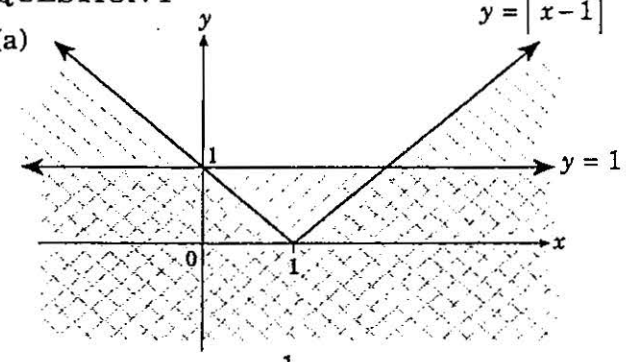
1995 HIGHER SCHOOL CERTIFICATE
SOLUTIONS
3 UNIT (ADDITIONAL) AND 3/4 UNIT (COMMON) MATHEMATICS

QUESTION 1

(a)



(b) $xy = 1 \quad \therefore \quad y = \dfrac{1}{x}$

$$\therefore \quad \int_1^4 y\,dx = \int_1^4 \frac{1}{x}\,dx$$
$$= \Big[\ln x\Big]_1^4$$
$$= \ln 4 - \ln 1$$
$$= \ln 4 \approx 1{\cdot}386.$$

(c) $$\lim_{x\to 0}\frac{\sin x}{5x} = \lim_{x\to 0}\frac{1}{5}.\frac{\sin x}{x}$$
$$= \frac{1}{5}\lim_{x\to 0}\frac{\sin x}{x}$$
$$= \frac{1}{5}\times 1$$
$$= \frac{1}{5}.$$

(d) $$2^{n+1} + 2^n = 2^n(2+1)$$
$$= 3\times 2^n \qquad \text{—①}$$

From ①: $$\frac{2^{1001}+2^{1000}}{3} = \frac{3\times 2^{1000}}{3}$$
$$= 2^{1000}.$$

(e) $$I = \int_0^1 6x\sqrt{9-x^2}\;dx$$

Put $u = 9 - x^2$

$\therefore \quad \dfrac{du}{dx} = -2x$

$\therefore \quad dx = -\dfrac{du}{2x}$ When $x = 0,\ u = 9$ and $x = 1,\ u = 8.$

$$\therefore \quad I = \int_9^8 6x.u^{\frac{1}{2}}.\left(-\frac{du}{2x}\right)$$
$$\therefore \quad = -3\int_9^8 u^{\frac{1}{2}}\,du$$
$$\therefore \quad = 3\int_8^9 u^{\frac{1}{2}}.du$$
$$= 3\left[\frac{2}{3}u^{\frac{3}{2}}\right]_8^9$$
$$= 3\left[\frac{2}{3}\left(9^{\frac{3}{2}} - 8^{\frac{3}{2}}\right)\right]$$
$$= 2\left(27 - 16\sqrt{2}\right).$$

QUESTION 2

(a) (i) $f(x) = x^3 + 5x^2 + 17x - 10$

$f(0) = -10$

$f(2) = 8 + 20 + 34 - 10$

$= 52.$

Since the function is continuous, and changes sign between 0 and 2, the root lies between 0 and 2.

