2017 NSW ESA Mathematics Exam Solutions

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Section I

1	2	3	4	5	6	7	8	9	10
A	D	C	A	B	D	B	A	C	A

Q1 $y=-\frac{2}{3}x-\frac{4}{3}$ A

Q2 D

Q3 $\frac{d}{dx}e^{x^2}=\frac{d\left(e^{x^2}\right)}{d\left(x^2\right)}\times\frac{d\left(x^2\right)}{dx}=2xe^{x^2}$ C

Q4 A

Q5 $\ln a=\ln b-\ln c=\ln\frac{b}{c}$, $\therefore\ a=\frac{b}{c}$ B

Q6 D

Q7 $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}=\frac{1}{\cos\theta\sin\theta}=\sec\theta\operatorname{cosec}\theta$ B

Q8 A

Q9 Line: $f'(x)=-2x+4$, $y=f(x)=-x^2+4x+c$

When $x=2$, $y=12$, $\therefore\ c=8$ C

Q10 The velocity changes from negative to positive once only when the graph crosses the horizontal axis. A

Section II

Q11a $\frac{2\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}=\frac{2\left(\sqrt{5}+1\right)}{4}=\frac{\sqrt{5}+1}{2}$

Q11b $\int(2x+1)^4\,dx=\frac{(2x+1)^5}{5\times2}+c=\frac{(2x+1)^5}{10}+c$

Q11c $\frac{d}{dx}\left(\frac{\sin x}{x}\right)=\frac{x\cos x-\sin x}{x^2}$

Q11d $\frac{d}{dx}\left(x^3\ln x\right)=3x^2\ln x+x^3\times\frac{1}{x}=x^2(3\ln x+1)$

Q11ei Area of $\Delta OAB=\frac{1}{2}\times6^2\times\sin30°=9\text{ cm}^2$

Q11eii Area of shaded segment $=\frac{30}{360}\times\pi\times6^2-9=3\pi-9\text{ cm}^2$

Q11f $(x-2)^2=4a(y-1)$ passes through $(0,4)$, $\therefore\ a=\frac{1}{3}$

$\therefore\ (x-2)^2=4\left(\frac{1}{3}\right)(y-1)$

Q11g $|3x-1|=2$, $3x-1=\pm2$, $3x=-1$ or 3, $x=-\frac{1}{3}$ or 1

Q11h $3-x\ge0$, $x\le3$, $\therefore$ the domain is $(-\infty,3]$.

Q12a $y=x^2+4x-7$, $\frac{dy}{dx}=2x+4=6$ at $(1,-2)$

Tangent: $y+2=6(x-1)$, $y=6x-8$

Q12b $V=2\times\int_0^2\pi y^2\,dx=2\pi\int_0^2\left(16-4x^2\right)dx$

$=2\pi\left[16x-\frac{4x^3}{3}\right]_0^2=\frac{128\pi}{3}$ cubic units

Q12c $a+4d=200$ and $2(2a+3d)=1200$, $\therefore\ a=360$ and $d=-40$

$\therefore$ the tenth term $=360+9(-40)=0$

Q12di $d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|(1)(-4)+(-1)(0)+(-2)|}{\sqrt{1^2+(-1)^2}}=3\sqrt{2}$ units

Q12dii The length of $CD=\sqrt{3^2+3^2}=3\sqrt{2}$ units

Area of the trapezium $=\frac{1}{2}\left(5\sqrt{2}+3\sqrt{2}\right)\times3\sqrt{2}=24$ square units

Q12ei $\Pr(\text{even})=\frac{2}{5}$

Q12eii $\Pr(\text{at least one even})=1-\Pr(\text{none})=1-\left(\frac{3}{5}\right)^3=\frac{98}{125}$

Q12eiii $\Pr(\text{even odd odd})=\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2=\frac{18}{125}$

