BOARD OF STUDIES
NEW SOUTH WALES

2002

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total marks – 100

Section I Pages 2–11

22 marks

- Attempt Questions 1–22
- Allow about 30 minutes for this section

Section II Pages 12–23

78 marks

- Attempt Questions 23–28
- Allow about 2 hours for this section

Section I

22 marks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

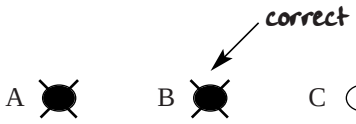
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



A ⊗ B ⊗ C ○ D ○

1 The results of a geography test are displayed in a stem-and-leaf plot.

2	3 3 4 5
3	5 6 6 6 7 7
4	1 2
5	0 0 4

What is the range of the data?

(A) 15

(B) 27

(C) 29

(D) 31

2 Which of the following is the correct simplification of $8x^3 - 5x^3$?

(A) $3x^6$

(B) $3x^3$

(C) $3x$

(D) 3

3 The Great Pyramid of Egypt has a square base of side 230 m. Its perpendicular height is 135 m.

What is the volume of the pyramid?

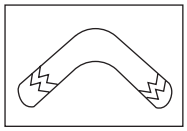
(A) $10\,350\ \text{m}^3$

(B) $1\,397\,250\ \text{m}^3$

(C) $2\,380\,500\ \text{m}^3$

(D) $7\,141\,500\ \text{m}^3$

4



Boomerang Council

RATES AND CHARGES NOTICE
1st July 2001 to 30th June 2002

Z Smith
14 The Crescent
Boomerang

Customer Ref No	1111111
Due Date	31/08/2001
Issue Date	17/07/2001
Assessment No	11111-00000-1

PROPERTY RATING CATEGORY	VALUER GENERAL'S LAND VALUE	VALUATION BASE DATE
Residential	$377 000	01/07/2000

RATES & CHARGES	RATEABLE VALUE OR QTY	CENTS IN $ OR CHARGE	AMOUNT
Residential rate	$377 000	0.272950	
Waste Mgt Chg 80 litres	1	$195.00	$195.00
		TOTAL PAYABLE	

Payments made after 29/06/2001 will not be shown on this notice.

What is the total payable?

(A) $1029.02

(B) $1224.02

(C) $102 902.15

(D) $103 097.15

5 Sarah has two packets of jelly beans. Each packet contains one black and five yellow jelly beans. Sarah takes one jelly bean from each packet without looking.

What is the probability that both of the jelly beans are black?

(A) $\frac{1}{36}$

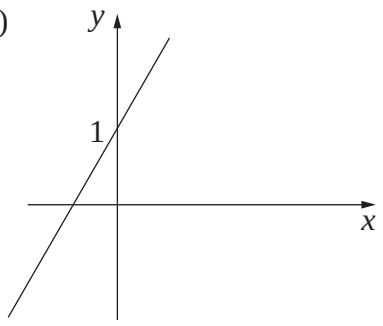
(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

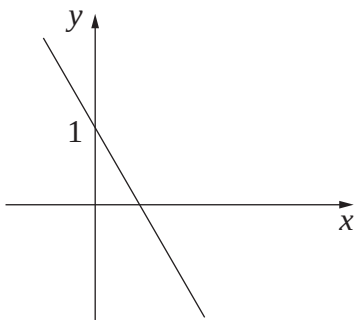
(D) $\frac{1}{3}$

6 Which one of the following could be the graph of $y = 3x + 1$?

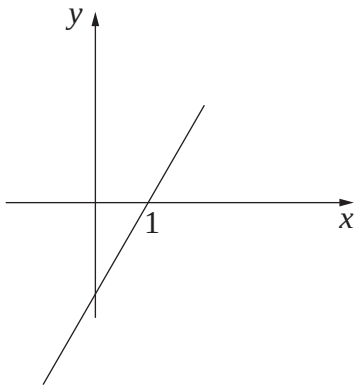
(A)



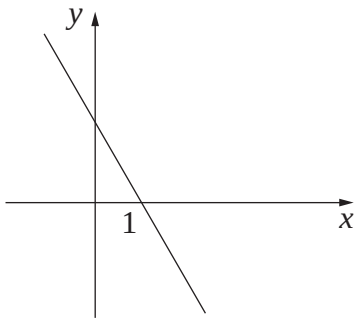
(B)



(C)



(D)



7 Richard has 2000 shares with a current market value of $4.80 each. During the past twelve months, Richard received a total dividend of $240.