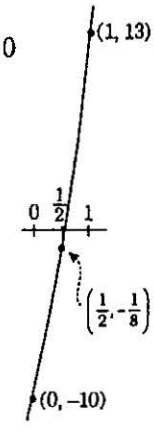
(ii) $f(1) = 1 + 5 + 17 - 10$

$= 13.$

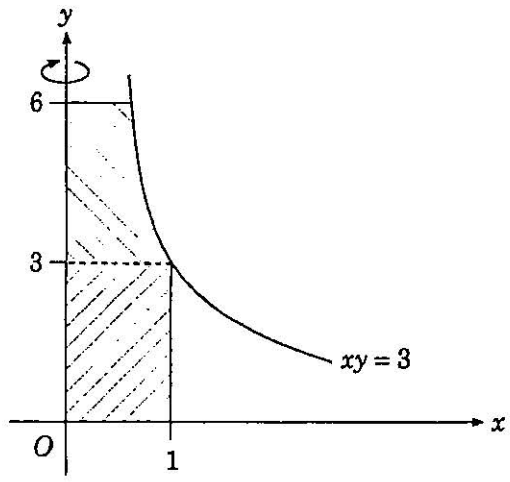
$\therefore$ The root lies between 0 and 1.

(iii) $$f\left(\frac{1}{2}\right) = \frac{1}{8} + \frac{5}{4} + \frac{17}{2} - 10$$
$$= -\frac{1}{8}.$$

Thus, the root is between $\frac{1}{2}$ and 1, so the end closer to the root is 1 (not 0).



(b)



The lower region is a circular cylinder, radius 1 unit, height 3 units.

$\therefore$ Volume $= \pi r^2 h$

$= \pi \times 1^2 \times 3$

$= 3\pi$ units^3.

Volume of the upper region is

$$\int_3^6 \pi x^2 \, dy = \pi \int_3^6 \left(\frac{3}{y}\right)^2 dy$$

$$= 9\pi \int_3^6 y^{-2} \, dy$$

$$= 9\pi \left[\frac{y^{-1}}{-1}\right]_3^6$$

$$= 9\pi \left[-\frac{1}{6} + \frac{1}{3}\right]$$

$$= \frac{9\pi}{6}$$

$$= \frac{3\pi}{2} \text{ units}^3.$$

$\therefore$ Volume $= 3\pi + \frac{3\pi}{2}$

$= \frac{9\pi}{2}$ units^3.

(c) $x^3 + 6x^2 - x - 30 = 0$

Let the roots be α, β, γ, where $\alpha = \beta + \gamma$, then

$$\alpha + \beta + \gamma = -\frac{b}{a} = -6$$

$\therefore$ $2\alpha = -6$, $\alpha = -3$,

and $\alpha\beta\gamma = -\frac{d}{a} = 30$

$\therefore$ $\beta\gamma = -10$,

and $\beta + \gamma = -3$.

By inspection, $\beta = -5$, $\gamma = 2$ (or $\beta = 2$, $\gamma = -5$).

$\therefore$ The three roots are $-5, -3, 2$.

QUESTION 3

(a) (i) The first code letter can be chosen eight ways. The second and third can also be chosen in each of eight ways.

$\therefore$ The number of different codes is

$$8 \times 8 \times 8 = 512$$

(ii) The first code letter can be chosen eight ways, the second only seven ways, and the third only 6 ways.

$\therefore$ The number of codes is

$$8 \times 7 \times 6 = 336$$

(iii) The number of ways of choosing 3 from 8 is:

$$^8C_3 = \frac{8 \times 7 \times 6}{1 \times 2 \times 3}$$

$$= 56.$$

(b) The general term is $^6C_n \, (x^2)^{6-n} \left(\frac{3}{x}\right)^n$.

If this is independent of x,

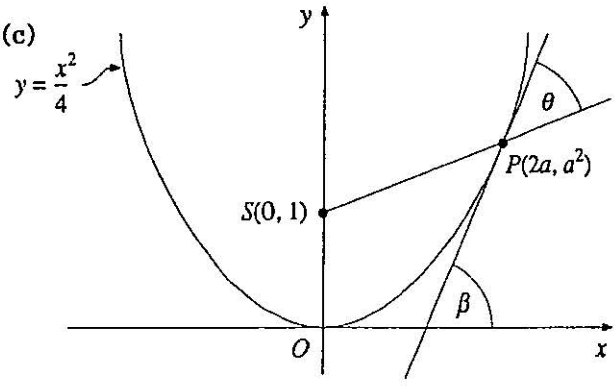
then $x^{2(6-n)} \times x^{-n} = x^0$

$\therefore$ $12 - 3n = 0$

$\therefore$ $n = 4$.

