nearest centimetre. His answers are 10 cm and 15 cm. Between what lower and upper values must the actual area of the rectangle lie?

(A) 10×15 cm^2 (lower) and 11×16 cm^2 (upper)

(B) 10×15 cm^2 (lower) and 10.5×15.5 cm^2 (upper)

(C) 9.5×14.5 cm^2 (lower) and 10×15 cm^2 (upper)

(D) 9.5×14.5 cm^2 (lower) and 10.5×15.5 cm^2 (upper)

19 The roof of the Sydney Opera House is covered with 1.056 million tiles. If each tile covers 175 cm^2, what area is covered by the tiles?

(A) 184.8 m^2

(B) 18 480 m^2

(C) 184 800 m^2

(D) 1 848 000 m^2

20 Iliana buys several items at the supermarket. The docket for her purchases is shown.

XYZ Supermarket

27/04/03 12.53

MILK 1L	$1.19
* PEPSI 1.25L	$1.29
* DISINFECTANT	$7.23
* TEA TREE OIL	$4.13
SPINACH	$1.64
SOUP	$1.57
TOTAL	$17.05

10% GST INCLUDED
IN COST OF TAXABLE ITEMS

* = TAXABLE ITEMS

What is the amount of GST included in the total?

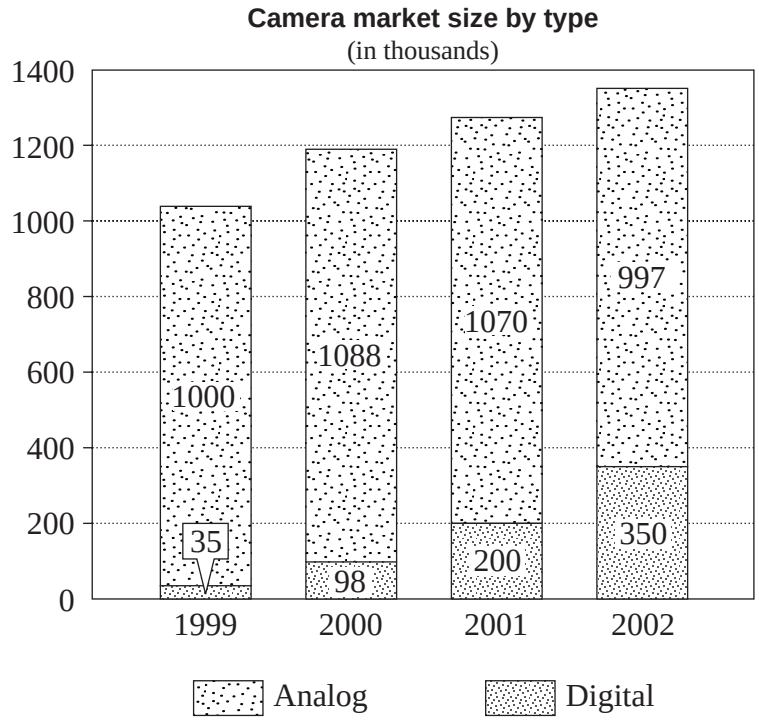
(A) $1.15

(B) $1.27

(C) $1.55

(D) $1.71

21 The graph below shows the numbers of the two major types of cameras, analog and digital, sold in Australia in the years 1999–2002.



Reproduced with the permission of the *Sydney Morning Herald*

In 2001, what percentage of the cameras sold were digital cameras? (To the nearest per cent.)