What is the current dividend yield on these shares?

(A) 0.025%

(B) 2%

(C) 2.5%

(D) 40%

8 Results for an aptitude test are given as *z*-scores. In this test Di gained a *z*-score of 3. The test has a mean of 55 and a standard deviation of 6.

What was Di's actual mark in this test?

(A) 57

(B) 58

(C) 64

(D) 73

9 The table shows monthly repayments for loans over 30 years.

		Loan amount			
		$100 000	**$150 000**	**$200 000**	**$250 000**
Interest rate per annum	**5.0%**	$537	$806	$1074	$1343
	5.5%	$568	$852	$1136	$1420
	6.0%	$600	$900	$1200	$1499
	6.5%	$633	$949	$1265	$1581
	7.0%	$666	$998	$1331	$1664
	7.5%	$700	$1049	$1399	$1749

James borrows $200 000 over a period of 30 years at 6.5% per annum. Repayments are to be made monthly according to the table.

How much would James repay over 30 years if the interest rate were to remain the same?

(A) $1265

(B) $37 950

(C) $390 000

(D) $455 400

10 The game of Beach Quidditch is played with a large hollow spherical ball made from gold vinyl. The diameter of the ball is 1.2 metres.

If the vinyl costs $32 per square metre, which of the following is closest to the cost of the vinyl for one ball?

(A) $29

(B) $145

(C) $232

(D) $579

11 At the end of 2000, Zara purchased a new computer for $4999. Use the declining balance method to determine the value of the computer at the end of 2002, assuming a depreciation rate of 40% per annum. (Answer to the nearest dollar.)

(A) $800

(B) $1000

(C) $1800

(D) $2000

12 In the town of Burrow the ages of the residents are normally distributed. The mean age is 40 years and the standard deviation is 12 years.

Approximately what percentage of the residents are younger than 52?

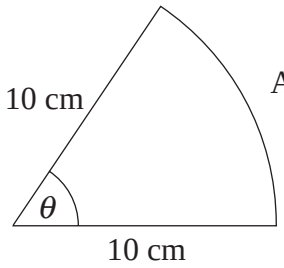
(A) 16%

(B) 32%

(C) 68%

(D) 84%

13 This is a sketch of a sector of a circle.



Arc length = 12 cm

NOT TO SCALE

Find the value of θ to the nearest degree.

(A) 47°

(B) 48°

(C) 68°

(D) 69°

14 Arrange the numbers 5.6×10^{-2}, 4.8×10^{-1}, 7.2×10^{-2} from smallest to largest.

(A) 5.6×10^{-2}, 7.2×10^{-2}, 4.8×10^{-1}

(B) 4.8×10^{-1}, 5.6×10^{-2}, 7.2×10^{-2}

(C) 7.2×10^{-2}, 5.6×10^{-2}, 4.8×10^{-1}

(D) 4.8×10^{-1}, 7.2×10^{-2}, 5.6×10^{-2}

15 Calculate the present value of an annuity in which \$1200 is invested at the end of every year for ten years and interest is paid annually at a rate of 5% per annum. (Answer to the nearest dollar.)

(A) \$1922

(B) \$9266

(C) \$15 093

(D) \$30 654

16 If $w = 2y^3 - 1$, what is the value of y when $w = 13$?

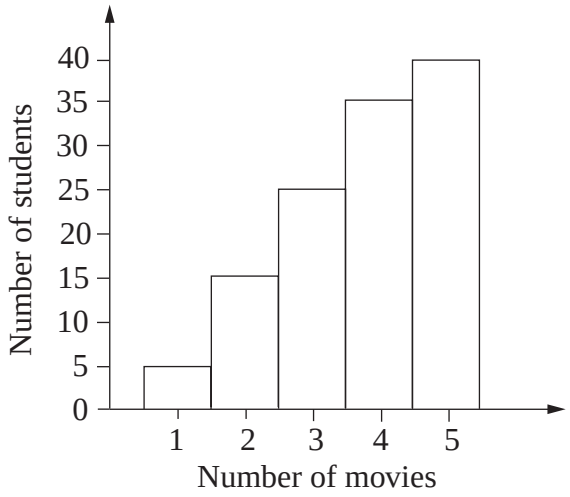
(A) $\frac{\sqrt[3]{14}}{2}$

(B) $\sqrt[3]{6}$

(C) $\sqrt[3]{7}$

(D) $\sqrt[3]{14}$

17 Students were surveyed about the number of movies they had watched in the last week. The results are shown in this *cumulative* frequency histogram.