$\therefore$ The term is $^6C_4 . x^4 . \frac{3^4}{x^4} = 1215$.

(c)



(i) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{2x}{4}$

$= \frac{x}{2}$

$= \frac{2a}{2}$ at $P(2a, a^2)$

$= a$.

But the gradient of the tangent at P is given by $\tan\beta$, since the tangent cuts the x–axis with angle β.

$\therefore$ $\tan\beta = a$.

(ii) Gradient of $SP = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{a^2 - 1}{2a - 0}$$

$$= \frac{a^2 - 1}{2a}$$

$$= \frac{1}{2}\left(a - \frac{1}{a}\right).$$

(iii) θ is the angle between two lines with gradients $m_2 = \tan\beta = a$ for the tangent and $m_1 = \dfrac{1}{2}\left(a - \dfrac{1}{a}\right)$ for SP.

$$\therefore \quad \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{a - \frac{1}{2}\left(a - \frac{1}{a}\right)}{1 + \frac{1}{2}\left(a - \frac{1}{a}\right).a}$$

$$= \frac{2a - a + \frac{1}{a}}{2 + a^2 - 1}$$

$$= \frac{a + \frac{1}{a}}{1 + a^2}$$

$$= \frac{(a^2 + 1)}{a(1 + a^2)}$$

$$= \frac{1}{a}.$$

(iv) $\tan(\theta + \beta) = \dfrac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$

$$= \frac{\frac{1}{a} + a}{1 - \frac{1}{a} \times a}$$

$$= \frac{\frac{1}{a} + a}{0}.$$

That is, $\tan(\theta + \beta)$ is undefined, hence $\theta + \beta = 90°$.

(v) ***Method 1***:

If $\theta = \beta = 45°$,

then $\quad a = \tan\beta = \tan 45° = 1,$

$\therefore \quad P(2a, a^2) = P(2, 1).$

Method 2:

If $\theta = \beta$, then from the diagram, $SP \parallel x$-axis.

$\therefore \quad P(2a, a^2) = (x, 1)$

$\therefore \quad a = 1$

so $\quad x = 2$

$\therefore \quad P(2, 1).$

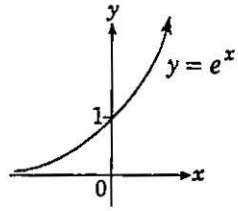
QUESTION 4

(a) $f(x) = \dfrac{e^x}{3 + e^x}$

$$f'(x) = \frac{(3 + e^x).e^x - e^x(e^x)}{(3 + e^x)^2}$$

$$= \frac{3e^x + e^{2x} - e^{2x}}{(3 + e^x)^2}$$

$$= \frac{3e^x}{(3 + e^x)^2}.$$



Since $\quad e^x \neq 0$

then $\quad f'(x) \neq 0$

$\therefore$ there are no stationary points.

(b) $f''(x) = \dfrac{3e^x(3 - e^x)}{(3 + e^x)^3}$

$= 0$ when $\quad 3 - e^x = 0$

that is, $\quad e^x = 3$

i.e., $\quad x = \ln 3.$

When $x = \ln 3$, $\quad y = \dfrac{e^x}{3 + e^x}$

$$= \frac{3}{3 + 3}$$

$$= \frac{1}{2}.$$

i.e., possible point of inflexion at $\left(\ln 3, \frac{1}{2}\right)$.

Now, to test sign change at $x = \ln 3$.

When $x < \ln 3$,

say $x = 1$, then $\quad 3 - e^1 > 0.$

When $x > \ln 3$,

say $x = 2$, then $\quad 3 - e^2 < 0.$

That is, $f''(x)$ will change sign.

Hence the inflexion is $\left(\ln 3, \frac{1}{2}\right)$.