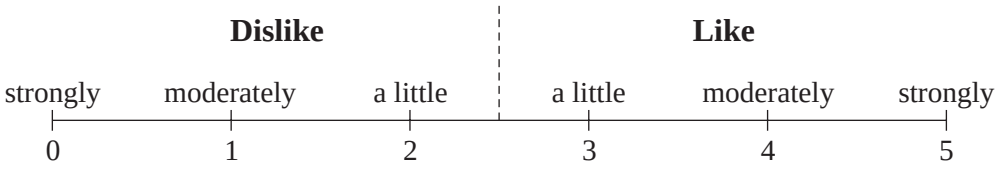
(A) 16%

(B) 19%

(C) 23%

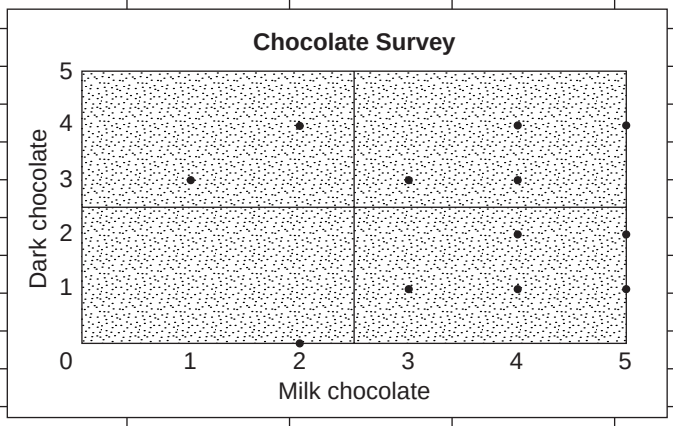
(D) 84%

22 Charlie surveyed 12 school friends to find out their preferences for chocolate. They were asked to indicate their liking for milk chocolate on the following scale.



They were also asked to do this for dark chocolate. Charlie displayed the results in a spreadsheet and graph as shown below.

	A	B	C	D	E	F	
1							
2			**Chocolate Survey Results**				
3							
4	Milk chocolate	Dark chocolate					
5	4	2					
6	3	3					
7	4	3					
8	2	4					
9	5	1					
10	4	1					
11	2	0					
12	1	3					
13	5	4					
14	4	4					
15	3	1					
16	5	2					
17							



Charlie assumes that these 12 students are representative of the 600 students at the school.

What is Charlie's estimate of the number of students in the school who **like** milk chocolate but **dislike** dark chocolate?

(A) 50

(B) 200

(C) 250

(D) 450

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

Question 23 (13 marks) Use a SEPARATE writing booklet.

(a) Keryn is designing a new watering system for the shrubs in her garden. She knows that each shrub needs 1.2 litres of water per day. To minimise evaporation, Keryn designs a system to drip water into a tube that takes the water to the roots.

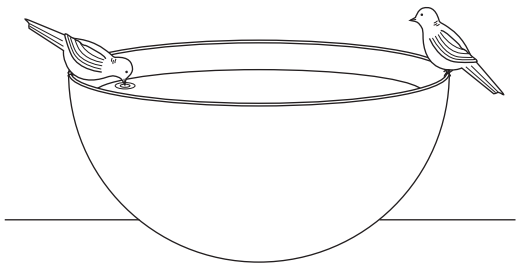
(i) What is the number of litres of water required daily for 13 shrubs? **1**

(ii) Keryn pays 94.22 cents per kilolitre for water. Calculate the total cost of watering 13 shrubs for one week. **1**

(iii) Keryn knows that $1\text{ mL} = 15$ drops. Find the number of drops that one shrub needs daily. **1**

(iv) How many drops per minute are required for one shrub if the system is in use for 10 hours per day? **1**

(b) In her garden, Keryn has a birdbath in the shape of a hemisphere (half a sphere). The internal diameter is 45 cm. **2**



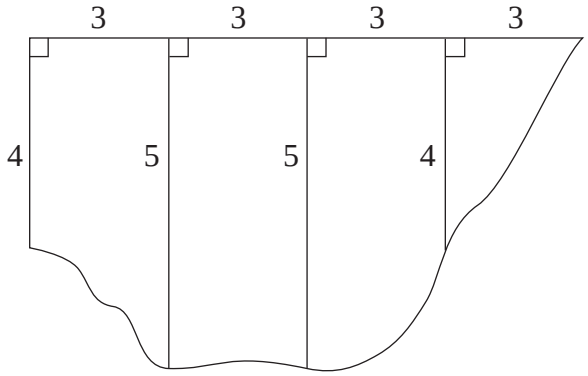
NOT TO SCALE

What is the internal surface area of this birdbath? (Give your answer to the nearest square centimetre.)

