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2016 NSW BOS Mathematics Extension 2 Solutions

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Section I

1	2	3	4	5	6	7	8	9	10
C	C	A	D	B	C	D	C	A	B

Q1 $f(-x)=f(x)$ C

Q2 C

$P(x)=x^3(x^2-1)-(x^2-1)=(x^2-1)(x^3-1)=(x-1)^2(x+1)(x^2+x+1)$

Q3 $e=0$ for a circle and $e<1$ for an ellipse
∴ the sum of the eccentricities <1. For other pairs, the sum > 1 A

Q4 Let $w=-a+bi$, $z=c+di$ where a, b, c and $d\in R^+$, $a>c$ and $b<d$.
$w-z=-(a+c)+(b-d)i$
Both real and imaginary parts of $w-z$ are negative. D

Q5 $\dfrac{1-i}{1+i}=\dfrac{(1-i)(1-i)}{(1+i)(1-i)}=\dfrac{-2i}{2}=-i$, clockwise rotation by $\dfrac{\pi}{2}$ B

Q6 $p(x+1)=(x+1)^{12}+(x+1)^{11}+(x+1)^{10}+(x+1)^9+(x+1)^8+\cdots$
Coefficient of $x^8={}^{12}C_8+{}^{11}C_8+{}^{10}C_8+{}^9C_8+1=715$ C

Q7 $xy=8$ has $y=x$ as an axis of symmetry. $xy=8$ intersects $y=x$ at $(2\sqrt{2},2\sqrt{2})$, ∴ distance of intersection from O is 4.
Rotate $xy=8$ clockwise by $\dfrac{\pi}{4}$ to obtain $x^2-y^2=k$ and the intersection is the x-intercept $(4,0)$. ∴ $k=4^2=16$. D

Q8 Horizontal component: $F\cos\theta-N\sin\theta=mr\omega^2$
Vertical component: $F\sin\theta+N\cos\theta=mg$ C

Q9 Let a and b be the side lengths of the rectangular slice.
$\dfrac{a-2}{8-2}=\dfrac{h}{4}$ and $\dfrac{b-3}{7-3}=\dfrac{h}{4}$, ∴ $a=\dfrac{3h}{2}+2$ and $b=h+3$
∴ $V=\displaystyle\int_0^4(h+3)\left(\frac{3h}{2}+2\right)dh$ A

Q10 $x+\dfrac{1}{x}=-1$, $x^2+x+1=0$, ∴ $x=cis\left(\dfrac{2\pi}{3}\right)$, $\dfrac{1}{x}=cis\left(-\dfrac{2\pi}{3}\right)$
or $x=cis\left(-\dfrac{2\pi}{3}\right)$, $\dfrac{1}{x}=cis\left(\dfrac{2\pi}{3}\right)$
∴ $x^{2016}=cis(0)=1$, $\dfrac{1}{x^{2016}}=cis(0)=1$, ∴ $x^{2016}+\dfrac{1}{x^{2016}}=2$ B

Section II

Q11ai $z=\sqrt{3}-i$, $|z|=2$, $\theta=\dfrac{-1}{\sqrt{3}}=-\dfrac{\pi}{6}$, ∴ $z=2\text{cis}\left(-\dfrac{\pi}{6}\right)$

Q11aii $z^6=2^6\text{cis}\left(-\dfrac{6\pi}{6}\right)=-2^6$, a real number

Q11aiii $n=3$, $z^3=2^3\text{cis}\left(-\dfrac{3\pi}{6}\right)=-8i$, a purely imaginary number