OR

Since the question states there is a point of inflexion, it must be where $f''(x) = 0$.

$\therefore$ The inflexion is at $\left(\ln 3, \frac{1}{2}\right)$.

(c) Since $e^x > 0$, $\quad 3 + e^x > e^x$

$$\therefore \quad f(x) = \frac{e^x}{3 + e^x} < 1 \text{ for all } x.$$

But since $e^x > 0$ and $3 + e^x > 0$

then $\quad \dfrac{e^x}{3 + e^x} > 0$ for all x.

i.e., $0 < f(x) < 1$ for all x.

(d) $f(x)=\dfrac{e^x}{3+e^x}=\dfrac{1}{3e^{-x}+1}$

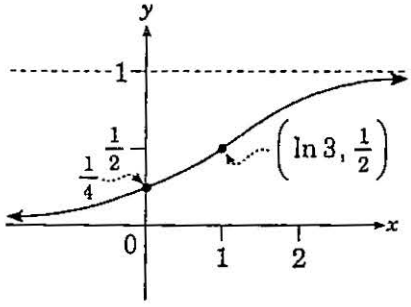
As $x\to\infty$, $e^{-x}\to 0$ $\quad\therefore\quad f(x)\to\dfrac{1}{0+1}$

$\therefore\ f(x)\to 1$ but is always less than 1.

As $x\to-\infty$, $e^x\to 0$ $\quad\therefore\quad f(x)\to\dfrac{0}{3+0}$

$\therefore\ f(x)\to 0$ but is always above zero.

(e)



Asymptotes at $y=0$ and $y=1$.

(f) $f(x)$ has an inverse function because it is an increasing function. Graphically, any horizontal line cuts it in one point only, so the inverse function (reflection in $y=x$) will satisfy the vertical line test.

(g) $f(x)=\dfrac{e^x}{3+e^x}$ i.e. $y=\dfrac{e^x}{3+e^x}$

$\therefore$ The inverse will be $x=\dfrac{e^y}{3+e^y}$

$\therefore\quad 3x+xe^y=e^y$

$e^y(1-x)=3x$

$e^y=\dfrac{3x}{1-x}$

$\therefore\quad y=\ln\left(\dfrac{3x}{1-x}\right)$

QUESTION 5

(a) (i) $\sin 2x=2\sin^2 x$

$\therefore\ 2\sin x\cos x-2\sin^2 x=0$

$2\sin x(\cos x-\sin x)=0$ —①

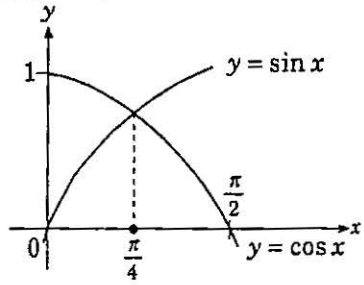
$\therefore\ \sin x=0$ or $\sin x=\cos x$

i.e. $\sin x=0$ or $\tan x=1$

$\therefore\ x=0,\pi,$ or $x=\dfrac{\pi}{4}$.

So $x=\dfrac{\pi}{4}$ is the only solution for $0<x<\pi$.

(ii) *Method 1*:



From ①, when $0<x<\dfrac{\pi}{4}$,

$\sin x>0$, and $\cos x>\sin x$

i.e., $2\sin x(\cos x-\sin x)>0$

$\therefore\ 2\sin x\cos x-2\sin^2 x>0$

$\sin 2x>2\sin^2 x$.

Method 2:

For $0\le x\le\dfrac{\pi}{4}$, $\sin 2x=2\sin^2 x$

only when $x=0,\dfrac{\pi}{4}$.

$\therefore\ \sin 2x>2\sin^2 x$ or

$\sin 2x<2\sin^2 x$ for $0<x<\dfrac{\pi}{4}$.