How many students said they watched four movies last week?

(A) 5

(B) 10

(C) 25

(D) 35

18 Amy buys a $1 ticket in a raffle. There are 200 tickets in the raffle and two prizes. First prize is $100 and second prize is $50.

Find Amy's financial expectation.

(A) –$1.00

(B) –$0.75

(C) –$0.25

(D) +$0.25

19 In one year, the population of a city increased by 20%. The next year, it decreased by 10%.

What was the percentage increase in the population over the two years?

(A) 8%

(B) 10%

(C) 15%

(D) 30%

20 Rob, Alex and Tan plan a swimming race against each other. Rob and Alex are each twice as likely as Tan to win the race.

What is the probability that Tan will win the race?

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

21 The sheets of paper Jenny uses in her photocopier are 21 cm by 30 cm. The paper is 80 gsm, which means that one square metre of this paper has a mass of 80 grams. Jenny has a pile of this paper weighing 25.2 kg.

How many sheets of paper are in the pile?

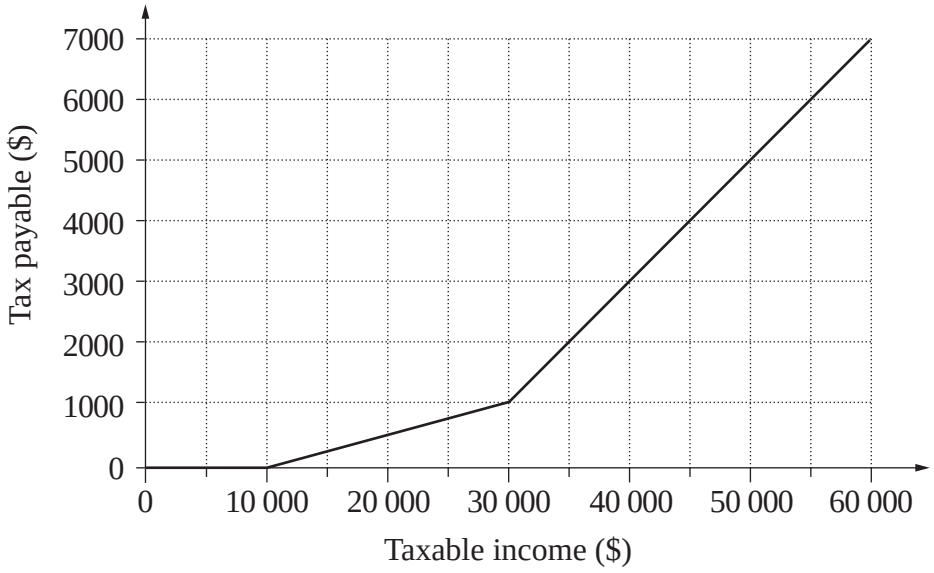
(A) 500

(B) 2000

(C) 2500

(D) 5000

22 The graph shows the tax payable for taxable incomes up to $60 000 in a proposed tax system.



How much of each dollar earned over $30 000 is payable in tax?

(A) 10 cents

(B) 12 cents

(C) 20 cents

(D) 23 cents

Section II

78 marks
Attempt Questions 23–28
Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

Question 23 (13 marks) Use a SEPARATE writing booklet.

(a) Jordan's gross pay is \$1500 per fortnight.

(i) Fortnightly deductions from Jordan's gross pay are: **1**

- \$269.17 for tax;
- \$7.88 for union fees;
- \$16.25 for private health insurance.

Calculate his fortnightly net pay.

(ii) Jordan is paid an annual leave loading of $17\frac{1}{2}\%$ of 4 weeks' gross pay. **1**

Calculate his annual leave loading.

(iii) Jordan visits Italy on his holidays. He pays €180 (180 euros) for a pair of boots. This price includes a value added tax of 20%.