Question 23 continues on page 13

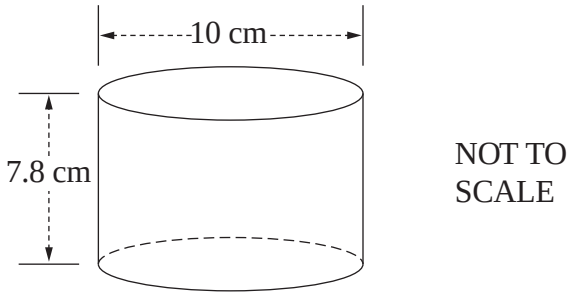
Question 23 (continued)

(c) A river has a cross-section as shown below, with measurements in metres. **2**



Calculate the area of this cross-section using Simpson's rule.

(d) Peta is designing an eight-cylinder racing engine. Each cylinder has a bore (diameter) of 10.0 cm and a stroke (height) of 7.8 cm, as shown below.



(i) Calculate the volume of each cylinder, correct to the nearest cubic centimetre. **2**

(ii) The capacity of the engine is the sum of the capacities of the eight cylinders. Does Peta's engine meet the racing requirement that the capacity should be under 5 litres? Justify your answer with a mathematical calculation. **3**

End of Question 23

Marks

Question 24 (13 marks) Use a SEPARATE writing booklet.

(a) Minh invests $24 000 at an interest rate of 4.75% per annum, compounded monthly. What is the value of the investment after 3 years? **2**

(b) Vicki earns a taxable income of $58 624 from her job with an insurance company. She pays $14 410.80 tax on this income.

(i) Vicki has a second job which pays $900 gross income per month. **1**

What is Vicki's total annual taxable income from both jobs, assuming that she has no allowable tax deductions?

(ii) Use the tax table below to calculate the total tax payable on her income from both jobs. **2**

Taxable income	*Tax payable*
$0–$6 000	NIL
$6 001–$22 000	18 cents for each $1 over $6 000
$22 001–$55 000	$2 880 plus 30 cents for each $1 over $22 000
$55 001–$66 000	$12 780 plus 45 cents for each $1 over $55 000
$66 001 and over	$17 730 plus 48 cents for each $1 over $66 000

(iii) Show that Vicki's monthly net income from her second job is $486.44. **3**

(iv) Vicki plans to take a holiday in two years time which she estimates will cost $12 000. At the end of each month, Vicki invests the net income from her second job in an account which pays 4% per annum, compounded monthly. **2**

Will she have enough in this account, immediately after the twenty-fourth payment, to pay for her holiday? Justify your answer with calculations.

Question 24 continues on page 15

Question 24 (continued)

(c) Zoë plans to borrow money to buy a car and considers the following repayment guide: **3**

Fortnightly car loan repayment guide

Amount borrowed	*Length of loan*		
	1 year	*2 years*	*3 years*
($)	($)	($)	($)
10 000	410	217	153
10 500	430	228	161
11 000	451	239	168
11 500	471	249	176
12 000	492	260	183
12 500	512	271	191
13 000	532	282	199
13 500	553	293	206
14 000	573	303	214
14 500	594	314	221
15 000	614	325	229
15 500	635	336	237
16 000	655	347	244

Zoë wishes to borrow $15 500 and pay back the loan in fortnightly instalments over two years.

What is the flat rate of interest per annum on this loan?

End of Question 24

Marks

Question 25 (13 marks) Use a SEPARATE writing booklet.

(a) A census was conducted of the 33 171 households in Sunnytown. Each household was asked to indicate the number of cars registered to that household. The results are summarised in the following table.

