Instructions: Time allowed 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

Mathematical tables will be supplied. Slide rules (or calculators) may be used.

QUESTION 1 (12 Marks)

(i) The perpendicular to the line $2x - y = 0$ which passes through $(-1, -1)$ meets the line at P. Find the coordinates of P.

(ii) Differentiate (a) $\dfrac{1}{(1+x)^2}$ (b) $x \sin x$

(iii) The first two terms of an arithmetic sequence are -48, -41. Find (a) the n-th term (b) the sum of the first n terms.

(iv) A number is selected at random from the integers 2 to 25. Find the probabilities that it is

(a) a perfect square (b) a prime

(c) either a prime or a perfect square.

{Express your answers as fractions}

QUESTION 2 (9 Marks)

(i) Write down primitives (indefinite integrals) of:

(a) $1 + \frac{1}{2}x$ (b) $x^{-3/2}$ (c) $\cos x + \sin x$

(ii) Evaluate (a) $\int_0^1 (1-x)^3 dx$ (b) $\int_1^e \frac{dx}{x}$

(iii) A particle moves on a line so that its distance s from the origin at time t is given by $s = t^3 - 9t^2 + 15t - 7$. Find when and where the particle first comes to rest.

QUESTION 3 (9 Marks)

(i) On a sketch indicate by shading the region in which $-1 < x + y < 1$

(ii) Find the equation of the curve $x^2 + y^2 - 6x - 2y - 6 = 0$ when the coordinate axes are translated to parallel axes through the point $(3, 1)$.

(iii) Find the equation of the parabola with vertex $(-1, -1)$ which passes through the origin and whose axis is parallel to the y- axis.

QUESTION 4 (10 Marks)

(i) Express $\cos 80^o + \cos 50^o$ in the form $2 \cos A \cos B$

(ii) Sketch (not on graph paper) $y = \cos 2x$ for $-\pi \leq x \leq \pi$

(iii) The three sides of a triangle ABC are AB = 3, AC = 5, BC = 7. Find the angle A.

QUESTION 5 (10 Marks)

(i) Find the stationary points of $y = x^4 - 18x^2 + 32$ and sketch the curve.

(ii) A particle moves along the x- axis with acceleration $\frac{d^2x}{dt^2} = 70 + 12t - 12t^2$. If the particle is initially at rest at the origin, i.e. $x = \frac{dx}{dt} = 0$ at $t = 0$, find:

(a) its maximum displacement in the positive direction;

(b) the time at which it again reaches the origin.

QUESTION 6 (10 Marks)

(i) For what values of x is $\sin^{-1}x$ defined?

(ii) Find the maximum value of $2x(1 - x)$

(iii) Find the range of the function f given by $f(x) = \sin^{-1}\{2x(1 - x)\}$ with domain $0 \leq x \leq 1$.

(iv) Differentiate $\log_e \cos x$.

QUESTION 7 (10 Marks)

(i) A ditch is 12 metres wide and its banks are vertical. Measurements of the depth (in metres) are taken at intervals of 3 metres across a section with the following results:

Distance from left bank	0	3	6	9	12
Depth	0.85	1.30	1.65	1.10	0.70

Calculate the area of this section of the ditch using Simpson's rule with five function values.

(ii) Use induction to prove that, for any positive integer n, $5^n - 1$ is divisible by 4.

QUESTION 8 (10 Marks)

A (0, 3, -9) B (-3, -3, -3), C (5, 1, 5) are three points referred to a Cartesian system of axes. Find:

(i) the length of BC;

(ii) the direction cosines of BC;

(iii) the angle ABC;

(iv) the equation of the plane through the origin perpendicular to BC;

(v) the equation of the sphere with BC as diameter.

QUESTION 9 (10 Marks)

(i) Four dice, each with faces labelled 1 to 6, are thrown in succession. Find the probability that the total score is 7. (Express your answer as a fraction.)

(ii) A machine produces bolts to meet certain specifications and 90% of the bolts produced meet these specifications. For a sample of 10 bolts find the probability that exactly 3 fail to meet the specifications. (Express your answer as a fraction.)

QUESTION 10 (10 Marks)

(i) Sketch (not on graph paper) the curve $y = e^x \cos x$ for $0 \leq x \leq \pi/2$ and find its maximum ordinate in this interval.

(ii) Tangents from the point P (x_0, y_0) touch the parabola $x^2 = 4y$ at Q and R. Prove that the midpoint T of QR is $(x_0, \frac{1}{2}x_0^2 - y_0)$. If P moves on the line $x - y = 1$, find the equation of the locus of T.