(1) What was the price of the boots before the tax was added? **1**

(2) How much is €180 in Australian dollars if \$A1 is worth €0.58? **1**

(b) (i) Katherine invests \$50 000 with Standard Credit Union for a term of 5 years. Her investment earns interest at 3.1% per annum compounded annually. **2**

How much will Katherine's investment be worth at the end of 5 years? Give your answer to the nearest dollar.

(ii) Katherine's sister Liz also has \$50 000 to invest for a term of 5 years. She invests with General Bank. Her investment earns interest at 3% per annum, compounded monthly. **2**

Which sister makes the better investment? Justify your answer with appropriate calculations.

Question 23 continues on page 13

Marks

Question 23 (continued)

(c) Minh wants to buy a hi-fi system from Advanced Sound Systems for $5000.

(i) Minh considers buying the system on hire purchase. **3**

What would Minh repay monthly if he were to buy the hi-fi system on the following hire purchase terms?

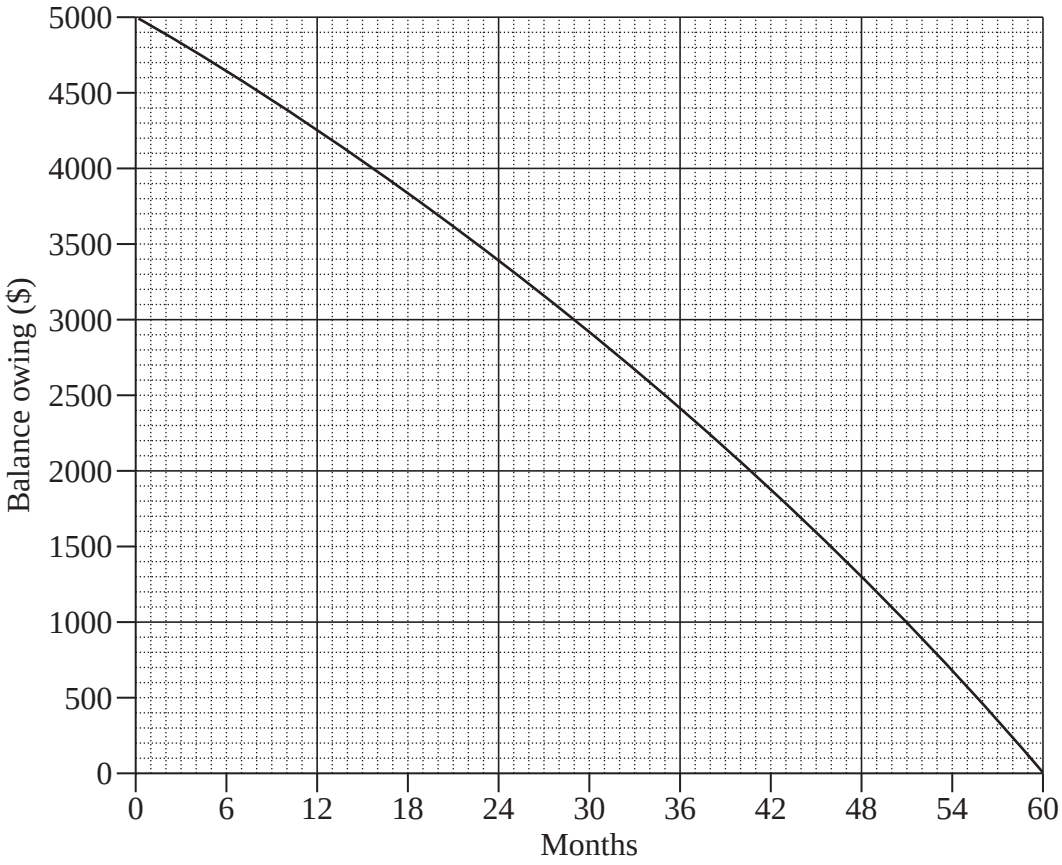
ADVANCED SOUND SYSTEMS
Hire Purchase Terms:
10% deposit
15% pa simple interest on the balance
Equal monthly repayments over 3 years

(ii) Minh decides instead to borrow the required $5000 from his local bank. The loan is a reducing balance loan over 5 years, with monthly repayments.

The graph shows the balance owing on the loan over time.