Number of cars	*Frequency*
0	2 735
1	12 305
2	13 918
3	3 980
4	233
Total	33 171

(i) (1) Determine the mode number of cars in a household. **1**

(2) Explain what is meant by *the mode number of cars in a household*. **1**

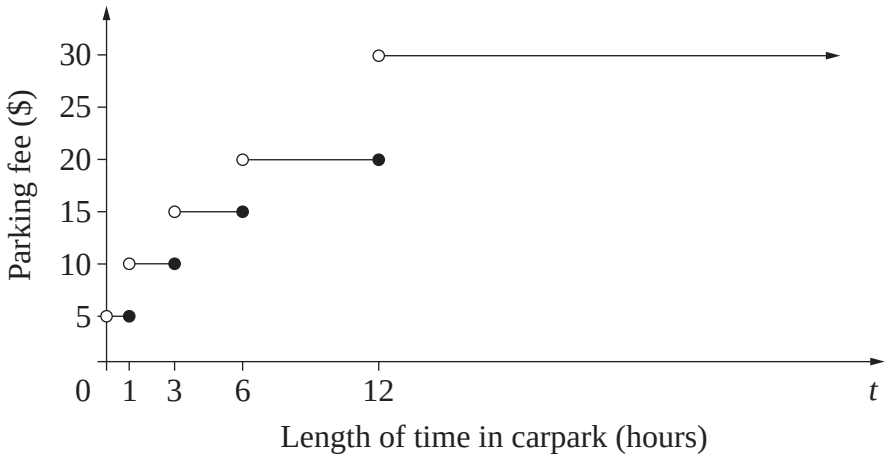
(ii) Sunnytown Council issued a 'free parking' sticker for each car registered to a household in Sunnytown. **2**

How many parking stickers were issued?

(iii) The council represented the results of the census in a sector graph. **1**

What is the angle in the sector representing the households with no cars? Give your answer to the nearest degree.

(iv) Visitors to Sunnytown Airport have to pay for parking. The following step graph shows the cost of parking for t hours. **1**



What is the cost for a car that is parked one evening from 6 pm to 8:30 pm?

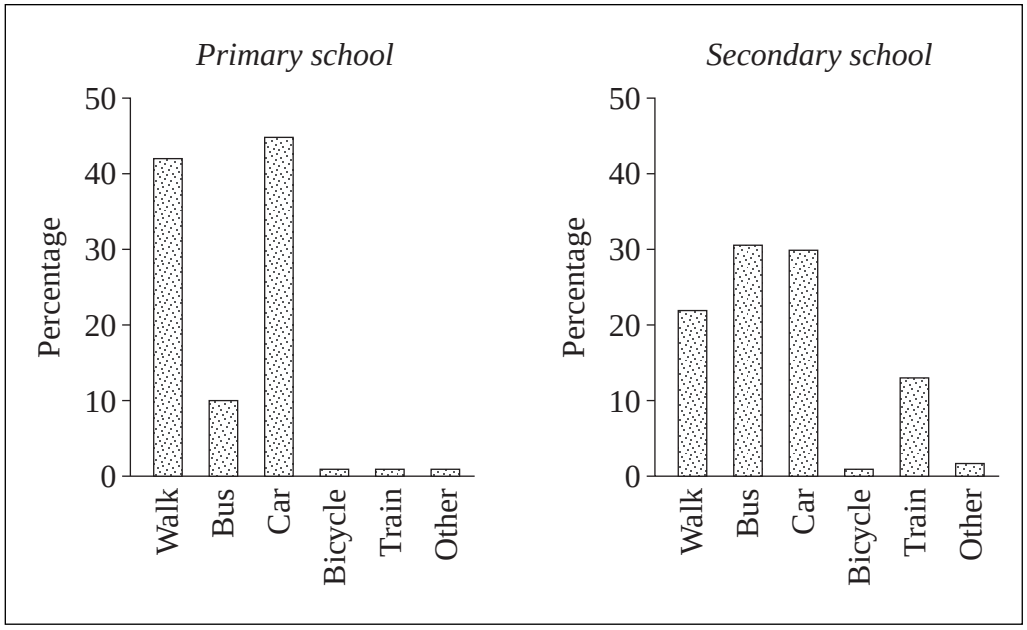
Question 25 continues on page 17

Question 25 (continued)

(b) Equal large numbers of primary and secondary school students in a city were surveyed about their method of travel to school.

The results are summarised in the relative frequency column graphs below.