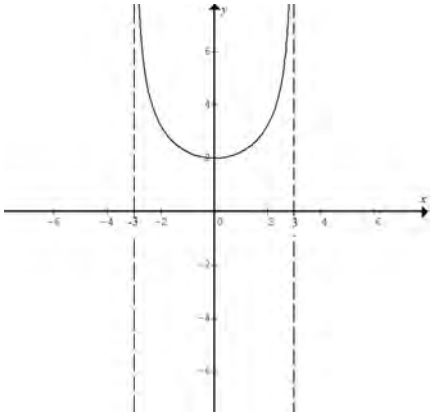
Q11b $\dfrac{d}{dx}\left(xe^{-2x}\right)=e^{-2x}-2xe^{-2x}$, ∴ $xe^{-2x}=\dfrac{1}{2}e^{-2x}-\dfrac{1}{2}\dfrac{d}{dx}\left(xe^{-2x}\right)$

$\displaystyle\int xe^{-2x}\,dx=\int\left(\frac{1}{2}e^{-2x}-\frac{1}{2}\frac{d}{dx}\left(xe^{-2x}\right)\right)dx=-\frac{1}{4}e^{-2x}-\frac{1}{2}xe^{-2x}+c$

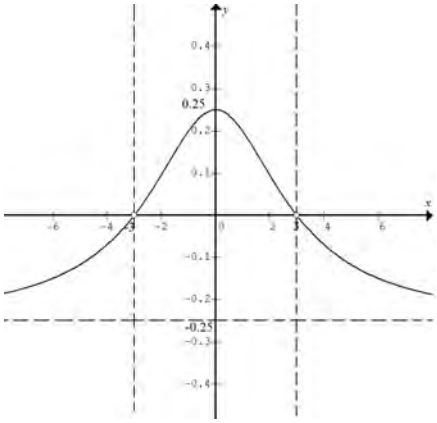
Q11c $x^3+y^3=2xy$, $3x^2+3y^2\dfrac{dy}{dx}=2x\dfrac{dy}{dx}+2y$

$3x^2-2y=2x\dfrac{dy}{dx}-3y^2\dfrac{dy}{dx}$, ∴ $\dfrac{dy}{dx}=\dfrac{3x^2-2y}{2x-3y^2}$

Q11di $y=\sqrt{f(x)}$



Q11dii $y=1/f(x)$



Q11e Domain: For $\sin^{-1}\left(\dfrac{x}{2}\right)$ to be defined, $-1\le\dfrac{x}{2}\le1$
∴ $-2\le x\le2$ is the domain
Range: For $-2\le x\le0$, $0\le y\le\pi$, for $0<x\le2$, $0<y\le\pi$
∴ the range is $0\le y\le\pi$.

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Q12ai $\frac{x^2}{9}+\frac{y^2}{4}=1$

Q12aii $4=9\left(1-e^2\right),\ e^2=\frac{5}{9},\ e=\frac{\sqrt{5}}{3}$

Q12aiii $\left(-\sqrt{5},0\right),\ \left(\sqrt{5},0\right)$

Q12aiv $x=-\frac{9\sqrt{5}}{5},\ x=\frac{9\sqrt{5}}{5}$

Q12bi $\frac{d}{dx}\left(xf(x)-\int xf'(x)dx\right)=xf'(x)+f(x)-xf'(x)=f(x)$

Q12bii Hence $\int f(x)dx=xf(x)-\int xf'(x)dx$

$\therefore\ \int \tan^{-1}x\,dx=x\tan^{-1}x-\int\frac{x}{1+x^2}dx=x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)+c$

Q12ci $z=\cos\theta+i\sin\theta$

$z^4=\cos 4\theta+i\sin 4\theta$

$z^4=(\cos\theta+i\sin\theta)^4=\cos^4\theta+4i\cos^3\theta\sin\theta-6\cos^2\theta\sin^2\theta$
$-4i\cos\theta\sin^3\theta+\sin^4\theta$

Equating the real parts: $\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$

Q12cii $\cos 4\theta=\cos^4\theta-6\cos^2\theta\left(1-\cos^2\theta\right)+\left(1-\cos^2\theta\right)^2$

$=8\cos^4\theta-8\cos^2\theta+1$

Q12di $xy=c^2,\ \frac{dy}{dx}=-\frac{c^2}{x^2}$

At $P\left(cp,\frac{c}{p}\right),\ \frac{dy}{dx}=-\frac{1}{p^2}$, .: gradient of the normal $=p^2$