(1) Use the graph to determine the balance owing after 2 years. **1**

(2) Use the graph to determine when the loan is half-paid. **1**



End of Question 23

Marks

Question 24 (13 marks) Use a SEPARATE writing booklet.

Jane and Sam are in a Geography class of 12 students. The class is going on a three-day excursion by bus.

(a) The students are asked to each pack one bag for the trip. The bags are weighed, and the weights (in kg) are listed in order as follows:

8, 9, 10, 10, 15, 18, 22, 25, 29, 35, 38, 41

(i) A bag is selected at random. What is the probability that the chosen bag weighs more than 30 kg? **1**

(ii) While Sam waits for the bus to be ready, he works out the five number summary for the weights of the bags: **2**

8, 10, 20, 32, 41

Using this five number summary, construct an accurate box-and-whisker plot to display the distribution of the weights of the bags.

(iii) Calculate the interquartile range of the weights. **1**

(b) While waiting in the carpark, Jane notices that some of the cars entering the carpark have headlights on. For each car, Jane notes whether or not the lights are on, and whether the driver is male or female.

Her results are presented in the two-way table below. There are two missing numbers at ***A*** and ***B***.

	Headlights on	*Headlights off*	*Total*
Male drivers	10	***A***	53
Female drivers	8	62	70
Total	***B***	105	

(i) Determine the values of ***A*** and ***B***. **2**

(ii) How many cars are included in this data set? **1**

(iii) What fraction of the cars had female drivers? **1**

(iv) Of the cars driven by women, what fraction had headlights on? **1**

Question 24 continues on page 15

Question 24 (continued)

(c) There is one seat at the back of the excursion bus that is very popular among the students.

Before the excursion, a draw is conducted to determine who will sit in the popular seat. The names of the 12 students are placed in a hat and 3 names are drawn without replacement. The first name drawn determines who will sit in the seat on the first day. The second name drawn determines who will sit in the seat on the second day. The third name drawn determines who will sit in the seat on the third day.

(i) What is the probability that Jane's name is the first drawn? **1**

(ii) What is the probability that Jane's name is the second drawn? **1**

(iii) What is the probability that Jane's name will NOT be one of the three names drawn from the hat? **2**

End of Question 24

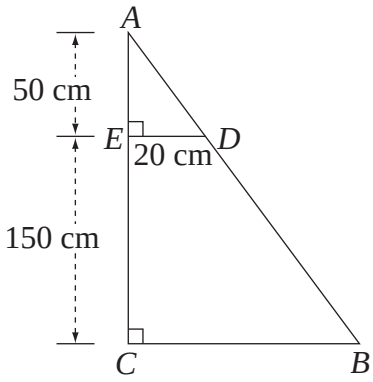
Please turn over

Marks

Question 25 (13 marks) Use a SEPARATE writing booklet.

(a) A shelf 20 cm wide is attached to a wall, under a light.

(i) The diagram shows the end view, *ED*, of the shelf attached to a wall *AC*.



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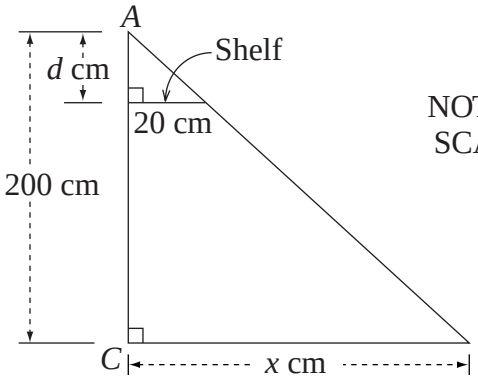
When the wall light at *A* is turned on, the shelf casts a shadow *CB* on the floor.

(1) Name a pair of similar figures in the diagram. **1**

(2) Calculate the enlargement factor between these two similar figures. **1**

(3) What is the length of the shadow *CB*? **1**

(ii) The shelf is moved to a new position *d* cm below the light. The length of the shadow is now *x* cm. **1**



NOT TO SCALE

Write down an equation relating *d* and *x*.

Question 25 continues on page 17

Question 25 (continued)

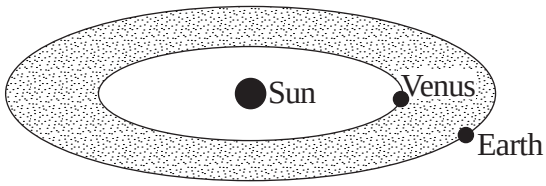
(b) The table shows the approximate coordinates for two cities.