Let $x=\dfrac{\pi}{6}$, say:

$\sin\dfrac{\pi}{3}=0{\cdot}87$ and $2\sin^2\dfrac{\pi}{6}=0{\cdot}5$

i.e., $\sin 2\left(\dfrac{\pi}{6}\right)>2\sin^2\left(\dfrac{\pi}{6}\right)$

$\therefore\ \sin 2x>2\sin^2 x$ for $0<x<\dfrac{\pi}{4}$.

(iii) Since $\sin 2x>2\sin^2 x$

$\sin 2x-2\sin^2 x>0$ for $0<x<\dfrac{\pi}{4}$.

$\therefore$ Area between the curves

$=\displaystyle\int_0^{\frac{\pi}{4}}\sin 2x-2\sin^2 x\,dx$

$=\displaystyle\int_0^{\frac{\pi}{4}}\sin 2x\,dx+\int_0^{\frac{\pi}{4}}(\cos 2x-1)\,dx$

$\left(\begin{array}{l}\text{Since}\quad \cos 2x=1-2\sin^2 x\\ \text{i.e.,}\quad -2\sin^2 x=\cos 2x-1.\end{array}\right)$

$=\left[-\dfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}}+\left[\dfrac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}-\left[x\right]_0^{\frac{\pi}{4}}$

$=-\dfrac{1}{2}\cos\dfrac{\pi}{2}+\dfrac{1}{2}\cos 0+\dfrac{1}{2}\sin\dfrac{\pi}{2}$

$-\dfrac{1}{2}\sin 0-\dfrac{\pi}{4}+0$

$=0+\dfrac{1}{2}+\dfrac{1}{2}-0-\dfrac{\pi}{4}$

$=1-\dfrac{\pi}{4}$ units^2.

(b) We are considering ten trials, so the terms of $(q+p)^{10}$ will be considered, where $p = 0{\cdot}015$, $q = 0{\cdot}985$.

(i) For exactly one win:

$$\text{Probability} = {}^{10}C_1\, q^9\, p^1$$
$$= 10 \times 0{\cdot}985^9 \times 0{\cdot}015$$
$$= 0{\cdot}130\,923\cdots \approx 0{\cdot}1309.$$

(ii) For at least one win, we want

$$1 - P(\text{no wins})$$
$$= 1 - {}^{10}C_0\, q^{10}\, p^0$$
$$= 1 - 0{\cdot}8597$$
$$= 0{\cdot}140\,269\cdots \approx 0{\cdot}1403.$$

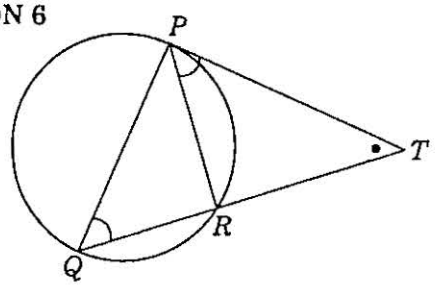
(iii) \$200 000 would need the initial \$8000 plus 24 no-wins, so for *more than* \$200 000 we need 25 no-wins.

Now, probability that it is not won is 0·985. So if not won for 25 times in sequence, it will have a probability of

$$(0{\cdot}985)^{25} = 0{\cdot}685\,339\cdots$$
$$\approx 0{\cdot}6853.$$

QUESTION 6

(a)



In ΔPRT, ΔQPT

$\angle PTR = \angle PTQ$ (same angle)

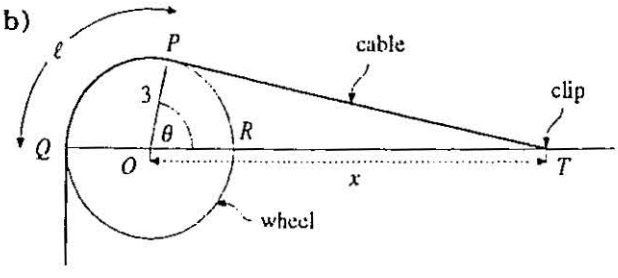
$\angle TPR = \angle PQR$ (angle between tangent and chord equals angle in the alternate segment)

$\therefore$ The triangles are equiangular, hence similar.