Equation of the normal: $y-\frac{c}{p}=p^2(x-cp),\ \frac{y}{p}-\frac{c}{p^2}=p(x-cp)$

$\frac{y}{p}-\frac{c}{p^2}=px-cp^2,\ px-\frac{y}{p}=cp^2-\frac{c}{p^2},\ px-\frac{y}{p}=c\left(p^2-\frac{1}{p^2}\right)$

Q12dii Multiply the last equation by $x\neq 0$:

$px^2-\frac{xy}{p}=c\left(p^2-\frac{1}{p^2}\right)x,\ px^2-c\left(p^2-\frac{1}{p^2}\right)x-\frac{c^2}{p}=0$

Sum of roots $=cp+cq=-\frac{-c\left(p^2-\frac{1}{p^2}\right)}{p}$

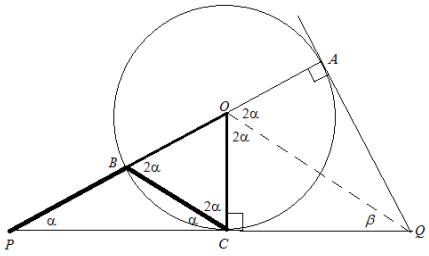
$\therefore\ q=\frac{\left(p^2-\frac{1}{p^2}\right)}{p}-p,\ q=-\frac{1}{p^3}$

Q13a $f(x)=x^x,\ \ln\left(f(x)\right)=x\ln x,\ \frac{d}{dx}\ln\left(f(x)\right)=\ln x+1$

To find the minimum of $\ln\left(f(x)\right)=x\ln x$, let $\ln x+1=0$, .: $x=\frac{1}{e}$

.: $f(x)=e^{x\ln x}$ has a minimum at $x=\frac{1}{e}$.

Q13b Let $\angle OPQ=\alpha$. The other relevant angles are shown in terms of α.



$\angle AOQ=\angle ABC$, .: $OQ\ //\ BC$, .: $\beta=\alpha$

.: ΔOPQ is isosceles, hence $OP=OQ$

Q13ci Vertical component: $T_1\times\frac{0.4}{0.5}-10M=0$, .: $T_1=12.5M$

Horizontal component: $T_1\times\frac{0.3}{0.5}+T_2=M\times 0.3\omega^2$

$T_2=M\times 0.3\omega^2-7.5M=0.3M\left(\omega^2-25\right)$

Q13cii $\frac{T_2}{T_1}>1,\ 0.024\left(\omega^2-25\right)>1,\ \omega^2-25>\frac{125}{3}$

$\omega^2>\frac{200}{3},\ \omega>\sqrt{\frac{600}{9}},\ \omega>\frac{10\sqrt{6}}{3}$ radians per second

Q13di $p(x)=ax^3+bx^2+cx+d,\ p'(x)=3ax^2+2bx+c$

For $p(x)$ to have one x-intercept only, $p'(x)=3ax^2+2bx+c\neq 0$ for all x

.: the discriminant of $p'(x)$ must be less than zero

.: $(2b)^2-4(3a)c<0$, .: $b^2-3ac<0$

Q13dii $p'(x)=3ax^2+2bx+c$. If $b^2-3ac=0$, then

$p'(x)=3ax^2+2bx+\frac{b^2}{3a}=3a\left(x^2+\frac{2b}{3a}+\frac{b^2}{9a^2}\right)=3a\left(x+\frac{b}{3a}\right)^2$

$p(x)=a\left(x+\frac{b}{3a}\right)^3+\text{constant}$

If $p\left(-\frac{b}{3a}\right)=0$, then the constant is zero and $p(x)=a\left(x+\frac{b}{3a}\right)^3$

.: the multiplicity of the root $x=-\frac{b}{3a}$ is 3.