City	*Latitude*	*Longitude*
Buenos Aires	35° S	60° W
Adelaide	35° S	140° E

(i) What is the time difference between Adelaide and Buenos Aires? (Ignore time zones.) **1**

(ii) Roy lives in Adelaide and his cousin Juan lives in Buenos Aires. Roy wants to telephone Juan at 7 pm on a Friday night, Buenos Aires time. At what time, and on what day, should Roy make the call? **2**

(c) (i) The orbits of Earth and Venus around the Sun are almost circular, and in the same plane. **2**



NOT TO SCALE

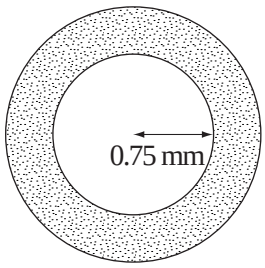
Earth is 1.496×10^8 km from the Sun.

Venus is 1.082×10^8 km from the Sun.

Treating the space between the orbits as an annulus, calculate its area. Write your answer in scientific notation correct to two significant figures.

(ii) Rearrange the formula for the area of an annulus, $A = \pi\left(R^2 - r^2\right)$, to make R the subject. **2**

(iii) A small metal washer is to be made in the shape of an annulus with inner radius 0.75 mm. **2**



NOT TO SCALE

The area of the face of the washer (shaded on the diagram) is to be 6.79 mm^2. Calculate the outer radius correct to two decimal places.

End of Question 25

Marks

Question 26 (13 marks) Use a SEPARATE writing booklet.

(a) After three small quizzes, Vicki has an average mark of 5. She wants to increase her average to 6. **2**

What mark must she score in the next quiz for her average mark to be exactly 6?

(b) Roxy selected 30 students at random from Year 12 at her high school, and asked each of them how many text messages they had sent from a mobile phone within the last day. The results are summarised in the following table.

Number of text messages sent	*Frequency*
0	3
1	3
2	4
3	4
4	9
5	7

(i) Calculate the mean number of text messages sent. (Give your answer correct to two decimal places.) **1**

(ii) Calculate the sample standard deviation. (Give your answer correct to two decimal places.) **1**

(iii) Determine the median number of text messages sent. **1**

(iv) Describe the skewness of the data. **1**

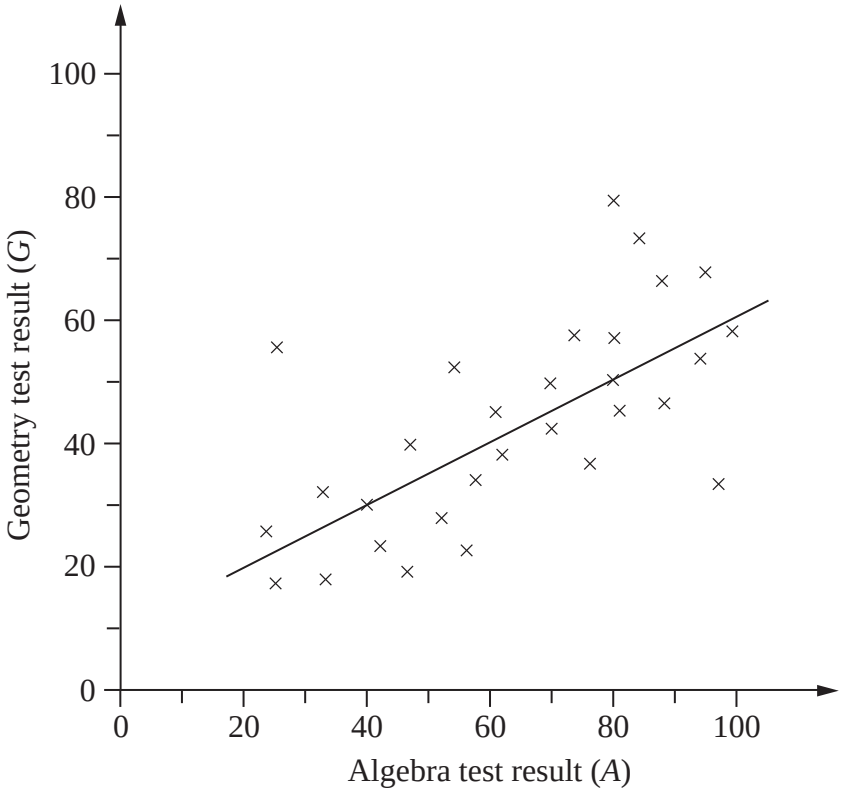
(v) There are 150 students in Year 12 at Roxy's school. Use the sample data in the table to estimate how many of these Year 12 students would have sent more than three text messages within the last day. **1**