Q12eiv $\Pr(\text{exactly one even})=3\times\frac{18}{125}=\frac{54}{125}$

Q13a $(x-4)^2+(x+4)^2-2(x-4)(x+4)\cos60°=13^2$ where $x-4>0$, i.e. $x>4$

$2x^2+32-\left(x^2-16\right)=169$, $x^2=121$, $\therefore\ x=11$

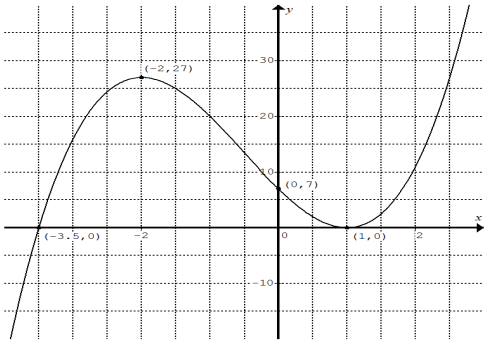
Q13bi $y=2x^3+3x^2-12x+7$, $\frac{dy}{dx}=6x^2+6x-12=6(x+2)(x-1)$

To find the stationary points, let $\frac{dy}{dx}=0$, $\therefore\ x=-2$ and $y=27$ or $x=1$ and $y=0$.

x	<-2	-2	$-2<x<1$	1	>1
dy/dx	positive	0	negative	0	positive
nature		max. t. p.		min. t. p.	

Stationary point $(-2,27)$ is a maximum turning point, and stationary point $(1,0)$ is a minimum turning point.

Q13bii $y = 2x^3 + 3x^2 - 12x + 7 = (x-1)^2(2x+7)$,
x-intercepts: at $x = -3.5$, $x = 1$, y-intercept: at $y = 7$



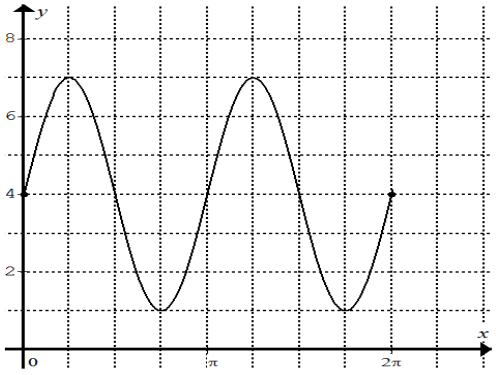
Q13biii $x < -2$ or $x > 1$

Q13c $t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$, $\left(t^{\frac{1}{3}} + 3\right)\left(t^{\frac{1}{3}} - 2\right) = 0$

$\therefore\ t^{\frac{1}{3}} + 3 = 0$ or $t^{\frac{1}{3}} - 2 = 0$, $\therefore\ t = -27$ or $t = 8$

Q13d $V = \int_0^{10} \frac{2t}{1+t^2}\,dt = \left[\ln\left(1+t^2\right)\right]_0^{10} = \ln(101)$

Q14a



Q14bi $\int_0^{\frac{\pi}{3}} \cos x\,dx = \left[\sin x\right]_0^{\pi/3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Q14bii $\int_0^{\frac{\pi}{3}} \cos x\,dx \approx \frac{\pi/3}{6}\left[\cos 0 + 4\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\right] = \left(\frac{3+4\sqrt{3}}{36}\right)\pi$

Q14biii $\left(\frac{3+4\sqrt{3}}{36}\right)\pi \approx \frac{\sqrt{3}}{2}$, $\therefore\ \pi \approx \frac{18\sqrt{3}}{3+4\sqrt{3}}$

Q14ci $C = Ae^{kt}$, $\frac{dC}{dt} = kAe^{kt}$, $kC = kAe^{kt}$, $\therefore\ C = Ae^{kt}$ satisfies $\frac{dC}{dt} = kC$.

Q14cii Let $e^{k(5730)} = 0.5$, $5730k = \ln 0.5$, $k = \frac{\ln 0.5}{5730} \approx -0.00012$

Q14ciii $e^{kt} = 0.9$, $kt = \ln 0.9$, $t = \frac{5730 \times \ln 0.9}{\ln 0.5} \approx 8.7 \times 10^2$ years

Q14d $2 \times \int_0^1 \left[k\left(1-x^2\right) - 2k\left(x^2-1\right)\right]dx = 8$

$k \times \int_0^1 \left(1-x^2\right)dx = \frac{4}{3}$, $\therefore\ k\left[x - \frac{x^3}{3}\right]_0^1 = \frac{4}{3}$, $\therefore\ k = 2$

Q15a $\angle AED = 90° - \frac{x°}{2}$
For AB // ED, $\angle BAE + \angle AED = 180°$, sum of co-interior angles.
$\therefore\ 3x + 90 - \frac{x}{2} = 180$, $x = 36$