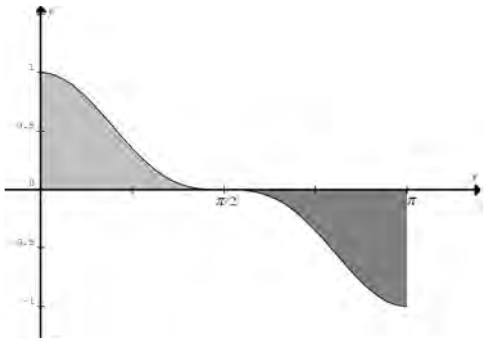
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Q14ai $\int \sin x \sin^2 x\,dx = \int \sin x\left(1-\cos^2 x\right)dx = \int -\left(1-u^2\right)du$

$= \dfrac{u^3}{3} - u + c = \dfrac{1}{3}\cos^3 x - \cos x + c$

Q14aii The graph of $y = \cos^m x$ is shown below for odd $m > 0$ in the interval $[0, \pi]$. The two shaded regions are equal in area.

$\therefore \displaystyle\int_0^{\pi} \cos^m x\,dx = \int_0^{\frac{\pi}{2}} \cos^m x\,dx + \int_{\frac{\pi}{2}}^{\pi} \cos^m x\,dx = 0$



Q14aiii $V = \displaystyle\int_0^{\pi} 2\pi x \sin^3 x\,dx = 2\pi \int_0^{\pi} x \sin^3 x\,dx$

Integration by parts:

$u = x$, $dv = \sin^3 x\,dx$, $du = dx$, $v = \dfrac{1}{3}\cos^3 x - \cos x$

$V = 2\pi\left(\left[\dfrac{1}{3}x\cos^3 x - x\cos x\right]_0^{\pi} - \displaystyle\int_0^{\pi}\left(\frac{1}{3}\cos^3 x - \cos x\right)dx\right)$

Since $\displaystyle\int_0^{\pi}\left(\frac{1}{3}\cos^3 x - \cos x\right)dx = 0$ from part aii,

$\therefore V = 2\pi\left(-\dfrac{\pi}{3} + \pi\right) = \dfrac{4\pi^2}{3}$ unit^3

Q14bi $I_n = \displaystyle\int_0^1 \frac{x^n}{\left(x^2+1\right)^2}dx$, let $x = \tan\theta$, $\dfrac{dx}{d\theta} = \sec^2\theta$, $\dfrac{d\theta}{dx} = \dfrac{1}{\sec^2\theta}$

When $x = 0$, $\theta = 0$; when $x = 1$, $\theta = \dfrac{\pi}{4}$

$I_0 = \displaystyle\int_0^1 \frac{1}{\left(\tan^2\theta+1\right)^2}dx = \int_0^1 \frac{1}{\left(\sec^2\theta\right)^2}dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sec^2\theta}d\theta = \int_0^{\frac{\pi}{4}} \cos^2\theta\,d\theta$

$= \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{4}}(1+\cos 2\theta)d\theta = \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}} = \frac{\pi}{8} + \frac{1}{4}$

Q14bii

$I_0 + I_2 = \displaystyle\int_0^1 \frac{1}{\left(x^2+1\right)^2}dx + \int_0^1 \frac{x^2}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{1+x^2}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{1}{x^2+1}dx$

$= \left[\tan^{-1} x\right]_0^1 = \dfrac{\pi}{4}$

Q14biii $I_4 = \displaystyle\int_0^1 \frac{x^4}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{\left(x^2+1\right)^2 - 2\left(x^2+1\right)+1}{\left(x^2+1\right)^2}dx$

$= \displaystyle\int_0^1 \left(1 - \frac{2}{x^2+1} + \frac{1}{\left(x^2+1\right)^2}\right)dx$

$= 1 - 2\left(I_0 + I_2\right) + I_0 = 1 - 2\times\dfrac{\pi}{4} + \dfrac{\pi}{8} + \dfrac{1}{4} = \dfrac{5}{4} - \dfrac{3\pi}{8}$