Method of travel to school



(i) Describe TWO differences in the method of travel between these primary school and secondary school students. **2**

(ii) Suggest a possible reason for ONE of these differences. **1**

(iii) There were 25 000 primary school students surveyed. How many of these students travelled to school by bus? **1**

(c) Results for an aptitude test are given as z-scores. In this test, Hardev gains a z-score of 1.

(i) Interpret Hardev's score with reference to the mean and standard deviation of the test. **2**

(ii) The scores for the test are normally distributed. What proportion of people sitting the test obtain a higher score than Hardev? **1**

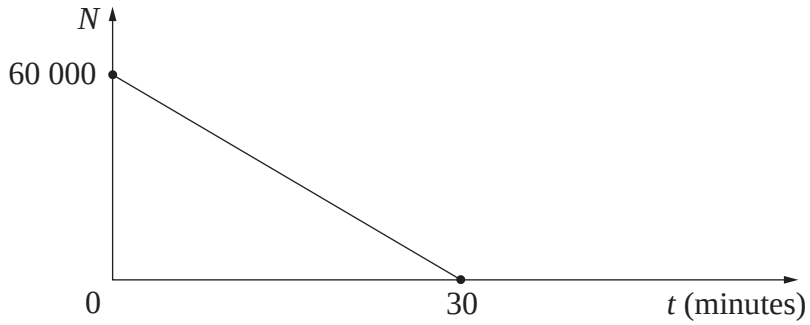
End of Question 25

Marks

Question 26 (13 marks) Use a SEPARATE writing booklet.

(a) At a World Cup rugby match, the stadium was filled to capacity for the entire game. At the end of the game, people left the stadium at a constant rate.

The graph shows the number of people (N) in the stadium t minutes after the end of the game.



The equation of the line is of the form $N = a - bt$, where a and b are constants.

(i) Write down the value of a, and give an explanation of its meaning. **2**

(ii) (1) Calculate the value of b. **1**

(2) What does the value of b represent in this situation? **1**

(iii) Rearrange the formula $N = a - bt$ to make t the subject. **2**

(iv) How long did it take 10 000 people to leave the stadium? **1**

(v) Copy or trace the graph of N against t shown above into your answer booklet. **2**

Suppose that 15 minutes after the end of the game, several of the exits had been closed, reducing the rate at which people left.

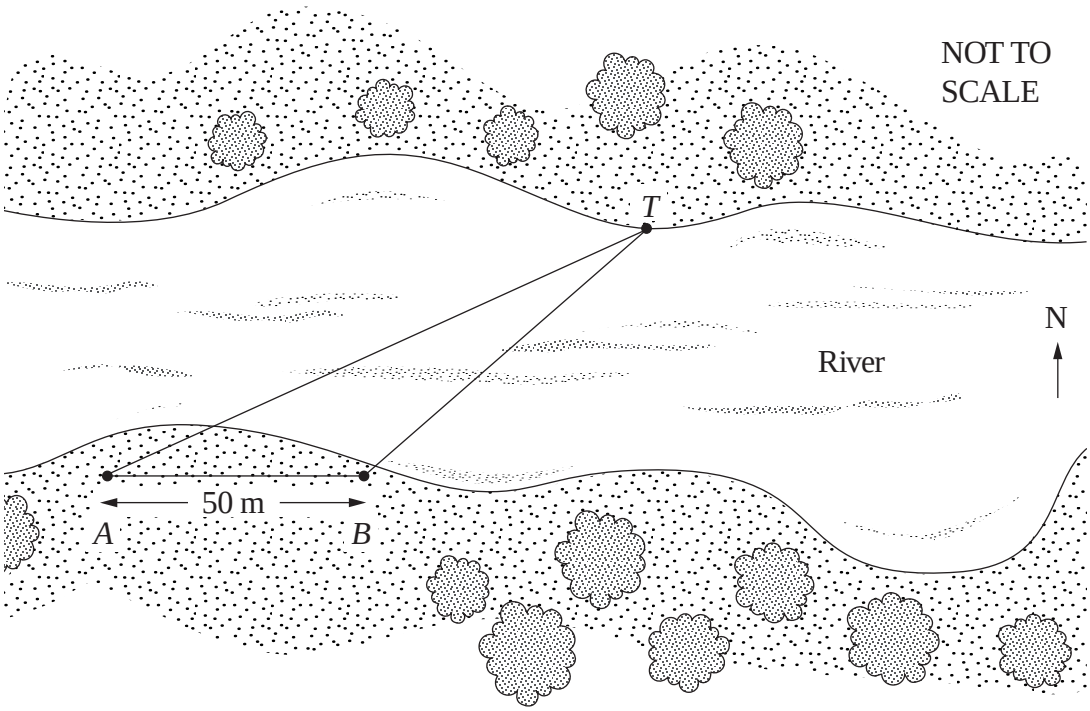
On the same axes, carefully draw another graph of N against t that could represent this new situation. Your new graph should show N from $t = 15$ until all the people had left the stadium.