$\therefore \quad \dfrac{PT}{QT} = \dfrac{RT}{PT}$ (corresponding sides in similar triangles)

$\therefore \quad PT^2 = QT \times RT.$

b)



(i) Radius $OP \perp$ tangent PT

$\therefore\ \Delta OPT$ is right angled

$$\therefore \quad \cos\theta = \frac{OP}{OT}$$
$$= \frac{3}{x}.$$

(ii) $\text{arc}\ \ell = r(\angle POQ)$

$= 3(\pi - \theta)$ since QOR is a straight line

$$= 3\left[\pi - \cos^{-1}\left(\frac{3}{x}\right)\right].$$

(iii) $$\frac{d\ell}{dx} = 3 \times \frac{d}{dx}\left(-\cos^{-1}\frac{3}{x}\right)$$

Put $u = \dfrac{3}{x} \Rightarrow \dfrac{du}{dx} = -3x^{-2}$

$$\therefore \quad \frac{d}{dx}\left(-\cos^{-1}\frac{3}{x}\right)$$
$$= \frac{d}{du}\left(-\cos^{-1}u\right) \times \frac{du}{dx}$$
$$= \frac{+1}{\sqrt{1-u^2}} \cdot (-3x^{-2})$$
$$= \frac{-3}{x^2\sqrt{1-\frac{9}{x^2}}}$$
$$= \frac{-3x}{x^2\sqrt{x^2-9}}$$

$$\therefore \quad \frac{d\ell}{dx} = \frac{-9}{x\sqrt{x^2-9}}.$$

The significance of the negative value is that ℓ is decreasing as x increases. That is, the length of cable in contact with the wheel decreases as x increases.

(iv) $s = \ell + PT$

Now, from (a),

$$PT = \sqrt{QT \times RT}$$
$$= \sqrt{(x+3)(x-3)}$$
$$= \sqrt{x^2-9}$$

OR

Since $\angle OPT = 90°$ (from (i)),

$$PT^2 + 3^2 = x^2$$

$$\therefore \quad PT = \sqrt{x^2-9}$$

$$\therefore \quad s = 3\left(\pi - \cos^{-1}\frac{3}{x}\right) + \sqrt{x^2-9}.$$

(v) We want $\frac{ds}{dt}$ when $x=10$, given that $\frac{dx}{dt}=2$. Now, $\frac{ds}{dt}=\frac{ds}{dx}\cdot\frac{dx}{dt}$

$$\therefore \frac{ds}{dx}=\frac{-9}{x\sqrt{x^2-9}}+\frac{1}{2}(x^2-9)^{-\frac{1}{2}}\times 2x$$

$$=\frac{-9}{x\sqrt{x^2-9}}+\frac{x}{\sqrt{x^2-9}}$$

$$=\frac{-9+x^2}{x\sqrt{x^2-9}}$$

$$=\frac{\sqrt{x^2-9}}{x}.$$

$$\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$$

$$=\frac{\sqrt{x^2-9}}{x}\times 2$$

$\therefore$ When $x=10$,

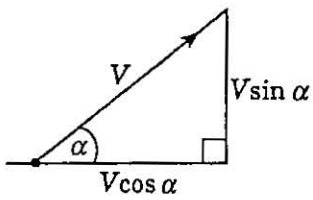
$$\frac{ds}{dt}=\frac{\sqrt{91}}{10}\times 2$$

$$=\frac{\sqrt{91}}{5}$$

$$\approx 1{\cdot}908 \text{ m/sec.}$$

QUESTION 7

(a) At $t=0$:



$$\ddot{x}=0$$

$\therefore \dot{x}=c_1$

and when $t=0$, $\dot{x}=V\cos\alpha$

$\therefore \dot{x}=V\cos\alpha$

$$x=Vt\cos\alpha+c_2$$

when $t=0$, $x=0$, $\therefore c_2=0$

$\therefore x=Vt\cos\alpha.$

$$\ddot{y}=-g$$

$\therefore \dot{y}=-gt+k_1$

and when $t=0$, $\dot{y}=V\sin\alpha$

$\therefore V\sin\alpha=k_1$

$\therefore \dot{y}=-gt+V\sin\alpha$

$$y=-\frac{1}{2}gt^2+Vt\sin\alpha+k_2$$

when $t=0$, $y=h$, $\therefore k_2=h$

$\therefore y=-\frac{1}{2}gt^2+Vt\sin\alpha+h.$

(b) Since $x=Vt\cos\alpha$

$$\therefore t=\frac{x}{V\cos\alpha}$$

$$\therefore y=-\frac{1}{2}g\cdot\frac{x^2}{V^2\cos^2\alpha}+V\sin\alpha\cdot\frac{x}{V\cos\alpha}+h$$

$$\therefore y=h+x\tan\alpha-x^2\cdot\frac{g}{2V^2\cos^2\alpha} \qquad \text{—①}$$

(c) Since the ball just clears the fence, we have when $x=R$, $y\geq h$

$$\therefore h+R\tan\alpha-R^2\frac{g}{2V^2\cos^2\alpha}\geq h$$

$$\therefore V^2\tan\alpha-\frac{Rg}{2\cos^2\alpha}\geq 0$$

$$\therefore V^2\geq\frac{Rg}{2\tan\alpha\cdot\cos^2\alpha}$$

i.e. $V^2\geq\frac{Rg}{2\sin\alpha\cos\alpha}$.

(d) When $x=r+R$, $y=0$. Substituting in ①:

$$\therefore 0=h+(R+r)\tan\alpha-(R+r)^2\frac{g}{2V^2\cos^2\alpha}$$

i.e. $\frac{(R+r)^2g}{2V^2\cos^2\alpha}=h+(R+r)\tan\alpha$

$$\therefore \frac{g}{2V^2\cos^2\alpha}=\frac{1}{(R+r)^2}\Big(h+(R+r)\tan\alpha\Big).$$

But $V^2\tan\alpha\geq\frac{Rg}{2\cos^2\alpha}$ (from (c))

$$\therefore \frac{g}{2V^2\cos^2\alpha}\leq\frac{\tan\alpha}{R}$$

$$\therefore \frac{1}{(R+r)^2}\Big(h+(R+r)\tan\alpha\Big)\leq\frac{\tan\alpha}{R}$$

$$\therefore R\Big(h+(R+r)\tan\alpha\Big)\leq(R+r)^2\tan\alpha$$

$$\therefore Rh+R(R+r)\tan\alpha\leq(R+r)^2\tan\alpha$$

$$\therefore Rh\leq\tan\alpha\Big((R+r)^2-R(R+r)\Big)$$

$$\therefore \tan\alpha\geq\frac{Rh}{(R+r)\Big(R+r-R\Big)}$$

i.e. $\tan\alpha\geq\frac{Rh}{r(R+r)}$.

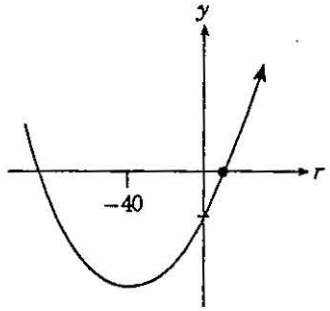
(e) When $V\leq 50$, $g=10$, $R=80$, and $h=1$.

Then, since $V^2\geq\frac{gR}{2\sin\alpha\cos\alpha}$

$$\therefore \sin 2\alpha\geq\frac{gR}{V^2},$$

and when $V\leq 50$, $\sin 2\alpha\geq\frac{10\times 80}{50^2}\geq\frac{8}{25}$.