Q14c It is true that $(x+1)(x-1)^2 \geq 0$ for $x \geq 0$

$\therefore x^3 - x^2 - x + 1 \geq 0$, $x^3 + 1 \geq x^2 + x$

Complete the squares by adding $2x\sqrt{x}$ to both sides:

$x^3 + 2x\sqrt{x} + 1 \geq x^2 + 2x\sqrt{x} + x$, $\therefore \left(x\sqrt{x}+1\right)^2 \geq \left(x+\sqrt{x}\right)^2$

Since $x\sqrt{x} + 1 \geq 1$ for $x \geq 0$, $\therefore x\sqrt{x} + 1 \geq x + \sqrt{x}$ for $x \geq 0$

Q15a Let $x = \alpha^2$, $x = \beta^2$ and $x = \gamma^2$ be the roots of the required cubic equation, $\therefore \alpha = \pm\sqrt{x}$

Since α is a root of $x^3 - 3x + 1 = 0$, $\therefore \left(\pm\sqrt{x}\right)^3 - 3\left(\pm\sqrt{x}\right) + 1 = 0$

$\left(\sqrt{x}\right)^3 - 3\sqrt{x} = \mp 1$, $\sqrt{x}(x-3) = \mp 1$

Squaring both sides: $x\left(x^2 - 6x + 9\right) = 1$, $x^3 - 6x^2 + 9x - 1 = 0$, the required cubic equation.

Q15bi $a = \dfrac{1}{2}\dfrac{dv^2}{dx} = -\dfrac{\mu^2}{x^2}$, $\dfrac{v^2}{2\mu^2} = \displaystyle\int \frac{-1}{x^2}dx = \frac{1}{x} + c$

At $x = b$, $v = 0$, $\therefore c = -\dfrac{1}{b}$, $\therefore \dfrac{v^2}{2\mu^2} = \dfrac{1}{x} - \dfrac{1}{b}$, $v^2 = 2\mu^2\left(\dfrac{1}{x} - \dfrac{1}{b}\right)$

For $0 < x \leq b$, the particle moves towards O,

$\therefore v = \dfrac{dx}{dt} = -\mu\sqrt{2}\sqrt{\dfrac{b-x}{bx}}$

Q15bii $x = b\cos^2\theta$, $\dfrac{dx}{d\theta} = -2b\sin\theta\cos\theta$

At $t = 0$, $x = b$, $\therefore \theta = 0$; at time t, $x = d$, $\therefore \theta = \cos^{-1}\sqrt{\dfrac{d}{b}}$

$\therefore \dfrac{dx}{dt} = -\mu\sqrt{2}\sqrt{\dfrac{b - b\cos^2\theta}{b^2\cos^2\theta}} = -\dfrac{\mu\sqrt{2}}{\sqrt{b}}\dfrac{\sin\theta}{\cos\theta}$

$\dfrac{dt}{dx}\times\dfrac{dx}{d\theta} = \dfrac{dt}{d\theta} = \dfrac{\sqrt{b}\cos\theta}{\mu\sqrt{2}\sin\theta}\times 2b\sin\theta\cos\theta = \dfrac{b\sqrt{2b}}{\mu}\cos^2\theta$

$\therefore t = \dfrac{b\sqrt{2b}}{\mu}\displaystyle\int_0^{\cos^{-1}\sqrt{\frac{d}{b}}} \cos^2\theta\,d\theta$

Q15biii Given $t = \dfrac{1}{\mu}\sqrt{\dfrac{b}{2}}\left(\sqrt{bd - d^2} + b\cos^{-1}\sqrt{\dfrac{d}{b}}\right)$

As $d \to 0$, $t \to \dfrac{1}{\mu}\sqrt{\dfrac{b}{2}}b\cos^{-1} 0 = \dfrac{1}{\mu}\sqrt{\dfrac{b}{2}}b\times\dfrac{\pi}{2} = \dfrac{\pi}{\mu}\sqrt{\left(\dfrac{b}{2}\right)^3}$