Question 26 continues on page 19

Marks

Question 26 (continued)

(b)



In the diagram above, the following measurements are given:

$\angle TAB = 30°$.
B is 50 m due east of A.
The bearing of T from B is 020°.

Copy or trace ΔABT into your answer booklet.

(i) Explain why $\angle ABT$ is 110°. **1**

(ii) Calculate the distance BT (to the nearest metre). **3**

End of Question 26

Marks

Question 27 (13 marks) Use a SEPARATE writing booklet.

(a) A celebrity mathematician, Karl, arrives in Sydney for one of his frequent visits. Karl is known to stay at one of three Sydney hotels.

Hotel *X* is his favourite, and he stays there on 50% of his visits to Sydney. When he does not stay at Hotel *X*, he is equally likely to stay at Hotels *Y* or *Z*.

(i) What is the probability that he will stay at Hotel *Z*? **1**

(ii) On his first morning in Sydney, Karl always flips a coin to decide if he will have a cold breakfast or a hot breakfast. If the coin comes up heads he has a cold breakfast. If the coin comes up tails he has a hot breakfast.

(1) List all the possible combinations of hotel and breakfast choices. **2**

(2) Give a brief reason why these combinations are not all equally likely. **1**

(3) Calculate the probability that Karl stays at Hotel *Z* and has a cold breakfast. **1**

(b) Two unbiased dice are thrown. The dice each have six faces. The faces are numbered 1, 2, 3, 4, 5 and 6.

(i) What is the probability that neither shows a 6? **1**

(ii) Dale plays a game with these dice. There is no entry fee. **3**

When the dice are thrown:

- Dale wins $20 if both dice show a 6.
- He wins $2 if there is only one 6.
- He loses $2 if neither shows a 6.

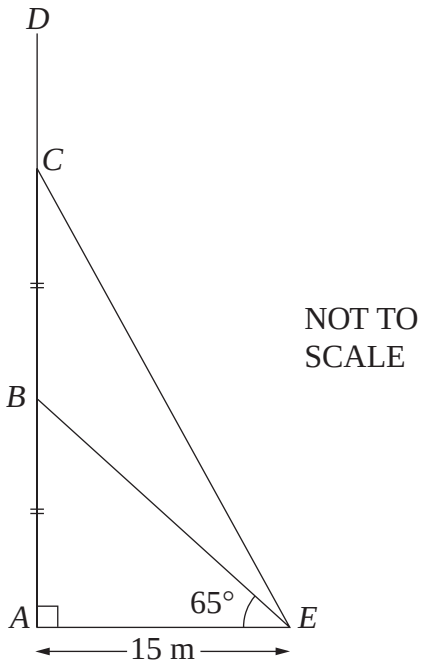
What is his financial expectation from this game?

Question 27 continues on page 21

Marks

Question 27 (continued)

(c) The diagram shows a radio mast AD with two of its supporting wires, BE and CE. The point B is half-way between A and C.



(i) Calculate the height AB in metres, correct to one decimal place. **2**

(ii) Calculate the distance CE in metres, correct to one decimal place. **2**

End of Question 27

Marks

Question 28 (13 marks) Use a SEPARATE writing booklet.

(a) Sandra is on holiday in Mexico and she plans to buy some silver jewellery. Various silver pendants are on sale. The cost varies directly with the square of the length of the pendant. A pendant of length 30 mm costs 130 Mexican pesos. **2**

How much does a pendant of length 40 mm cost? (Answer correct to the nearest Mexican peso.)