Question 26 continues on page 19

Marks

Question 26 (continued)

(c) A class of 30 students sat for an algebra test and a geometry test. The results were displayed in a scatterplot, and a line of fit was drawn, as shown.



(i) How many students scored less than 30 on the algebra test? **1**

(ii) Calculate the gradient of the line of fit drawn. **1**

(iii) What is the equation of the line of fit drawn? **2**

(iv) Describe the correlation between geometry test results and algebra test results. **1**

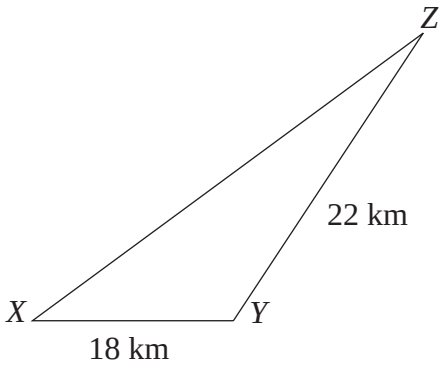
(v) Mitchell looked at the scatterplot and said: 'In this class, all students who are near the top in algebra are also near the top in geometry'. Explain why his statement is incorrect. **1**

End of Question 26

Marks

Question 27 (13 marks) Use a SEPARATE writing booklet.

(a) In the diagram X, Y and Z represent the locations of three towns. The town Y is due east of X, and the bearing of Z from Y is 046°.



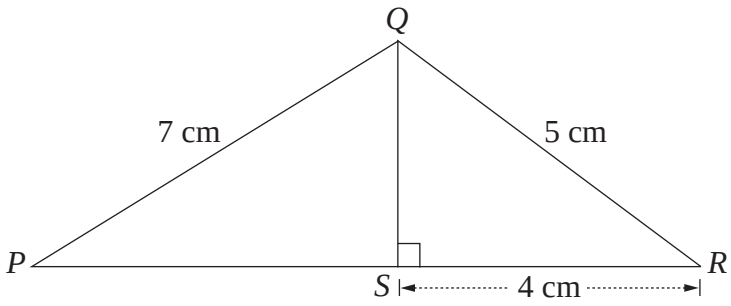
NOT TO SCALE

(i) Find the size of $\angle XYZ$. **1**

(ii) Find the distance XZ correct to one decimal place. **2**

(iii) What is the bearing of Y from Z? **1**

(b)



NOT TO SCALE

(i) Find the perimeter of ΔPQR. (Give your answer to one decimal place.) **3**

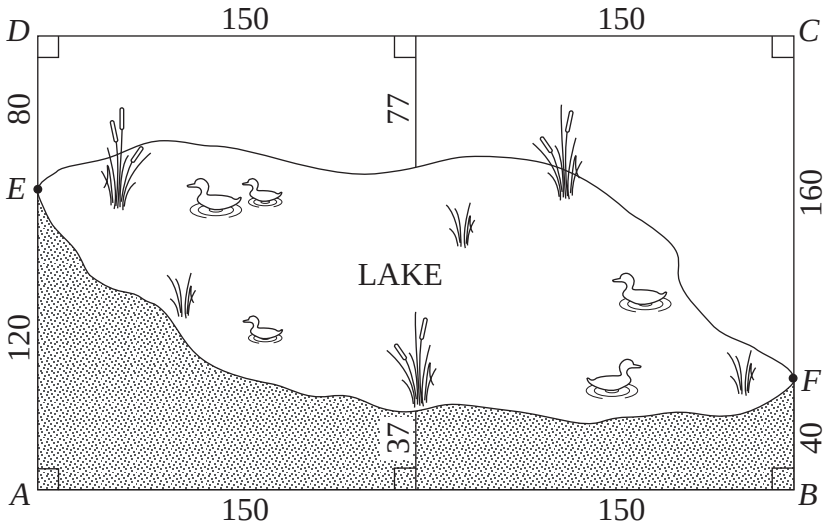
(ii) Find the size of $\angle QPS$ to the nearest degree. **1**

Question 27 continues on page 21

Marks

Question 27 (continued)

(c) In order to find the area of a lake, Bob took some measurements (in metres) and drew the following diagram.