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Q15ci Let $\dfrac{3!}{x(x+1)(x+2)(x+3)}=\dfrac{a_0}{x}+\dfrac{a_1}{x+1}+\dfrac{a_2}{x+2}+\dfrac{a_3}{x+3}$

$$\frac{3!}{x(x+1)(x+2)(x+3)}=\left(a_0(x+1)(x+2)(x+3)+a_1x(x+2)(x+3)\right.$$
$$\left.+a_2x(x+1)(x+3)+a_3x(x+1)(x+2)\right)/\,x(x+1)(x+2)(x+3)$$

Let $x\to 0,-1,-2,-3$ to obtain

$a_0=\dfrac{3!}{(x+1)(x+2)(x+3)}=\dfrac{3!}{(1)(2)(3)}=1$, $a_1=\dfrac{3!}{(-1)(1)(2)}=-3$,

$a_2=\dfrac{3!}{(-2)(-1)(1)}=3$ and $a_3=\dfrac{3!}{(-3)(-2)(-1)}=-1$ respectively.

$\therefore\ \dfrac{3!}{x(x+1)(x+2)(x+3)}=\dfrac{1}{x}-\dfrac{3}{x+1}+\dfrac{3}{x+2}-\dfrac{1}{x+3}$

Q15cii

$$\frac{n!}{x(x+1)(x+2)\cdots(x+n)}=\frac{a_0}{x}+\frac{a_1}{x+1}+\cdots+\frac{a_k}{x+k}+\cdots+\frac{a_n}{x+n}$$

Let $x\to -k$,

$a_k=\dfrac{n!}{x(x+1)(x+2)\cdots(x+n)}$ (without $(x+k)$ in the denominator)

$=\dfrac{n!}{-k(-k+1)(-k+2)\cdots(-k+n)}$ (negative when k is odd)

$=\dfrac{(-1)^k n!}{k(k-1)(k-2)\cdots(1)(1)(2)\cdots(n-k)}=\dfrac{(-1)^k n!}{k!(n-k)!}=(-1)^k\binom{n}{k}$

Q15ciii $\dfrac{n!}{x(x+1)(x+2)\cdots(x+n)}=\dfrac{a_0}{x}+\dfrac{a_1}{x+1}+\cdots+\dfrac{a_n}{x+n}$

Let $x=1$,

$$\frac{a_0}{1}+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1}=\frac{n!}{(n+1)!}=\frac{1}{n+1}$$

$$1\binom{n}{0}-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+\frac{(-1)^n}{n+1}\binom{n}{n}=\frac{1}{n+1}$$

$$1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+\frac{(-1)^n}{n+1}=\frac{1}{n+1}$$

$\therefore\ \lim\limits_{n\to\infty}\left(1-\dfrac{1}{2}\binom{n}{1}+\dfrac{1}{3}\binom{n}{2}-\dfrac{1}{4}\binom{n}{3}+\cdots+\dfrac{(-1)^n}{n+1}\right)=\lim\limits_{n\to\infty}\dfrac{1}{n+1}=0$

Q16ai $|z|=1$ and $|w|=1$

$1+z+w=0$, $(1+\cos\theta+\cos\alpha)+(\sin\theta+\sin\alpha)i=0$

$\therefore\ \sin\theta+\sin\alpha=0$ and $\cos\theta+\cos\alpha=-1$

$\therefore\ \sin\theta=-\sin\alpha=\sin(-\alpha)$ or $\sin(\pi-\alpha)$

$\therefore\ \theta=-\alpha$ Note: $(\pi-\alpha)$ does not satisfy $\cos\theta+\cos\alpha=-1$.

$\therefore\ \cos(-\alpha)+\cos\alpha=-1$, $\cos\alpha+\cos\alpha=-1$, $\cos\alpha=-\dfrac{1}{2}$

$\therefore\ \alpha=\pm\dfrac{2\pi}{3}$ and $\theta=\mp\dfrac{2\pi}{3}$

$\therefore$ 1, z and w are on the unit circle centred at O, and separated by $\dfrac{2\pi}{3}$.