(b) In 2002, the population of Mexico was approximately 103 400 000.

(i) The growth rate of Mexico's population is estimated to be 1.57% per annum. If y represents the estimated number of people in Mexico at a time x years after 2002, write a formula relating x and y in the form $y = b(a^x)$. Use appropriate values for a and b in your formula. **2**

(ii) Using your formula, or otherwise, find an estimate for the size of Mexico's population two years after 2002. Express your answer to the nearest thousand. **2**

(c) (i) While Sandra is on holiday she visits countries where the Fahrenheit temperature scale is used. She knows that the correct way to convert from Celsius to Fahrenheit is: **1**

'Multiply the Celsius temperature by 1.8, then add 32.'

Find the value of A in the following table.

Celsius	*Fahrenheit*
5	41
15	59
A	77

(ii) Peter uses the following method to approximate the conversion from Celsius to Fahrenheit: **2**

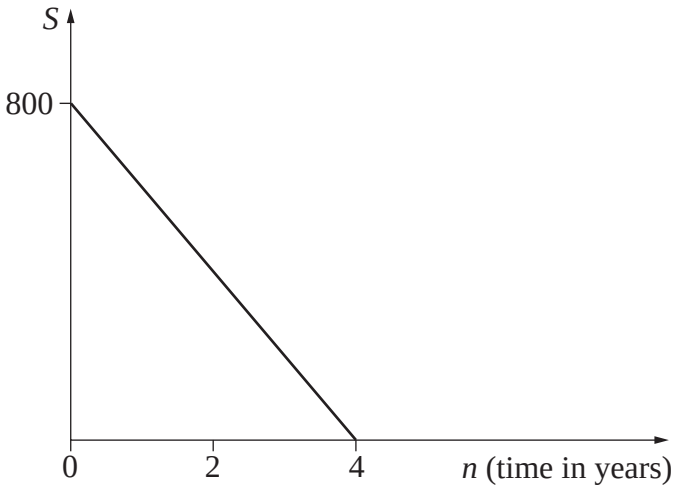
'Add 12 to the Celsius temperature, then double your result.'

Express Peter's rule as an algebraic equation. Use C for the Celsius temperature and F for the approximate Fahrenheit temperature.

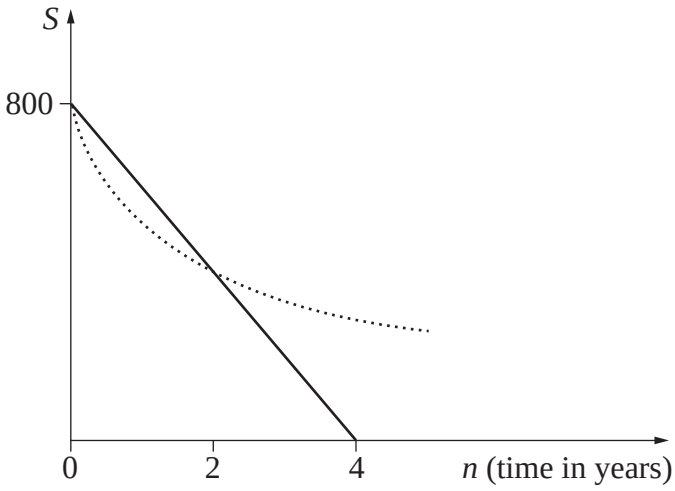
Question 28 continues on page 23

Question 28 (continued)

(d) While Sandra is on holidays she buys a digital camera for $800. Using the straight-line method of depreciation, the salvage value, S, of the camera will be zero in four years time, as shown on the graph below. **4**



Using the declining balance method of depreciation, at a rate of R% per annum, the salvage value is represented by the dotted curve on the graph below.



NOT TO SCALE

Both methods give the same salvage value after two years. Find the value of R.

End of paper

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FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$I = Prn$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$A = P(1+r)^n$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$S = V_0 - Dn$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$S = V_0(1-r)^n$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$y = mx + b$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$