NOT TO SCALE

(i) Use Simpson's Rule to find the shaded area *ABFE*. **2**

(ii) Calculate the area of the lake. **3**

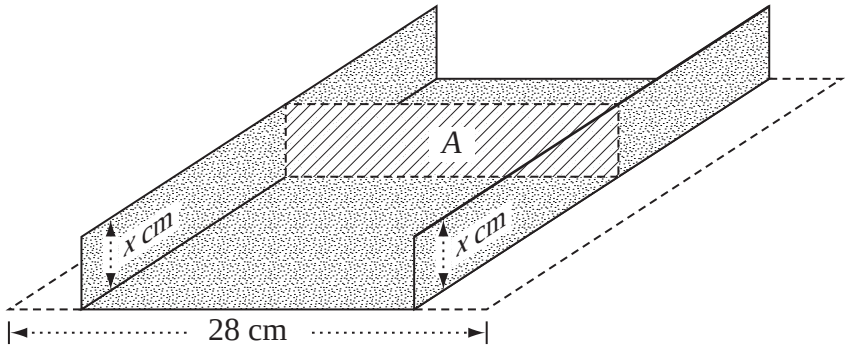
End of Question 27

Please turn over

Marks

Question 28 (13 marks) Use a SEPARATE writing booklet.

(a) A long rectangular sheet of metal 28 cm wide is to be made into a gutter by turning up sides of equal height x cm, perpendicular to the base.

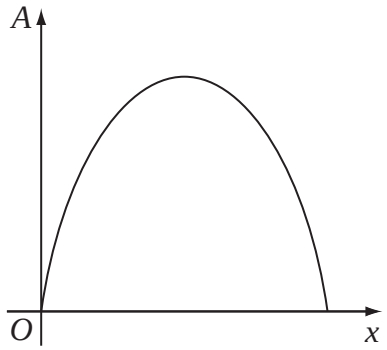


(i) Show that a formula for the cross-sectional area, A, of the gutter is **1**

$$A = 28x - 2x^2.$$

(ii) Explain why the formula in part (i) is only valid for values of x between 0 and 14. **1**

(iii) The graph of A against x, for values of x between 0 and 14, is a parabola, as shown. **2**



What is the maximum value of A?

Question 28 continues on page 23

Marks

Question 28 (continued)

(b) In 2001, when Toby was in Year 9, he started earning money by juggling at children's parties. He charged \$50 per party.

(i) Write a formula for the amount, Q (in dollars), that Toby had earned in 2001 after he had juggled at n parties. **1**

(ii) By the end of 2001 Toby had juggled at 30 parties. **3**

Draw the straight line graph of Q against n, with n on the horizontal axis and Q on the vertical axis. Use your ruler to draw the axes, and mark a scale on each axis.

(iii) Before Toby started juggling at parties he spent \$300 on juggling equipment. On your graph in part (b) (ii) draw a horizontal line through the point on the vertical axis where $Q = 300$. Give an interpretation of the point at which this horizontal line crosses the straight line graph of Q against n. **1**

(iv) Toby has a long-term plan. When he has finished Year 12, he wants to go to university for 3 years and then take a back-packing trip around the world. **4**

By the end of 2001 Toby had saved \$900 from his earnings, and sketched out the following plan for saving money:

- *In Year 9 (2001), I saved \$900 (interest included).*
- *For each year while still at school, save 30% more than I saved the previous year (interest included).*
- *At the end of Year 12 (2004), deposit total savings in an account paying 4% per annum interest, compounded annually.*
- *Add \$2500 to the account at the end of each of the three years while at university.*

Toby's goal is to have \$15 000 in his account at the end of 2007.

Will he achieve this goal if he follows the above plan? Show all your calculations to justify your answer.

End of paper

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FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$I = Prn$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$A = P(1+r)^n$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$S = V_0 - Dn$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$S = V_0(1-r)^n$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$y = mx + b$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$