$\therefore$ 1, z and w form the vertices of an equilateral triangle.

Q16aii Given $1+z+w=0$

Let $z_1=2iz$ and $z_2=2iw$,

$\therefore\ 2i+z_1+z_2=i2+i(2z)+i(2w)=2i(1+z+w)=0$

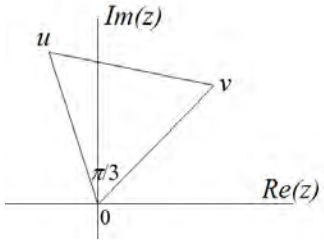
$\therefore\ 2i=2i\times 1$ is $2|1|$ and rotation of 1 by $\dfrac{\pi}{2}$ anticlockwise about O.

$z_1=2iz$ is $2|z|$ and rotation of z by $\dfrac{\pi}{2}$ anticlockwise about O.

$z_2=2iw$ is $2|w|$ and rotation of w by $\dfrac{\pi}{2}$ anticlockwise about O.

$\therefore\ 2i$, z_1 and z_2 form the vertices of an equilateral triangle.

Q16bi



$u=v\text{cis}\left(\dfrac{\pi}{3}\right)$, $u^3=v^3\text{cis}\,\pi=-v^3$

$\therefore\ u^3+v^3=(u+v)(u^2-uv+v^2)=0$

Since $u+v\neq 0$, $\therefore\ u^2-uv+v^2=0$, $u^2+v^2=uv$

Q16bii $v=1$, $u=\text{cis}\left(\dfrac{\pi}{3}\right)=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

Q16ci If Tom and any one of the remaining $(n-1)$ people have each other's hat, and the other $(n-2)$ people have other people's hats, the number of such derangements $=(n-1)D(n-2)$

Q16cii Tom or any person can pick another person's hat and the remaining $(n-1)$ people have other people's hats, the number of such derangements $=(n)D(n-1)$. However, this count includes the situation (as in part i) that the person swaps hats with one of the remaining $(n-1)$ people in $D(n-1)$ ways.

$\therefore$ number of such derangements

$=nD(n-1)-D(n-1)=(n-1)D(n-1)$

$\therefore\ D(n)=(n-1)D(n-1)+(n-1)D(n-2)$

$=(n-1)[D(n-1)+D(n-2)]$

Q16ciii $D(n)=(n-1)[D(n-1)+D(n-2)]$

$D(n)=nD(n-1)-D(n-1)+(n-1)D(n-2)$

$\therefore\ D(n)-nD(n-1)=-[D(n-1)-(n-1)D(n-2)]$

Q16civ

$D(2)-2D(1)=1$

$D(3)-3D(2)=-[D(2)-2D(1)]=-1$

$D(4)-4D(3)=-[D(3)-3D(2)]=1$

$D(5)-5D(4)=-[D(4)-4D(3)]=-1$ etc

$\therefore\ D(n)-nD(n-1)=(-1)^n$ for $n>1$

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Q16cv Given $D(1)=0$

$D(1)=1!\sum_{r=0}^{1}\frac{(-1)^r}{r!}=1-1=0$, true for $n=1$

Assume that $D(k)=k!\sum_{r=0}^{k}\frac{(-1)^r}{r!}$

From part iv, $D(n)=nD(n-1)+(-1)^n$

$\therefore\ D(k+1)=(k+1)D(k)+(-1)^{k+1}=(k+1)k!\sum_{r=0}^{k}\frac{(-1)^r}{r!}+(-1)^{k+1}$

$=(k+1)!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^k}{k!}\right)+(-1)^{k+1}$

$=(k+1)!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^k}{k!}+\frac{(-1)^{k+1}}{(k+1)!}\right)$

$\therefore\ D(k+1)=(k+1)!\sum_{r=0}^{k+1}\frac{(-1)^r}{r!}$

$\therefore\ D(n)=n!\sum_{r=0}^{n}\frac{(-1)^r}{r!}$ for all integers $n\geq 1$

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