Also, if it hits the ground outside the fence



$$\tan\alpha \geq \frac{Rh}{(R+r)r}$$

$$\therefore\ (R+r)r \geq \frac{Rh}{\tan\alpha}$$

$$\therefore\ r^2+80r \geq \frac{80}{\tan\alpha}$$

i.e. $r^2+80r-\dfrac{80}{\tan\alpha} \geq 0$

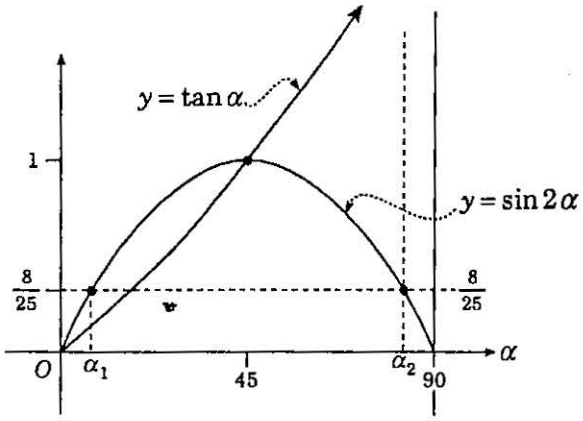
$$\therefore\ r = \frac{-80 \pm \sqrt{6400+\frac{320}{\tan\alpha}}}{2}$$

at extreme values.

For least r, we want $\tan\alpha$ to be a max.

Method 1:

We also know $\sin 2\alpha \geq \dfrac{8}{25}$.



Now, $\sin 2\alpha \geq \dfrac{8}{25}$ for $\alpha_1 \leq \alpha \leq \alpha_2$.

From the diagram, $\tan\alpha$ is a max for $\alpha = \alpha_2$, that is, for

$$\sin 2\alpha = \frac{8}{25}$$

where 2α is 'obtuse',

$$\therefore\quad 2\alpha = 160{\cdot}3375°$$

$$\alpha = 80{\cdot}6685°.$$

Hence, $r = \dfrac{-80 \pm \sqrt{6400+\frac{320}{\tan 80{\cdot}67°}}}{2}$

$$= \frac{-80 \pm 80{\cdot}3279}{2}$$

$$= \frac{0{\cdot}3279}{2}$$

$$= 0{\cdot}163\,98$$

Distance $\approx 0{\cdot}16$ m.

Method 2:

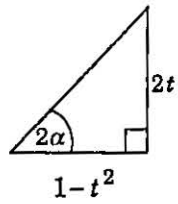
Put $t = \tan\alpha$, $\quad \tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$\therefore\ \sin 2\alpha = \dfrac{2t}{1+t^2}$ $\quad = \dfrac{2t}{1-t^2}$

$\therefore\ \dfrac{2t}{1+t^2} \geq \dfrac{8}{25}$

$\therefore\quad 50t \geq 8+8t^2$

$\therefore\ 8t^2-50t+8 \leq 0$



Now $8t^2-50t+8 = 0$

when $t = \dfrac{50 \pm \sqrt{2500-4\times 64}}{16}$

$$= \frac{50 \pm 47{\cdot}37\cdots}{16}$$

$$= 6{\cdot}0856\cdots \text{ or } 0{\cdot}16\cdots$$

$\therefore\ 8t^2-50t+8 \leq 0$ for $0{\cdot}16\cdots \leq t \leq 6{\cdot}08\cdots$

$\therefore$ max $\tan\alpha = 6{\cdot}0856\cdots$

$\therefore$ As before,

$$r = \frac{-80 \pm \sqrt{6400+\frac{320}{6{\cdot}0856\cdots}}}{2}$$

$$= 0{\cdot}163\,98 \text{ m}$$

Distance $\approx 0{\cdot}16$ m.