Q15bi Amount (\$) at the start of the first month after the deposit = X
Amount at the end of the first month after the withdrawal:
$M_1 = X\left(1 + \frac{0.042}{12}\right) - 2500 = 1.0035X - 2500$

Amount at the start of the second month after the deposit = $M_1 + X$
Amount at the end of the second month after the withdrawal:
$M_2 = (M_1 + X)1.0035 - 2500 = (1.0035X - 2500 + X)1.0035 - 2500$
$= X\left(1.0035^2 + 1.0035\right) - 2500\left(1.0035 + 1\right)$

Q15bii $M_3 = X\left(1.0035^3 + 1.0035^2 + 1.0035\right) - 2500\left(1.0035^2 + 1.0035 + 1\right)$
$M_{48} = X\left(1.0035^{48} + 1.0035^{47} + ... + 1.0035\right) - 2500\left(1.0035^{47} + ... + 1\right) = 80\,000$
$X = \frac{80\,000 + 2500\left(1.0035^{47} + ... + 1\right)}{\left(1.0035^{48} + 1.0035^{47} + ... + 1.0035\right)} \approx 4019.42$

Q15ci $v_1 = \int_0^t \left(6t + e^{-t}\right)dt + 3 = \left[3t^2 - e^{-t}\right]_0^t + 3 = 3t^2 - e^{-t} + 4$

Q15cii $3t^2 - e^{-t} + 4 = 6t + 1 - e^{-t}$, $3t^2 - 6t + 3 = 0$, $3(t-1)^2 = 0$
$\therefore\ v_1 = v_2$ when $t = 1$

Q15ciii $x_1 = \int_0^t \left(3t^2 + 4 - e^{-t}\right)dt = \left[t^3 + 4t + e^{-t}\right]_0^t = t^3 + 4t + e^{-t} - 1$

$x_2 = \int_0^t \left(6t + 1 - e^{-t}\right)dt = \left[3t^2 + t + e^{-t}\right]_0^t = 3t^2 + t + e^{-t} - 1$, given $x = 0$ when $t = 0$ for both.
Let $t^3 + 4t + e^{-t} - 1 = 3t^2 + t + e^{-t} - 1$ for $t > 0$.
$\therefore\ t^3 - 3t^2 + 3t = 0$, $t\left(t^2 - 3t + 3\right) = 0$
The discriminant of $t^2 - 3t + 3 = 0$ is negative, $\therefore$ no solution to the equation. The two particles do not meet for $t > 0$.

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Q16ai

$L = \ell_{AE} + \ell_{EB} = \sqrt{x^2 + 5^2} + \sqrt{7^2 + (9-x)^2} = \sqrt{x^2 + 25} + \sqrt{49 + (9-x)^2}$

Q16aii

$\frac{dL}{dx} = \frac{x}{\sqrt{x^2 + 25}} - \frac{9-x}{\sqrt{49 + (9-x)^2}} = 0$, $\frac{x}{\sqrt{x^2 + 25}} = \frac{9-x}{\sqrt{49 + (9-x)^2}}$

$\therefore \sin\alpha = \sin\beta$

Q16aiii The two triangles are similar, $\therefore \frac{x}{5} = \frac{9-x}{7}$, $x = \frac{45}{12}$

Q16aiv If B is reflected in the line where the river runs, then AEB is a straight line, giving a minimum for L.

Q16b $\frac{a}{1-r} = 2$ for $-1 < r < 1$, $\therefore -1 < -r < 1$, $0 < 1 - r < 2$

$\therefore 0 < 2(1-r) < 4$, $\therefore 0 < a < 4$

Q16ci ΔBMD and ΔBCE are similar since $MD // CE$.

$\therefore \frac{BM}{BD} = \frac{BC}{BE}$, $\therefore \frac{BD}{BE} = \frac{BM}{BC} = \frac{1}{2}$ since M is the midpoint of BC

$\therefore D$ is the midpoint of BE

$\therefore BD = DE = DA + AE$, and given $BD = DA + AC$

$\therefore AC = AE$, $\therefore \Delta ACE$ is isosceles.

Q16cii $\angle BAF = \angle AEC$, corresponding angles are equal

$\angle FAC = \angle ACE$, alternate angles are equal

$\angle ACE = \angle AEC$, equal angles of an isosceles triangle

$\therefore \angle BAF = \angle FAC$, $\therefore AF$ bisects $\angle BAC$

Please inform mathline@itute.com re conceptual and/or mathematical errors.

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